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보단조 가법 구간치 범함수와 구간치 쇼케이적분에 관한 연구(II)

On comonotonically additive interval-valued functionals and interval-valued Choquet integrals(II)

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요 약

이 논문에서는 Schmeidler[14]와 Narukawa[12]에 나오는 보단조 가법 실수치 범함수 개념의 일반화인 보단조 가법 구간치 범함수를 정의하고 그들의 성질을 연구한다. 또한 보단조 가법 구간치 범함수와 구간치 쇼케이적분이 적당한 함수공간 상에서 서로간의 관계를 조사한다. 수의 값을 갖는 함수들의 쇼케이적분을 생각하고자 한다. 이러한 구간 수의 값을 갖는 함수들의 성질들을 조사한다.

Abstract

In this paper, we will define comonotonically additive interval-valued functionals which are generalized comonotonically additive real-valued functionals in Schmeildler[14] and Narukawa[12], and prove some properties of them. And we also investigate some relations between comonotonically additive interval-valued functionals and interval-valued Choquet integrals on a suitable function space, cf.[9,10,11,13].

Key Words : fuzzy measures, interval-valued Choquet integrals, comonotonically additive functionals, Hausdorff metric.

1. Introduction.

It was well-known that closed set-valued functions had been used repeatedly in many papers [1,2,4,5,6,7,8, 9,13,16,17]. Using these properties, we have been studied some characterizations of closed set-valued Choquet integrals in [5,6] and convergence therems for interval-valued Choquet integrals in [7,8]. We will define comonotonically additive interval-valued functional which generalize the concept of a comonotonically additive functionals in [12] and study some properties of them. And we also investigate some relations between comonotonically additive interval-valued functionals and interval-valued Choquet integrals under sufficient conditions.

In section 2, we list various definitions and notations which are used in the proof of our results. In section 3, using these definitions and properties, we investigate functionals represented by interval-valued Choquet integrals and define comonotonically additive interval-valued functional on a suitable class of interval number-valued functions. We also investigate some relations between comonotonically additive interval-valued functionals and interval-valued Choquet integrals.

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2. Definitions and preliminaries.

A fuzzy measure on a measurable space (X,Ω) is an extended real-valued function $\mu:\Omega\rightarrow[0,\infty]$ satisfying
(i) $\mu(\emptyset)=0$,
(ii) $\mu(A)\leq\mu(B)$, whenever $A,B\in\Omega$, $A\subset B$.
A fuzzy measure μ is said to be autocontinuous from above[resp.,below] if $\mu(A\cup B_n)\rightarrow\mu(A)$[resp., $\mu(A\sim B_n)\rightarrow\mu(A)$] whenever $A\in\Omega$, $\{B_n\}\subset\Omega$ and $\mu(B_n)\rightarrow 0$. If μ is autocontinuous both from above and from below, it is said to be autocontinuous. Recall that a function $f:X\rightarrow[0,\infty]$ is said to be measurable if $\{x|f(x)>\alpha\}\in\Omega$ for all $\alpha\in(-\infty,\infty)$.

Definition 2.1([3,10,11,12,15]) (1) The Choquet integral of a measurable function f with respect to a fuzzy measure μ is defined by

$$(C)\int f d\mu = \int_0^\infty \mu(\{x|f(x) > r\})dr$$

where the integral on the right-hand side is an ordinary one.

(2) A measurable function f is called Choquet integrable if the Choquet integral of f can be defined and its value is finite.

Throughout the paper, R^+ will denote the interval $[0,\infty)$,

$$I(R^+) = \{[a,b] | a,b \in R^+ \text{ and } a \le b\}.$$

Then a element in $I(R^+)$ is called an interval number. On the interval number set, we define; for each pair $[a,b],[c,d] \in I(R^+)$ and $k \in R^+$,

$$[a,b]+[c,d]=[a+c,b+d],$$
$$[a,b]\cdot[c,d]=[a\cdot c,b\cdot d],$$
$$k[a,b]=[ka,kb],$$
$$[a,b]\le[c,d] \text{ if and only if } a\le b \text{ and } c\le d,$$

Then $(I(R^+),d_H)$ is a metric space, where d_H is the Hausdorff metric defined by

$$d_H(A,B)=\max\{\sup_{x\in A}\inf_{y\in B}|x-y|, \sup_{y\in B}\inf_{x\in A}|x-y|\}$$

for all $A,B \in I(R^+)$. We note that $[a,b]<[c,d]$ if and only if $[a,b]\le[c,d]$ or $[a,b]\ne[c,d]$. It is easily to show that for $[a,b]$, $[c,d]\in I(R^+)$,

$$d_H([a,b],[c,d])=\max\{|a-c|,|b-d|\}.$$

Let $C(R^+)$ be the class of closed subsets of R^+. Throughout this paper, we consider a closed set-valued function $F: X \to C(R^+)\setminus\{\emptyset\}$ and an interval number-valued function $F: X\to I(R^+)\setminus\{\emptyset\}$. We denote that $d_H-\lim_{n\to\infty} A_n = A$ if and only if $\lim_{n\to\infty} d_H(A_n,A)=0$, where $A\in I(R^+)$ and $\{A_n\}\subset I(R^+)$.

Definition 2.2([1,2]) A closed set-valued function F is said to be measurable if for each open set $O\subset R^+$,

$$F^{-1}(O)=\{x\in X | F(x)\cap O \ne \emptyset\}\in\Omega.$$

Definition 2.3([1,2]) Let F be a closed set-valued function. A measurable function $f: X\to R^+$ satisfying

$$f(x)\in F(x) \text{ for all } x\in X$$

is called a measurable selection of F.

We say $f: X\to R^+$ is in $L_c^1(\mu)$ if and only if f is measurable and $(C)\int f\,d\mu<\infty$. We note that "$x\in X$ $\mu-a.e.$" stands for "$x\in X$ μ-almost everywhere". The property $P(x)$ holds for $x\in X$ $\mu-a.e.$ means that there is a measurable set A such that $\mu(A)=0$ and the property $P(x)$ holds for all $x\in A^c$, where A^c is the complement of A.

Definition 2.4([9,10,11,12]) Let f,g be measurable nonnegative functions. We say that f and g are comonotonic, in symbol $f\sim g$ if and only if

$$f(x)<f(x') \Rightarrow g(x)\le g(x') \text{ for all } x,x'\in X.$$

Theorem 2.5([9,10,11,12]) Let f,g,h be measurable functions. Then we have

(1) $f\sim f$,
(2) $f\sim g \Rightarrow g\sim f$,
(3) $f\sim a$ for all $a\in R^+$,
(4) $f\sim g$ and $f\sim h \Rightarrow f\sim(g+h)$.

Theorem 2.6([9,10,11,12]) Let f,g be nonnegative measurable functions.

(1) If $f\le g$, then $(C)\int f d\mu \le (C)\int g d\mu$.
(2) If $f\sim g$ and $a,b\in R^+$, then
$$(C)\int(af+bg)d\mu = a(C)\int f d\mu + b(C)\int g d\mu.$$

Definition 2.7 ([5,6,7,8]) (1) Let F be a closed set-valued function and $A\in\mathfrak{I}$. The Choquet integral of F on A is defined by

$$(C)\int_A F d\mu = \{(C)\int_A f d\mu \mid f\in S_c(F)\}$$

where $S_c(F)$ is the family of $\mu-a.e.$ Choquet integrable selections of F, that is,

$$S_c(F)=\{f\in L_c^1(\mu) \mid f(x)\in F(x)\ x\in X\ \mu-a.e.\}.$$

(2) A closed set-valued function F is said to be Choquet integrable if $(C)\int F d\mu \ne \emptyset$.

(3) A closed set-valued function F is said to be Choquet integrably bounded if there is a function $g\in L_c^1(\mu)$ such that
$\|F(x)\| = \sup_{r\in F(x)}|r| \le g(x)$ for all $x\in X$.

Instead of $(C)\int_X F d\mu$, we will write $(C)\int F d\mu$. Let us discuss some properties of interval-valued Choquet integrals which mean Choquet integrals of measurable interval number-valued functions.

Assumption (A) For each pair $f,g\in S_c(F)$, there exists $h\in S_c(F)$ such that $f\sim h$ and $(C)\int g d\mu=(C)\int h d\mu$.

We consider the following classes of interval number-valued functions;

$$\mathfrak{I} = \{F \mid F: X \to I(R^+) \text{ is measurable and Choquet integraby bounded}\}$$

and

$$\mathfrak{I}_1 = \{F \in \mathfrak{I} \mid F \text{ satisfies the assuption(A)}\}.$$

Remark 2.8 Let m be the Lebesgue measure on R^+ and $\mu = m^2$. If

$$F(x) = \begin{cases} [0,1] & \text{for } x \in [0,1] \\ \{0\} & \text{else} \end{cases}$$

then clearly we have that $F \in \mathfrak{I}$. Now, we prove that F satisfies the assumption(A). Let $f, g \in S_c(F)$. We can put $c = (C)\int g d\mu$ and

$$h(x) = \begin{cases} c & \text{for } x \in [0,1] \\ 0 & \text{else} \end{cases}.$$

Then we have $f \sim h$ if we claim that for all x, $x' \in R^+$, $f(x) < f(x') \Rightarrow h(x) \le h(x')$. In fact, we can easily prove this if we take the five cases; (i) $x < x' < 0$, (ii) $x < 0 < x' < 1$, (iii) $0 < x < x' < 1$, (iv) $0 < x < 1 < x'$, (v) $1 < x < x'$. So, we also have

$$\begin{aligned} (C)\int h d\mu &= \int_0^\infty \mu(\{x \mid h(x) > a\}) da \\ &= \int_0^c \mu(\{x \mid h(x) > a\}) da \\ &= c = (C)\int g d\mu. \end{aligned}$$

Thus $F \in \mathfrak{I}_1$.

Theorem 2.9([8]) If $F \in \mathfrak{I}_1$, then we have

(1) $cF \in \mathfrak{I}_1$ for all $c \in R^+$,

(2) $(C)\int F d\mu = [(C)\int f_* d\mu, (C)\int f^* d\mu]$

where $f^*(x) = \sup\{r \mid r \in F(x)\}$ and $f_*(x) = \inf\{r \mid r \in F(x)\}$.

We recall that f^*, f_* are Choquet integrable selections of F in [8].

3. Comonotonically additive interval-valued functionals.

We assume that X is a locally compact Hausdorff space and the class Ω_1 of its Borel subsets. Let K^+ the set of continuous nonnegative functions defined on X with compact support.

Definition 3.1 ([12,13,14]) Let ℓ be a real-valued functional on K^+.

(1) ℓ is comonotonically additive if and only if

$$f \sim g \Rightarrow \ell(f+g) = \ell(f) + \ell(g) \text{ for all } f, g \in K^+.$$

(2) ℓ is positively homogeneous if and only if

$$\ell(af) = a\ell(f) \text{ for all } a \in R^+ \text{ and } f \in K^+.$$

(3) ℓ is monotonic if and only if

$$f \le g \Rightarrow \ell(f) \le \ell(g) \text{ for all } f, g \in K^+.$$

Definition 3.2 ([12]) A fuzzy measure μ is said to be outer regular if and only if for all $B \in \Omega_1$, $\mu(B) = \inf\{\mu(O) \mid O$ is an open set such that $O \supset B\}$.

Theorem 3.3 ([12]) For a comonotonically additive, positively homogeneous and monotonic functional ℓ on K^+, there exists a outer regular fuzzy measure μ on Ω_1 such that for all $f \in K^+$, $\ell(f) = (C)\int f d\mu$.

Since the Choquet integral with respect to every fuzzy measure is a comonotonically additive, positively homogeneous and monotonic functional, we have the following corollary.

Corollary 3.4 ([12]) For every fuzzy measure μ, there exists a outer regular measure μ_r such that for every $f \in K^+$,

$$(C)\int f d\mu = (C)\int f d\mu_r.$$

We consider interval-valued Choquet integrals with respect to fuzzy measure and will define comonotonically additive, positively homogeneous and monotonic interval-valued functional on the class $\mathfrak{I}_1$ of interval number-valued functions.

Definition 3.5 Let $F, G \in \mathfrak{I}$. We say that F and G are comonotonic, in symbol, $F \sim G$ if and only if

(i) $f^*(x) < f^*(x') \Rightarrow g^*(x) \le g^*(x')$ for all $x, x' \in X$, and
(ii) $f_*(x) < f_*(x') \Rightarrow g_*(x) \le g_*(x')$ for all $x, x' \in X$,
where
$f^*(x) = \sup\{F(x)\}$, $f_*(x) = \inf\{F(x)\}$, $g^*(x) = \sup\{G(x)\}$, and $g_*(x) = \inf\{G(x)\}$.

From Definition 3.5, clearly we have the following theorem.

Theorem 3.6 Let $F, G \in \mathfrak{I}$. Then we have

(1) $F \sim F$,
(2) $F \sim G \Rightarrow G \sim F$,

(3) $F \sim A$ for all $A \in I(R^+)$,
(4) $F \sim G,\quad F \sim H \Rightarrow F \sim G+H$.

Theorem 3.7 Let $F, G \in \mathfrak{I}_1$. If $F \sim G$, then we have

$$(C)\int (F+G)d\mu = (C)\int Fd\mu + (C)\int Gd\mu.$$

Proof. It is easily to show that if we put $h^*(x) = \sup(F+G)(x)$ and $h_*(x) = \inf(F+G)(x)$, then $h^* = f^* + g^*$ and $h_* = f_* + g_*$. We note that f^* and f_* [respectively, g^* and g_*] are Choquet integrable selections of F [respectively, G]. By Theorem 2.9,

$$(C)\int (F+G)\, d\mu = [\,(C)\int h_* d\mu, (C)\int h^* d\mu]$$

.

Definition 3.5 implies that $f^* \sim g^*$ and $f_* \sim g_*$. Thus, by Theorem 2.5,

$$(C)\int h^* d\mu = (C)\int f^* d\mu + (C)\int g^* d\mu \text{ and}$$
$$(C)\int h_* d\mu = (C)\int f_* d\mu + (C)\int g_* d\mu.$$

Therefore, we have

$$\begin{aligned}(C)\int (F+G)\, d\mu &= [\,(C)\int h_* d\mu, (C)\int h^* d\mu] \\ &= [\,(C)\int f_* d\mu + (C)\int g_* d\mu, (C)\int f^* d\mu + (C)\int g^* d\mu] \\ &= [\,(C)\int f_* d\mu, (C)\int f^* d\mu] + [\,(C)\int g_* d\mu, (C)\int g^* d\mu] \\ &= (C)\int Fd\mu + (C)\int Gd\mu.\end{aligned}$$

Clearly, Theorem 2.6 and the definition of order $\leq$ on $I(R^+)$ imply the following theorem.

Theorem 3.8 Let $F, G \in \mathfrak{I}_1$. Then we have
(1) $(C)\int aFd\mu = a(C)\int Fd\mu$ for all $a \in R^+$,
(2) if $F \leq G$, then $(C)\int Fd\mu \leq (C)\int Gd\mu$.

We consider the class of interval number-valued functions with continuous selections;

$$\mathfrak{I}_2 = \{F \in \mathfrak{I}_1 \mid S_c(F) \subset K^+\}.$$

Definition 3.9 (1) A mapping $T: \mathfrak{I}_2 \to I(R^+)$ is said to be an interval-valued functional.
(2) An interval-valued functional T is comonotonically additive if and only if

$$F \sim G \Rightarrow T(F+G) = T(F) + T(G).$$

(3) An interval-valued functional T is positively homogeneous if and only if

$$T(aF) = aT(F) \text{ for all } a \in R^+.$$

(4) An interval-valued functional T is monotonic if and only if for each pair $F, G \in \mathfrak{I}_2$,

$$F \leq G \Rightarrow T(F) \leq T(G).$$

Definition 3.10 Let $\ell : K^+ \to R^+$ be a real-valued functional. A mapping $T_\ell : \mathfrak{I}_2 \to I(R^+)$ is said to be an interval-valued functional induced by ℓ if for all $F \in \mathfrak{I}_2$,

$$T_\ell(F) = \{\, \ell(f) \mid f \in S_c(F)\,\}.$$

Theorems 2.9, 3.7 and 3.8 imply the following theorem.

Theorem 3.11 If $T: \mathfrak{I}_2 \to I(R^+)$ is defined by $T(F) = (C)\int Fd\mu$ for all $F \in \mathfrak{I}_2$, then T is a comonotonicallly additive, positively homogeneous, monotonic interval-valued functional.

Theorem 3.12 Let $\ell : K^+ \to R^+$ be a comonotonicallly additive, positively homogeneous, monotonic functional. If T_ℓ an interval-valued functional induced by ℓ, there exists a outer regular fuzzy measure μ on Ω_1 such that for all $F \in \mathfrak{I}_2$,

$$T_\ell(F) = (C)\int Fd\mu = [\, \ell(f_*), \ell(f^*)],$$

where $f^*(x) = \sup\{F(x)\}$ and $f_*(x) = \inf\{F(x)\}$.

Proof. Since ℓ is a comonotonicallly additive, positively homogeneous, monotonic functional on K^+, by Theorem 3.3, there exists a outer regular fuzzy measure μ on Ω_1 such that for all $f \in K^+$, $I(f) = (C)\int fd\mu$. For all $F \in \mathfrak{I}_2$,

$$\begin{aligned}T_\ell(F) &= \{\, \ell(f) \mid f \in S_c(F)\} \\ &= \{(C)\int fd\mu \mid f \in S_c(F)\,\} \\ &= (C)\int Fd\mu \\ &= [\,(C)\int f_* d\mu, (C)\int f^* d\mu] \\ &= [\, \ell(f_*), \ell(f^*)]\end{aligned}$$

Theorem 3.13 Let $\ell : K^+ \to R^+$ be a real-valued functional. If ℓ is comonotonically additive, positively homogeneous and monotonic, so is T_ℓ.

Proof. By Theorem 3.12, T_ℓ is an interval-valued functional on $\mathfrak{I}_2$. If $F, G \in \mathfrak{I}_2$, and $F \sim G$, then

$$\begin{aligned}&T_\ell(F+G) \\ &= [\, \ell(\inf(F+G)), \ell(\sup(F+G))] \\ &= [\, \ell(f_* + g_*), \ell(f^* + g^*)] \\ &= [\, \ell(f_*) + \ell(g_*), \ell(f^*) + \ell(g^*)] \\ &= [\, \ell(f_*), \ell(f^*)] + [\, \ell(g_*), \ell(g^*)] \\ &= T_\ell(F) + T_\ell(G)\end{aligned}$$

We note that $F \sim G$ means $f_* \sim g_*$ and $f^* \sim g^*$. Thus, T_ℓ

is comonotonically additive. Similarly, we can prove that T_ℓ is positively homogeneous and monotonic.

We remark that the interval-valued Choquet integral with respect to every fuzzy measure is comonotonically additive, positively homogeneous, monotonic as in Theorem 3.11. Theorem 3.12 means that an interval-valued functional induced by a comonotonicallly additive, positively homogeneous, monotonic functional on K^+ is represented by the interval-valued Choquet integral with respect to a outer regular fuzzy measure.

4. Conclusion.

We obtained that a comonotonically additive, positively homogeneous, monotonic interval-valued functional on $\mathfrak{I}_2$ was represented as a interval-valued Choquet integral with respect to fuzzy measure in Theorem 3.12.

They will be used in the following applications : (1) subjectively prbability, Choquet expectations, and expected utility without additivity in a sample space of interval-valued variables, (2) capacity measure which are represented by interval-valued Choquet integrals, and (3) ambiguity measure related with interval number inferences. Futhermore, using these properties, we can define fuzzy number-valued Choquet integral and Choquet expectation and subjectively probability of fuzzy number valued random variables.

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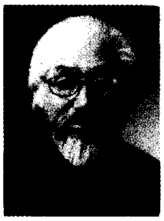

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