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Wavelet Neural Network Based Indirect Adaptive Control of Chaotic Nonlinear Systems

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Abstract

In this paper, we present a indirect adaptive control method using a wavelet neural network (WNN) for the control of chaotic nonlinear systems without precise mathematical models. The proposed indirect adaptive control method includes the off-line identification and on-line control procedure for chaotic nonlinear systems. In the off-line identification procedure, the WNN based identification model identifies the chaotic nonlinear system by using the serial-parallel identification structure and is trained by the gradient-descent method. And, in the on-line control procedure, a WNN controller is designed by using the off-line identification model and is trained by the error back-propagation algorithm. Finally, the effectiveness and feasibility of the proposed control method is demonstrated with applications to the chaotic nonlinear systems.

Key Words : Chaos control; Indirect adaptive control; Wavelet neural networks; Gradient-descent method; Error back-propagation algorithm

1. Introduction

Chaos control is one of the topics gaining great importance and attention in physics and engineering publications. Although the model descriptions of some chaotic nonlinear systems are simple, the dynamic behaviors are complex. Recently, many researchers have managed to use modern elegant theories to control chaotic nonlinear systems, and most of them are based on the exact chaotic nonlinear model (differential equations) [1]. The conventional control techniques such as feedback control, optimal control, and robust control are introduced to control the chaotic nonlinear systems, and these kinds of techniques confirm the effectiveness of chaos control [2-4]. But most of these techniques can be applied to control chaotic nonlinear systems when the exact or at least the approximate mathematical model for chaotic nonlinear systems is available. To overcome this shortcoming of them, the direct/indirect adaptive control methods can be used for controlling chaotic nonlinear systems [5].

On the other hand, the intelligent control techniques based on neural networks and fuzzy logic have been developed to control chaotic nonlinear systems [6]. Even though these intelligent control strategies have shown the effectiveness especially for unknown chaotic system, they have some drawbacks which derive from their own inherent characteristics. Therefore, wavelet neural network (WNN), which combines the advantages of wavelet transform (WT) and neural networks, is proposed recently to approximate, modeling and control nonlinear functions with local nonlinearities and fast variations as an alternative to multi-layer perceptron neural network (MLPN) or radial basis function network (RBFN) [7]. WNN has one hidden layer and can be applied to various applications because of the intrinsic properties of WT, which has an excellent time-frequency analysis ability.

In this paper, we propose WNN based indirect adaptive control method, which has better control performance than direct adaptive control due to inherent characteristics of chaotic nonlinear systems such as sensitivities of initial state and sampling time parameters. In our design method, WNN based indirect adaptive control system performs two procedure, which are the off-line identification and on-line control procedures. Firstly, a WNN based identification model identify the chaotic nonlinear systems by using the gradient-descent method through off-line process. And then, a WNN controller controls the chaotic nonlinear systems whose precise mathematical models are unavailable by using the off-line identification model and the parameters of a WNN controller are trained by the error back- propagation algorithm. Finally, in order to evaluate the performance of our controller, we apply the proposed method to the representative continuous-time chaotic nonlinear systems (Duffing and Lorenz systems).

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2. Wavelet Neural Networks

The theory of wavelets was first proposed by Mallat in the field of multi-resolution analysis (MRA) [7]. A family of wavelets is constructed by translations and dilations performed on a single fixed function called the mother wavelet. A wavelet ϕ_j is derived from its mother wavelet ϕ:

$$\phi_j(z) = \phi\left(\frac{x - m_j}{d_j}\right) \tag{1}$$

where, its translation factor m_j and its dilation factor d_j ($d_j > 0$) are real numbers. And, we choose the first derivative of a Gaussian function as a mother wavelet:

$$\phi(x) = -x\exp\left(-\frac{1}{2}x^2\right) \tag{2}$$

2.1 Configuration of a WNN

Figure 1 shows the configuration of a WNN, which has N_i inputs, 1 output, and N_w wavelets. And, the dotted lines of Fig. 1 represent the connection lines between the input nodes and wavelet nodes.

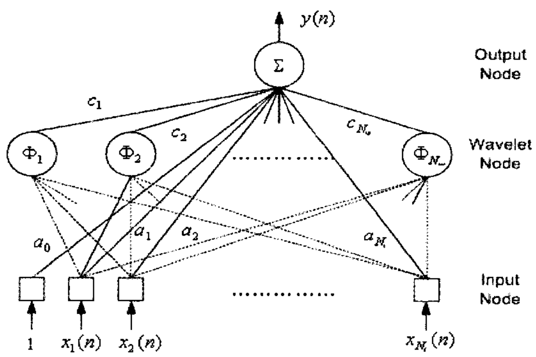

Fig. 1. Configuration of a WNN

Each wavelet of wavelet nodes is given by the product of the mother wavelets as follows:

$$\Phi_j(\mathbb{X}) = \prod_{k=1}^{N_i}\phi(z_{jk}),\ with\ z_{jk} = \frac{x_k - m_{jk}}{d_{jk}} \tag{3}$$

where, $k = 1, \cdots, N_i$, $j = 1, \cdots, N_w$.

In Fig. 1, the output of a WNN is composed by each wavelet and parameters as follows:

$$y = \Psi(\mathbb{X},\theta) = \sum_{j=1}^{N_w} c_j\Phi_j(\mathbb{X}) + a_0 + \sum_{k=1}^{N_i} a_k x_k \tag{4}$$

where, a_0 and a_k are connection weight between input nodes and output nodes. c_j is connection weight between wavelet nodes and output nodes, and θ is the set of adjustable parameters:

$$\theta = \{a_0, a_k, c_j, m_{jk}, d_{jk}\} \tag{5}$$

2.2 Training method of a WNN

As usual, the training is based on the minimization of the following quadratic cost function:

$$J(\theta(n)) = \frac{1}{2}(y_d(n) - y(n))^2 = \frac{1}{2}e^2(n) \tag{6}$$

where, $y(n)$ is the output value of n-th WNN and $y_d(n)$ is desired output value.

The minimization is performed by the following iterative gradient-descent method:

$$\begin{aligned}\theta(n+1) &= \theta(n) - \Delta\theta(n) \\ &= \theta(n) - \eta\frac{\partial J(\theta(n))}{\partial\theta(n)}\end{aligned} \tag{7}$$

where, η is the learning rate of a WNN.

The partial derivative of the cost function with respect to $\theta(n)$ is

$$\frac{\partial J(\theta(n))}{\partial\theta(n)} = -e(n)\frac{\partial y(n)}{\partial\theta(n)} \tag{8}$$

where, $\partial y(n)\ \partial\theta(n)$ is the gradient of the plant output $y(n)$, with respect to parameters set $\theta(n)$, and the components of this vector are

- parameter a_0

$$\frac{\partial y(n)}{\partial a_0} = 1 \tag{9}$$

- direct connection parameters a_k

$$\frac{\partial y(n)}{\partial a_k} = x_k \tag{10}$$

- weights c_j

$$\frac{\partial y(n)}{\partial c_j} = \Phi_j(\mathbb{X}) \tag{11}$$

- translations m_{jk}

$$\frac{\partial y(n)}{\partial m_{jk}} = -\frac{c_j}{d_{jk}}\frac{\partial\Phi_j(\mathbb{X})}{\partial z_{jk}} \tag{12}$$

where,

$$\frac{\partial\Phi_j(\mathbb{X})}{\partial z_{jk}} = \phi(z_{j1})\phi(z_{j2})\cdots\dot{\phi}(z_{jk})\cdots\phi(z_{jN_i}),$$

$$\dot{\phi}(z_{jk}) = \frac{d\phi(z_{jk})}{dz_{jk}} = (z_{jk}^2 - 1)\exp\left(-\frac{1}{2}z_{jk}^2\right)$$

• dilations d_{jk}

$$\frac{\partial y(n)}{\partial d_{jk}} = -\frac{c_j}{d_{jk}} z_{jk} \frac{\partial \Phi_j(\mathbb{X})}{\partial z_{jk}} \tag{13}$$

3. Indirect Adaptive Control

In this section, we describe the chaotic nonlinear systems and the design method of the WNN identification model and controller based on the indirect adaptive control technique.

3.1 Chaotic nonlinear systems

In this paper, we consider the Duffing and Lorenz systems, which are the representative continuous-time chaotic nonlinear systems [9].

The solution to the Duffing equations is often used as an example of a classic chaotic system. The state equation of the Duffing system is the follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} y \\ a_1 x - x^3 - a_2 y + b\cos(wt) + u \end{pmatrix} \tag{14}$$

where typically, $a_1 = 1.1$, $a_2 = 0.4$, $b = 2.1$ and $w = 1.8$.

Figure 2 shows a 2-dimensional system for Duffing strange attractor.

And, we discuss another continuous-time chaotic nonlinear system. The state equation of the Lorenz system with x, y and z as state variables is expressed by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \sigma(y-x) \\ rx - y - xz \\ xy - bz + u \end{pmatrix} \tag{15}$$

where, $\sigma = 10$, $b = 8/3$, $r = 28$.

Figure 3 shows the strange attractor for the Lorenz chaotic system.

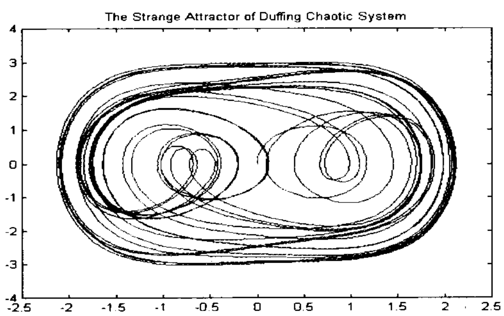

Fig. 2. Chaotic attractor of Duffing system

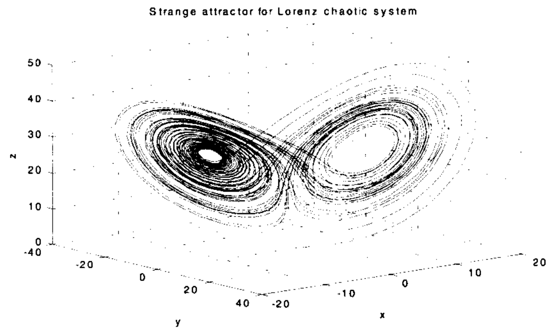

Fig. 3. Chaotic attractor of Lorenz system

3.2 System identification

The problem of identification consists of setting up a suitably parameterized identification model and adjusting the parameters of the model to optimize a performance function based on the error between the plant and the identification model outputs.

The identification model is divided into two types as follow: 1) Parallel Identification Model: the output of identification model is fed back into the identification model, and 2) Series-Parallel Identification Model: the output of plant is fed back into the identification model as shown in Fig. 4.

In this paper, we identify the chaotic nonlinear systems using the series-parallel identification model with a good performance and convergence abilities.

The training of WNN identification model is performed by using the gradient-descent method, as described in Section 2.2.

3.3 Design of WNN controller

The control process is an on-line process, which uses the WNN trained by the identification process, as described in Section 3.2.

Since the given systems are uncertain, we assume that the closed-loop system output data are available on-line for the controller. We take the approach that employs an on-line system identification unit, where the WNN identifier is based on a suitable nonlinear auto-regressive moving average (NARMA) real-time

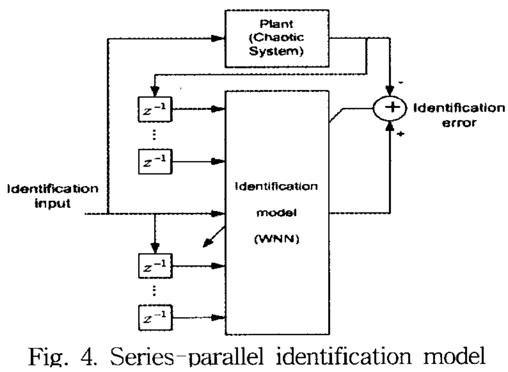

Fig. 4. Series-parallel identification model

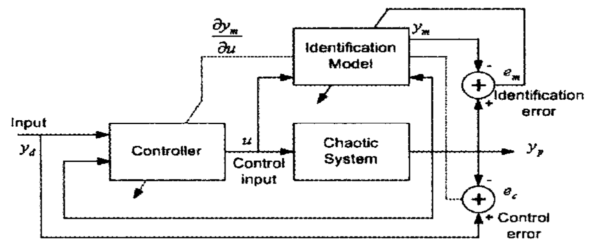

Fig. 5. The configuration of overall control system

modeling method, and a WNN controller. The overall configuration of a WNN controller based on the indirect adaptive control technique is shown in Fig. 5, where the output y_p is to be controlled to track the reference, y_d.

The identification error, e_m, where is defined as an error between the output of identification model and the output of chaotic nonlinear system, tunes the parameters of a WNN identifier using the gradient-descent method, as described in Section 2.2. The parameters of the WNN controller are adjusted by using the error back-propagation algorithm, where the difference between the output of chaotic nonlinear system and the reference signal is used as a control error, e_c.

Since the output error e_u, of the controller output layer can not be obtained directly from the plant for training the parameters of a WNN controller, the parameters of a WNN controller are updated by using the Jacobian of the identification model with respect to control input, u.

Our purpose is to select optimal control input u in order to minimize the following quadratic cost function:

$$J(\tilde{\theta}(n)) = \frac{1}{2}(y_p(n) - y_d(n))^2 = \frac{1}{2}e_c^2(n) \tag{16}$$

where, $y_p(n)$ and $y_d(n)$ are the n-th output of the chaotic nonlinear system and the n-th reference signal, respectively. The $e_c(n)$ is the control error signal and $\tilde{\theta}(n)$ is the parameter set of a WNN controller for training the controller as follows:

$$\tilde{\theta} = \{\tilde{a}_0, \tilde{a}_k, \tilde{c}_j, \tilde{m}_{jk}, \tilde{d}_{jk}\} \tag{17}$$

And, the minimization problem is to train the parameter set, θ, as follows:

$$\begin{aligned}\tilde{\theta}(n+1) &= \tilde{\theta}(n) - \Delta\tilde{\theta}(n)\\ &= \tilde{\theta}(n) - \eta\frac{\partial J(\tilde{\theta}(n))}{\partial\tilde{\theta}(n)}\end{aligned} \tag{18}$$

where, $\partial J(\tilde{\theta}(n))/\partial\tilde{\theta}(n)$ is the gradient of the quadratic cost function, $J(\tilde{\theta}(n))$, with respect to the parameter set, $\tilde{\theta}(n)$, and is calculated by the following equation.

$$\begin{aligned}\frac{\partial J(\tilde{\theta}(n))}{\partial\tilde{\theta}(n)} &= e_c(n)\frac{\partial y_m(n)}{\partial\tilde{\theta}(n)}\\ &= e_c(n)\frac{\partial y_m(n)}{\partial u(n)}\frac{\partial u(n)}{\partial\tilde{\theta}(n)}\end{aligned} \tag{19}$$

where, $\partial y_m(n)/\partial u(n)$ is the Jacobian of a WNN identification model with respect to the control signal, $u(n)$, and is calculated by

$$\begin{aligned}\frac{\partial y_m(n)}{\partial u(n)} &= \frac{\partial y_m(n)}{\partial \mathbb{X}}\frac{\partial \mathbb{X}}{\partial u(n)}\\ &= \sum_{j=1}^{N_w}\frac{c_j}{d_{jk}}\frac{\partial\Phi_j(\mathbb{X})}{\partial z_{jk}} + a_k \Big|_{k=N_s+1}\end{aligned} \tag{20}$$

where, $\partial\mathbb{X}/\partial u(n)$ is the column vector as follows:

$$\begin{matrix}\leftarrow N_s \rightarrow \;\; \leftarrow N_e \rightarrow\\ [0, 0, \cdots, 0, 1, 0, \cdots, 0]^T\end{matrix} \tag{21}$$

where, N_s and N_e are the number of the past output of plant and the number of control input as the input of a WNN identification model, respectively.

Finally, the partial derivative, $\partial u(n)/\partial\tilde{\theta}(n)$, of the control signal, $u(n)$, with respect to the parameter set of a WNN controller, $\tilde{\theta}(n)$, can be calculated by using the Eqns. from Eqn. (9) to Eqn. (13).

4. Simulation Results

In this section, we present some simulation results to validate the proposed indirect adaptive control scheme for continuous- time chaotic nonlinear systems. In order to evaluate the performance of the proposed controller, we compares the results of a WNN based indirect adaptive control with those of a neural network (NN) based indirect adaptive control [10].

4.1 Identification and control of the Duffing system

In order to identify the Duffing system, the WNN identification model must need the identification input. In this paper, the identification input is selected as $1.0e^{-3}cos(t)$ through enough simulations.

In the identification of the Duffing system, we define the initial system state as (1, 0) and the learning rate is chosen as 0.01. Also, the sampling time is chosen as 0.02. The simulation environments of the proposed WNN identification model and [10] are shown in Table 1. Figure 6 shows the identification result of Duffing system.

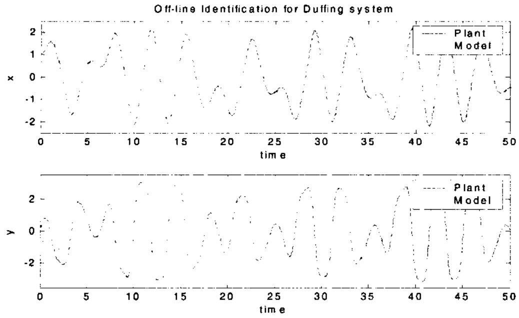

Fig. 6. The identification result for Duffing system

In this paper, we use the mean-squared error (MSE) as the performance index. The MSEs for system identification are given in Table 2.

Table 1. The simulation environments of the ID model

WNN model (ours)	[redacted]	5
		2
		2
		0.02
		0.01
		10000
NN model ([10])		5, 5
		2
		1
		0.02
		0.0001
		180000

Table 2. Off-line identification errors for Duffing system

	[redacted]	
[redacted]	0.0063	0.013

From the above results, we can see that the WNN identification model shows a good identification result, and it is fast and effective, as compared with the NN identification model.

The control objective for the Duffing system is to follow the unstable periodic solution of the Duffing system. In other words, as the value of b in Eqn. (14) varies, the Duffing system may have either a chaotic or a periodic solution. In the tracking control of the Duffing system, we define the reference signal as one periodic solution in the case of $b = 2.3$. The simulation environments of the proposed WNN controller and [10] are shown in Table 3. And, Fig. 7 shows the on-line identification result and tracking control result. Finally, the MSEs for control performance are given in Table 4.

From the above results, we can see that a WNN based indirect adaptive control shows a better final control performance as compared with the NN based indirect adaptive control.

Table 3. The simulation environments of the controller

WNN (ours)	[redacted]	3
		1
		1
		0.02
		0.05
NN ([10])		5, 5
		2
		1
		0.02
		0.01

Table 4. Mean-squared errors for Duffing system

	[redacted]			
[redacted]	0.0008	0.0389	0.4155	0.0568

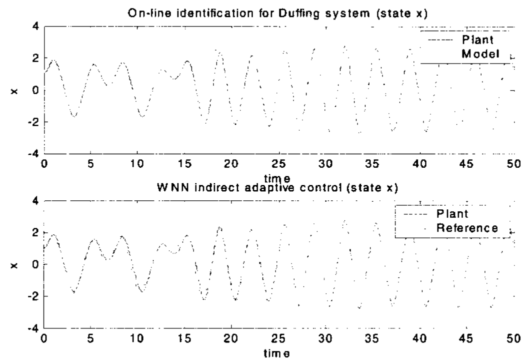

Fig. 7. On-line identification and control results

4.2 Identification and control of Lorenz system

In order to identify the Lorenz system, the identification input of a WNN identification model is selected as same value used in simulation of the Duffing system. In the identification of the Lorenz system, we define the initial system state as (0.1673, 0.5651, 0.9854) and the used parameters of the identification model are shown in Table 5. And, Fig. 8 and Table 6 show the off-line identification result and the MSEs of off-line identification, respectively.

In the off-line identification results, a WNN identification model shows a better performance as compared with the NN identification model.

When regulating the Lorenz system, the reference signal is selected as (-8.4853, -8.4853, 27). Table 7 shows the simulation environments of WNN and NN controllers, and Fig. 9 shows the regulation control result of the Lorenz system. Finally, the errors of control performance are given in Table 8.

From the above regulation control results, we can see that although on-line identification errors of a WNN based indirect adaptive control are more than those of a NN based indirect adaptive control, the control results of a WNN based indirect adaptive control show a better final control performance as compared with the NN based indirect adaptive control.

Table 5. The simulation environments of the ID model

WNN model (ours)	Number of wavelet function	5
	Number of input state	2
	Number of past input	2
	Sampling time	0.01
	Learning rate	0.0001
	Learning iteration	20000
NN model ([10])	No. of neurons (1st & 2nd hidden layer)	20, 10
	Number of input state	2
	Number of past input	1
	Sampling time	0.01
	Learning rate	0.0001
	Learning iteration	600000

Table 6. Off-line identification errors for Lorenz system

	Off-line ID error (WNN, ours)	Off-line ID error (NN, [10])
State x	0.5745	0.7936
State y	0.9115	1.9308
State z	0.6481	2.6655

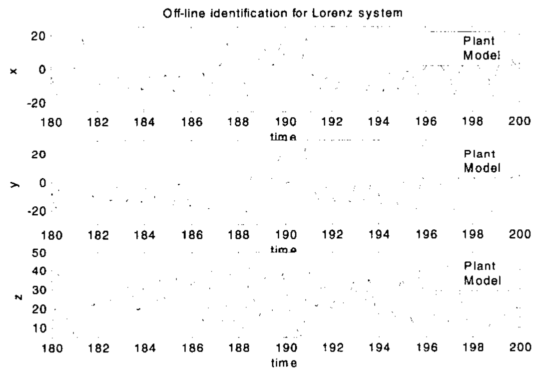

Fig. 8. The identification result for Lorenz system

Table 7 The simulation environments of the controller

WNN (ours)	Number of wavelet function	5
	Number of input state	2
	Number of past input	2
	Sampling time	0.01
	Learning rate	0.00001
NN ([10])	No. of neurons (1st & 2nd hidden layer)	40, 20
	Number of input state	0
	Number of past input	1
	Sampling time	0.01
	Learning rate	0.000001

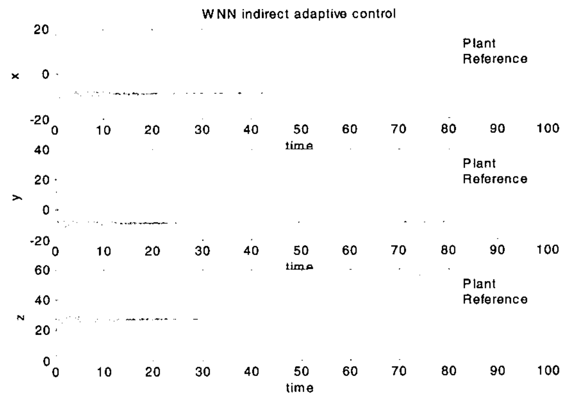

Fig. 9. The regulation control result for Lorenz system

Table 8. The errors for Lorenz system

	WNN (ours)		NN ([10])	
	ID error	Control error	ID error	Control error
State x	0.2403e-3	-0.1831	-1.8718e-6	-0.6223
State y	0.9353e-3	-0.1832	2.9487e-6	-0.6223
State z	-0.0420e-3	-0.0002	4.1499e-6	2.8335e-7

5. Conclusions

In this paper, we have presented the indirect adaptive control of the chaotic nonlinear systems using the WNN. The proposed control method includes the identification and control process for chaotic nonlinear systems. The identification process for chaotic nonlinear systems is an off-line process, which utilizes the serial-parallel structure of WNN. The control process is an on-line process, which uses the WNN trained by the identification process. the gradient-descent method and error back-propagation algorithm are used for training of identification and control for chaotic nonlinear systems, respectively. And the effectiveness and feasibility of the proposed control method have been demonstrated with application to the Duffing system and Lorenz system, which are two representative continuous-time chaotic nonlinear systems. From the simulation results, we have shown that a WNN based indirect adaptive control has more accurate control performance than the NN based indirect adaptive control. Also, we have verified that the proposed WNN based indirect adaptive control scheme worked well for various chaotic nonlinear systems. Finally, further study on extending of the algorithm proposed in this paper is the stability analysis of the overall system and the feasibility for real nonlinear control systems.

References

[1] G. Chen and X. Dong, "From Chaos to Order-Perspectives and Methodologies in Controlling Chaotic Nonlinear Dynamical Systems," International Jour. of Bifurcation and Chaos, Vol. 3, No. 6, pp. 1363-1409, 1993.

[2] E. Ott, C. Grebogi and J. A. York, "Controlling Chaos," Physical Review Letters, Vol. 64, No. 11, pp. 1196-1199, 12 March 1990.

[3] E. A. Jackson and A. W. Hübler, "Periodic Entrainment of Chaotic Logistic Map Dynamics," Physica D, Vol. 44, pp. 407-420, 1990.

[4] G. Chen and X. Dong, "On Feedback Control of Chaotic Continuous-Time Systems," IEEE Trans. on Circuits and Systems, Vol. 40, No. 9, pp. 591-601, September 1993.

[5] K. S. Park, J. B. Park, Y. H. Choi, G. Chen, "Generalized Predictive Control of Discrete-Time Chaotic System," International Jour. of Bifurcation and Chaos, Vol. 8, No. 7, pp. 1591- 1597, 1998.

[6] K. B. Kim, J. B. Park, Y. H. Choi and G. Chen, "Control of Chaotic Nonlinear Systems Using Radial Basis Function Network Approximators," Information Sciences, Vol. 130, pp. 165-183, 2000.

[7] Y. Oussar, I. Rivals, L. Personnaz and G. Dreyfus, "Training Wavelet Networks for Nonlinear Dynamic Input-Output Modeling," Neurocomputing, Vol. 20, pp. 173-188, 1998.

[8] S. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Transform," IEEE Trans. on Pattern Anal. Machine Intell., Vol. 7, No. 7, pp. 674-693, 1989.

[9] S. H. Strogatz, Nonlinear Dynamics and Chaos, Addison Wesley Co., 1994.

[10] K. H. Oh, Y. H. Choi, J. B. Park and K. Y. Lim, "Design of Neural Network Controller for Continuous-Time Chaotic Nonlinear Systems," Jour. of the Institute of Electronics Engineers of Korea SC, Vol. 39, No. 1, pp. 51-65, 2002

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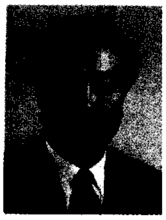

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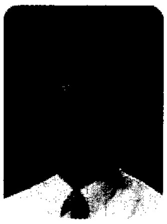

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