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# Block Error Performance Analysis of Mobile Multimedia Communication System in Nakagami Fading Channel

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ABSTRACT

The block error probabilities of noncoherent frequency shift keying in a Nakagami fading channel are presented in this paper. The channel fading speed, slow or fast. is consider in evaluating block error probabilities. The effectiveness of diversity combing and error correction coding in improving block error performance is examined. The effect of cochannel interference on block error performance is also studied in this paper.

Key word : Block Error, Fast Nakagami Fading, Cochannel Interference, Diversity, Coding

## I. INTRODUCTION

Bit error probabilities are often examined for a particular channel environment and a particular modulation format. In the case of Nakagami fading, the bit error probabilities have been derived for CFSK (coherent frequency-shift keying) [1], [2] NFSK (noncoherent frequency-shift keying) [1], [2] and M-phase CPSK (coherent phase-shift keying) [3] signals. In data communication applications, the expressions of block error probabilities are important in evaluating system performance. For example, in a system with automatic repeat request(ARQ), a block error may result in a retransmission and system throughput efficiency is closely related to the block error probability. Note that, in fading channels, bit errors in a block are usually correlated and the expression of the block error probability can not be obtained directly from the bit error probability.

Previous works on block error probabilities have been conducted considering mobile radio channels with Rayleigh [4]-[6] and Rician [7] fading characteristics. This paper investigates the block error probabilities of NFSK signals in a Nakagami fading environment. The Nakagami distribution [8] is chosen to characterize the fading channel because it taken the Rayleigh distribution as a special case, approximates the Rician distribution well, models fading conditions which are more or less severe than that of Rayleigh, and more importantly, fits experimental data better than Rayleigh or Rician distribution [9], [10]

This paper also extends the work in [4]-[7] the following aspects. Firstly, the effect of channel fading speed, slow · or fast is considered,. Two extremes are examined. In a slow fading case, the signal-to-noise ratio is assumed to be constant over one block duration and under fast fading, two adjacent bits are considered to be faded independently [11]. The latter situation could result when the bits in a block are interleaved. Secondly a comparative study of diversity combing and error-correction coding is presented. The effectiveness of diversity and coding is examined considering fading depth and fading speed. Thirdly, cochannel interference considered on evaluating block error probabilities. This is of great importance in studying cellular system performance since frequency reuse in cellular system result in cochannel interference

Section Ⅱ derives the expression of the NFSK block error probabilities in a Nakagami fading channel. In Section Ⅲ, the NFSK performance is·

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examined considering fading depth/speed and diversity/coding techniques. The cochannel interference effect is also studied in this section. Conclusion are given in Section Ⅳ.

## Ⅱ. DERIVATION OF BLOCK ERROR PROBABILITY

### A. Nakagami Fading Model

A Nakagami distribution characterize channels with different fading depth through a parameter called amount of fading, AF (AF is defined as the ratio of the variance of the received energy to the square of the mean of the received energy [10]). The AF of a signal is assumed to be 1/m. The signal envelope, $\alpha$, is a random variable with a Nakagami probability density function (pdf) [8], i.e.,

$$P_a(a) = \frac{2}{\Gamma(m)}\left(\frac{m}{2R}\right)^m a^{2m-1}\exp\left(-\frac{m}{2R}a^2\right) \quad (1)$$

where Γ( · ) is the Gamma function, R is the mean signal power, and m≥1/2. The signal power, $\gamma$ is equal to $(1/2)\alpha^2$ and the pdf of $\gamma$ is found to be

$$P_r(r) = P_a(a)\frac{da}{d\gamma} = \frac{1}{\Gamma(m)}\left(\frac{m}{R}\right)^m r^{m-1} \cdot \exp\left(-\frac{m}{2R}r\right) \quad (2)$$

which is a Gamma pdf. Note that (1) and (2) degenerate into Rayleigh and exponential pdf, respectively, when m=1.

### B. NFSK Block Error Probabilities

In deriving block error probabilities, we need to calculate the probability of more than M bit error in a block of N bit, $p(M,N)$. The bit error probability of NFSK with a signal-to-noise $\gamma$ is

$$p_e(\gamma) = \frac{1}{2}\exp\left(-\frac{\gamma}{2}\right) \quad (3)$$

in a Gaussian noise channel and the corresponding probability $p(M,N)$ conditional on $\gamma$ is

$$\begin{aligned} p(r,M,N) &= \sum_{i=M+1}^{N}\binom{N}{i}p_e{}^i(r)[1-p_e(r)]^{N-1} \\ &= \sum_{i=M+1}^{N}\binom{N}{i}\binom{i-1}{M}\frac{(-1)^{i-M-1}}{2^i}\exp\left(-r\frac{i}{2}\right) \end{aligned} \quad (4)$$

In a Nakagami fading channel signal-to-noise ratio is a random variable with a pdf $p_r(r)$ in the form of (2) with a mean signal to noise ratio R. Assuming that the channel fading is very slow so that the signal-to-noise ratio remains constant over an entire block, we have for a Nakagami fading channel

$$\begin{aligned} p(M,N) = &\sum_{i=M+1}^{N}\binom{N}{i}\binom{i-1}{M}\frac{(-1)^{i-M-1}}{2^i} \\ &\cdot \int_0^\infty \exp\left(-\gamma\frac{i}{2}\right)p_r(r)dr \end{aligned} \quad (5)$$

The integration in (5) is derived as

$$\begin{aligned} \int_0^\infty \exp\left(-\gamma\frac{i}{2}\right)p_r(r)dr &= \frac{1}{\Gamma(m)}\left(\frac{m}{R}\right)^m\int_0^\infty r^{m-1} \\ &\quad\cdot \exp\left(-\left(\frac{i}{2}+\frac{m}{R}\right)r\right)dr \\ &= \left(1+\frac{Ri}{2m}\right)^{-m} \end{aligned} \quad (6)$$

which leads to

$$\begin{aligned} p(M,N) = &\sum_{i=M+1}^{N}\binom{N}{i}\binom{i-1}{M} \\ &\cdot \frac{(-1)^{i-M-1}}{2^i}\left(1+\frac{Ri}{2m}\right)^{-m} \end{aligned} \quad (7)$$

In the absence of error-correction coding, the NFSK block error probability under Nakagami fading therefore

$$p_e = p(0,N) = \sum_{i=M+1}^{N}\binom{N}{i}\frac{(-1)^{i-1}}{2^i}\left(1+\frac{Ri}{2m}\right)^{-m} \quad (8)$$

## Ⅲ. PERFORMANCE EVALUATION

### 3.1. Effect of Fading Amount

The most important parameter in the Nakagami fading model is m which is directly related to the fading amount. For m=1/2, the fading distribution is a one sided Gaussian distribution. Rayleigh fading corresponds to m=1. As m increases, AF decreases. When m tends to infinity, the channel becomes nonfading. The block error probability curves of

NFSK are shown in Fig 1 for various fading conditions (m=1/2, 1, 2, 10, 20, ∞). The block length is assumed to be N=31 bits.

### 3.2. Effect of Fast Fading

In deriving the block error probability expression, (8), a slow Nakagami fading channel is considered. In the case of fast fading where the signal strength varies continuously such that two adjacent bits in a block are faded independently, we have

$$p(M,N) = \sum_{i=M+1}^{N} \binom{N}{i} p_e^{\,i}(1-p_e)^{N-i} \qquad (9)$$

where $p_e$ is the bit error probability under Nakagami fading giving by

$$p_e = \int_0^{\infty} p_e(r)\, p_r(r)\, dr = \frac{1}{2}\left(1+\frac{R}{2m}\right)^{-m} \qquad (10)$$

The block error probability under Fast fading is

$$p_e = p(0,N) = \sum_{i=1}^{N} \binom{N}{i} p_e^{\,i}(1-p_e)^{N-i} \qquad (11)$$

or simply

$$p_e = 1-(1-p_e)^N \qquad (12)$$

Fig 2 shows the block error probability curves (N=31 bits) for various fading parameter m under fast fading. Compared to Fig 1. it is observed, that, for a given value of m, the block error probability under fast fading is always higher than that under slow fading, especially when m is small (large amount of fading). The difference between the two diminishes with m approaching ∞.

### 3.3. Comparison of Diversity and Coding

Both diversity combing and error-correction coding can be used to eliminate channel fading effects. The block error probability of NFSK with error correction coding (correction of up to M bit error within of N bits) can be evaluated using (7) and (9) for slow and fast fading respectively.

Assuming a maximal ratio combining approach with L identical branches, the diversity effect is examined as follows. The output signal-to-noise ratio after maximal ratio combining is equal to the sum of the signal-to-noise ratios of the various combining branches. It can be shown that the pdf of the resulted signal-to -noise ratio is

$$P_r(r) = \frac{1}{\Gamma(mL)} \left(\frac{m}{R}\right)^{mL} r^{mL-1} \exp\left(-\frac{m}{R} r\right) \qquad (13)$$

Following the derivations in Section Ⅱ, the block error probability under slow fading is found to be

$$P_c = \sum_{i=1}^{N} \binom{N}{i} \frac{(-1)^{i-1}}{2^i} \left(1+\frac{Ri}{2m}\right)^{-mL} \qquad (14)$$

For fast fading, (12) holds with

$$P_c = \frac{1}{2}\left(1+\frac{R}{2m}\right)^{-mL} \qquad (15)$$

Fig.3 and 4 show the effects of diversity combining under slow and fast fading conditions respectively. The effects of error-correction coding are shown in Fig.5 and 6. Diversity combining is effective under both slow and fast fading in improving block error performance. When m, is very large (for example, m=10), however, the diversity effect is minimal since the channel tends to nonfading. Comparing Fig.5 and Fig.6, it is observed that error-correction coding is much more effective in the fast fading case than that in the slow fading case. Note also that coding improves block error performance even when m is large which differs from diversity combining.

### 3.4. Effect of Cochannel Interference

Cochannel interference exists in cellular systems in which a frequency band is reused in different cells. In evaluating the performance of a system with cochannel interference, we need to examine both the desired signal and cochannel interferers. The statistics of the desired signal are character -ized by the pdf given in (2). A cochannel inter- ferer can also be characterized using (2) but with fading parameter m, and mean power Y. Assuming that there are I cochannel interferers with identical statistical characteristics, the pdf of the composite interference power, z is found to be

$$P_z(z) = \left(\frac{m_y}{Y}\right)^{m_y I} \frac{z^{m_y I-1}}{\Gamma(m_y I)} \exp\left(-\frac{m_y}{Y} z\right) \qquad (16)$$

The pdf of the signal-to-interference power ratio r, is found as [12]

$$P_r(r)=\frac{\Gamma(m+m_yI)}{\Gamma(m)\Gamma(m_yI)}\left(\frac{m}{X}\right)^{m}\left(\frac{m_y}{Y}\right)^{m_yIr^{m-1}} \cdot\left(\frac{mr}{X}+\frac{m_y}{Y}\right)^{-m-m_yI} \qquad (17)$$

In deriving the block error probability under slow fading, we average the block error probability in a Gaussian noise channel over all possible values of signal-to-noise ratio (see (5)). The method can also be used to derive the block error probability when the signal is subject to interfering in a fading environment [2]. Following this approach, the block error probability of NFSK under slow Nakagami fading and interfering can be found as

$$P_c=\frac{\Gamma(m+m_yI)}{\Gamma(m_yI)}\sum_{i=1}^{N}\binom{N}{i} \frac{(-1)^{i-1}}{2^i}\Psi\left(m,1-m_yI,\frac{m_yXi}{2mY}\right) \qquad (18)$$

where $\Psi(\alpha,\beta;s)$ is a degenerated hypergeometric function defined as[13]

$$\Psi(\alpha,\beta;s)=\frac{1}{\Gamma(\alpha)}\int_0^{\infty}\exp(-st)t^{\alpha-1} \cdot(1+t)^{\beta-\alpha-1}dt \qquad (19)$$

## Ⅳ. CONCLUSIONS

The block error probabilities of NFSK signals under slow and fast Nakagami fading are derived in this paper. The application of diversity and coding to improve block error performance is investigated and the cochannel interference effect is also studied. The effect of fading amount on block error performance is observed. It is shown that diversity improves block error performance under both slow and fast fading while coding performs well only under fast fading. However, diversity is effective only when the fading amount is large while coding is effective even when the fading amount is small.

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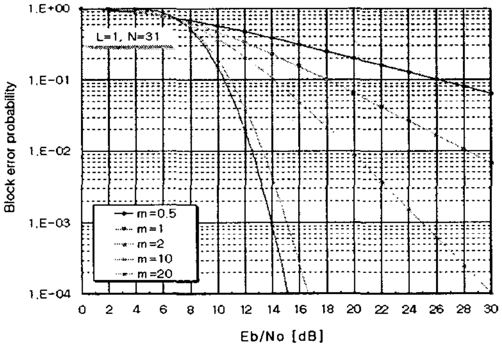

Fig. 1 Block error probability for NFSK over Nakagami slow fading channels

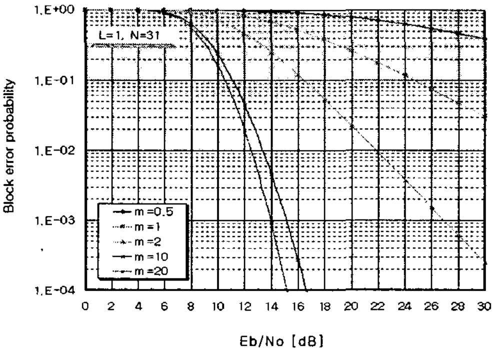

Fig. 2 Block error probability for NFSK over Nakagami fast fading channels

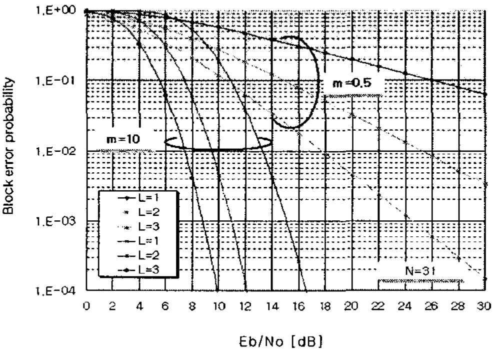

Fig. 3 Performance comparison of Diversity Nakagami slow fading channels

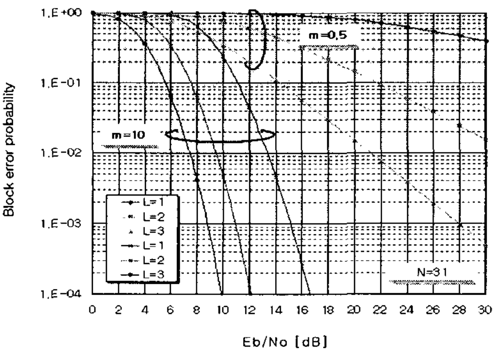

Fig. 4 Performance comparison of Diversity Nakagami fast fading channels

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