

A Framework for Determining Minimum Load Shedding for Restoring Solvability Using Outage Parameterization

Hwachang Song* and Byongjun Lee**

Abstract - This paper proposes a framework for determining the minimum load shedding for restoring solvability. The framework includes a continuation power flow (CPF) and an optimal power flow (OPF). The CPF parameterizes a specified outage from a set of multiple contingencies causing unsolvable cases, and it traces the path of solutions with respect to the parameter variation. At the nose point of the path, sensitivity analysis is performed in order to achieve the most effective control location for load shedding. Using the control location information, the OPF for locating the minimum load shedding is executed in order to restore power flow solvability. It is highlighted that the framework systematically determines control locations and the proper amount of load shedding. In a numerical simulation, an illustrative example of the proposed framework is shown by applying it to the New England 39 bus system.

Keywords: Nonlinear interior point method, optimal power flow, preventive and corrective control, voltage stability

1. Introduction

Voltage stability is one of the main factors limiting the secure region in the operation and planning of power systems today [1-3]. In deregulated environments, power systems have been experiencing an increase in uncertainty in terms of voltage stability. This is the result of diverse power transactions and benefit based operational schemes different from the vertically integrated environments of the past. Thus, to ensure the secure operation of systems, effective control strategies should be established against voltage collapse.

Control actions against voltage collapse can be classified into two categories, preventive and corrective control. Preventive control is used to prevent voltage instability before it actually occurs, and corrective control is a means to stabilize an unstable system after the occurrence of severe outages/ disturbances. When control strategies are being determined, the aim of the control can be any of the following [4]:

- Minimization of the number of control components,
- Minimization of control costs,
- Minimization of load shedding.

The first aim listed can be applied in the case of both preventive and corrective control. The second aim might be only used for preventive control, and the last aim is

adequate for corrective control, which is performed by forcing the system dynamic trajectory to come to the point of a stable equilibrium when it is unstable. Due to the fact that the solvability issue determines controls for restoring a network solution, it is closely related to corrective control against voltage collapse. The paper focuses on this minimum load shedding solvability problem considering the number of control locations.

The framework proposed in this paper includes two modules, outage continuation power flow (OCPF) and solvability optimal power flow (SOPF). The OCPF module is used to trace the path of a power flow solution with respect to a parameter representing a specified outage. OCPF mainly utilizes the concept of the branch parameter variation based CPF proposed by Flueck [5], but in this paper the concept is extended in order to deal with combined multiple outages of generating units and branches. In the framework, the main objective of OCPF is to determine a set of effective control locations using sensitivity information acquired at the nose point of the solution path. To obtain the minimum amount of load shedding at the selected control locations, SOPF is performed, which is developed with a nonlinear interior point method (NIPM). It should be noted that the framework can determine the effective control locations systematically and come up with the proper amount of load shedding for any combined multiple outages. In the case study, an example of the framework using the New England 39 bus system is shown.

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2. Formulations in the Framework

In power system operation, unsolvable cases occur when severe contingencies are applied such as multiple outages of main transmission lines and/or main generating units. Unsolvability is difficult for power system engineers to analyze because of the nonexistence of a power flow solution; thus, determining effective locations for restoring solvability is also difficult. When control strategies are decided for severe multiple outages, a useful control means must be load shedding. This section describes formulations in the framework for the minimum load shedding problem to restore power flow solvability.

The fundamental formulation that needs to be solved in the framework is as follows:

$$\begin{aligned} \min \quad & \sum_i P_{si} l_i \\ \text{s.t.} \quad & g(x) = 0 \\ & h_{\min} \leq h(x) \leq h_{\max} \end{aligned} \quad (1)$$

where x is the vector of state variables, $g(\cdot)$ and $h(\cdot)$ correspond to the function vectors for network equations and inequality constraints including load shedding constraints, and h_{\min} and h_{\max} represent the lower and upper limits of $h(\cdot)$. In the objective function of (1), P_{si} denotes the amount of load shedding at bus i , and l_i is a binary variable (1 or 0) that indicates whether bus i is available for load shedding.

In (1), the most important issue is how to select the set of locations whose loads are shed. As mentioned in the introduction of this paper, the goals for this kind of problem are not only to minimize the total amount of control but also the number of control locations. In unsolvable cases, however, it is difficult to obtain analytical information about which locations are effective for restoring solvability by only using conventional power flow. Therefore, in this paper, a continuation power flow tool is developed in order to analyze unsolvable cases after severe multiple contingencies have occurred. The tool uses the concept of branch parameter variation based CPF [5], but it is extended to take into account multiple outages of branches and generating units combined. That tool is referred to in this paper as the outage continuation power flow (OCPF).

3.1 OCPF

For development of the OCPF, a parameter, ζ , representing an outage or multiple outages is introduced, and the power flow equations are reformulated with the outage parameter. In the formulation, ζ is 0 in the normal

state and 1 following outage application. An example of ζ -V curves is shown in Fig. 1.

For a branch outage, the coefficient of the corresponding branch is parameterized with ζ as presented in Fig. 2. For a generating unit outage, the type of the corresponding bus is changed from PV into PQ, unless reactive power of the unit doesn't reach the maximum reactive output; then, active and reactive power of the unit is shifted to the predetermined set of alternate generators. This concept is similar to that applied in modified continuation power flow [6].

First, the reformulation for a branch outage is described. When a branch outage is considered, power injection equations of corresponding buses are changed as follows:

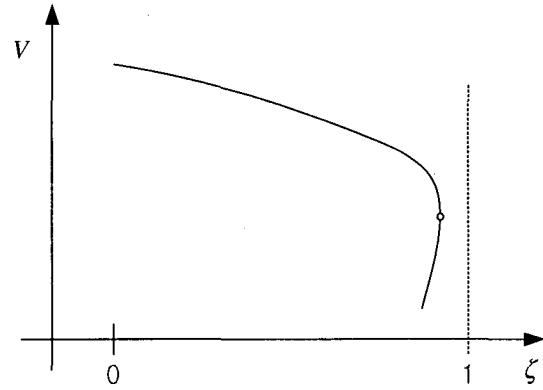


Fig. 1 An example of ζ -V curves

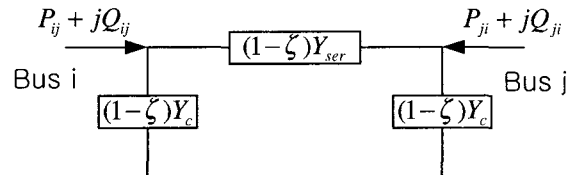


Fig. 2 Parameterization of a branch outage with ζ

$$\begin{aligned} P_{Ti} = & V_i \left(\sum_{k=1, \neq i, j} (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) V_j \right) \\ & + V_i (1 - \zeta) (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) V_j \\ & + V_i^2 (G_{ii, \text{others}} + (1 - \zeta)(g_c - G_{ij})) \end{aligned} \quad (2a)$$

$$\begin{aligned} Q_{Ti} = & V_i \left(\sum_{k=1, \neq i, j} (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) V_j \right) \\ & + V_i (1 - \zeta) (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) V_j \\ & - V_i^2 (B_{ii, \text{others}} + (1 - \zeta)(b_c - B_{ij})) \end{aligned} \quad (2b)$$

$$Y_{ser} \equiv G_{ij} + jB_{ij}, \quad Y_c \equiv g_c + jb$$

where G_{ik} and B_{ik} are actual and imaginary parts of the (i,k) component of the Y bus matrix, V_i and V_j are voltage magnitudes of bus i and j , and δ_{ij} denotes voltage angle difference between bus i and j . The injection equations for bus j are reformulated in the same way.

In the case of a generating outage, active and reactive generations of corresponding buses are modified as follows:

For the outage bus i in a generating unit outage,

$$\begin{aligned} P_{Gi} &= (1-\zeta)P_{Gio} \\ Q_{Gi} &= (1-\zeta)Q_{Gio} \\ \Delta P_{Gi} &= \zeta P_{Gio} \end{aligned} \quad (3a)$$

For a generating bus j in the predetermined set for the following ΔP_{Gi} ,

$$P_{Gj} = K_{Gj} \Delta P_{Gi} \quad (3b)$$

where P_{Gio} and Q_{Gio} denote the active and reactive generation of bus i before the corresponding outage occurs, and ΔP_{Gi} is the amount of active generation change of the bus. In (3.b) K_{Gj} describes the generation fraction factor of bus j .

As a specified set of outages to be applied is determined, the corresponding set of equations using (2) and (3) replaces the power flow equations. In (4), a set of power flow equations of bus i is shown.

$$\begin{aligned} f_{Pi} &= P_{Ti}(\zeta) + P_{Li} - P_{Gi}(\zeta) = 0 \\ f_{Qi} &= Q_{Ti}(\zeta) + Q_{Li} - Q_{Gi} = 0 \end{aligned} \quad (4)$$

where the subscripts T , L , and G denote injection, load and generation respectively, and f_{Pi} and f_{Qi} represent active and reactive power mismatch equations of bus i respectively.

Applying the locally parameterized continuation method [6, 7], the reformulated power flow equations including the outage parameter are solved. The continuation method is composed of predictor and corrector. In predictor, the initial guess of the next solution is determined using the tangent vector of the known solution. In corrector, the next solution is calculated using the Newton-Raphson method. It is noted that OCPF is developed to deal with multiple contingencies of branches and generating units combined as well as a single outage.

3.2 Sensitivity at the nose point

To determine adequate locations for load shedding in unsolvable cases, this paper uses ζ sensitivity with respect

to load shedding at a bus obtained at the maximum point of a constructed ζ -V curve. The parametric sensitivity is calculated based on the normal vector information at the margin boundary [8, 9]. The formulation used to achieve the sensitivity is as follows:

$$\begin{aligned} -\frac{\partial \zeta^*}{\partial P'_{Li}} &= \left(\frac{\partial \zeta^*}{\partial P_{Li}} + \tan \phi_i \frac{\partial \zeta^*}{\partial Q_{Li}} \right) \\ \frac{\partial \zeta^*}{\partial P_{Li}} &= -\left(v_p \quad v_q \right)^T \left(\frac{\partial f_p / \partial P_{Li}}{\partial f_q / \partial P_{Li}} \right) / d \\ \frac{\partial \zeta^*}{\partial Q_{Li}} &= -\left(v_p \quad v_q \right)^T \left(\frac{\partial f_p / \partial Q_{Li}}{\partial f_q / \partial Q_{Li}} \right) / d \\ &\quad \left(v_p \quad v_q \right)^T \left(\frac{\partial f_p / \partial \zeta}{\partial f_q / \partial \zeta} \right) \end{aligned} \quad (5)$$

where f_p and f_q are the vectors of active and reactive power mismatch equations respectively, v_p and v_q represent sub-vectors of the zero right eigenvector determined at the nose point and ϕ_i describes the power factor of bus i where load is decreased.

3.3 SOPF

SOPF is employed to obtain the amount of load shedding through solving the OPF formulation of (1). Once control locations are determined, binary variables are not obstacles any more in (1); that is, the formulation becomes a nonlinear programming (NLP) problem. To solve the NLP, in this paper, a nonlinear interior point method (NIPM) is used. This method is not much different from the established methods in the literature [10-12]. Using NIPM is effective for minimum load shedding solvability problems because it is able to cope with ill-conditioned voltage troubles [10].

3. Solution Procedure of the Framework

This section describes the solution procedure used in the framework. The flowchart of the framework for determining minimum load shedding to restore solvability is shown in Fig. 3.

As illustrated in Fig. 3, a set of contingencies including severe multi contingencies is presented. In the framework, a contingency is chosen from the contingency set, and power flow is calculated to confirm whether the case is unsolvable. If the case is solvable, another contingency is selected out of the contingency set. Otherwise, OCPF is performed to detect the nose point and then ζ sensitivity

with respect to load shedding is obtained. Using the sensitivity, control locations are determined in order to obtain the proper amount of load shedding of the locations. Lastly, SOPF is executed. The above procedures are repeated until contingency is eliminated from the set.

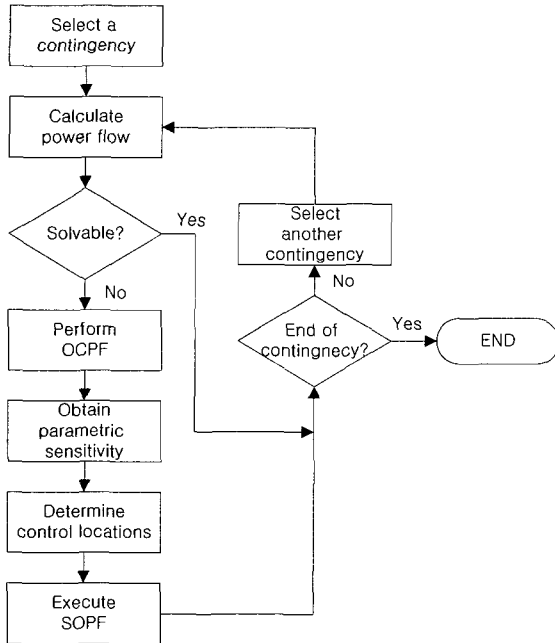


Fig. 3 Flowchart of the framework for minimum load shedding to restore solvability

4. Case Study

In this section, the simulation results are shown applying the proposed framework to the New England 39 bus system. The one line diagram of the test system is shown in Fig. 4, and total load at the base case is $61.41+j19.38$ [pu]. In the simulation, two contingencies are considered as unsolvable cases. One is outage of generating unit at bus 39 and the other is outage of the same unit as well as that of lines 10-11.

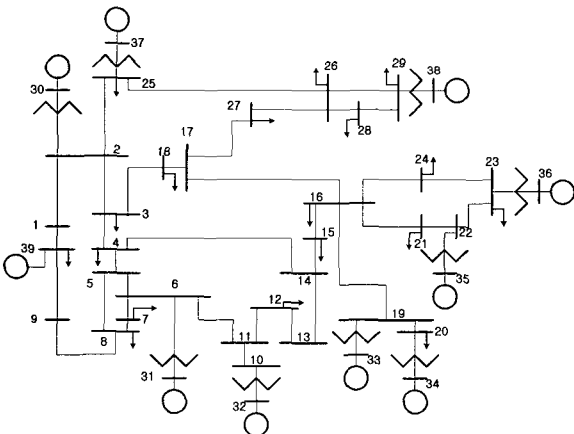


Fig. 4 One line diagram of New England 39 bus system

In accordance with the framework procedure, power flow is performed after applying each contingency, and both of them are unsolvable; thus, the ζ -V curve of each case is constructed with OCPF. Both cases contain outage of the generator at bus 39, so active power output of the unit needs to be shifted to the specified set of other generating units. In the simulation, the set includes 9 other generators, and active generation of each unit is selected using the rate of current output with respect to the sum of active generation. Fig. 5 shows the ζ -V curves of bus 12 at the two contingent cases. As shown in Fig. 5, the second case is more severe. At the maximum points of the ζ -V curves, the values of ζ are 0.8433 and 0.8305.

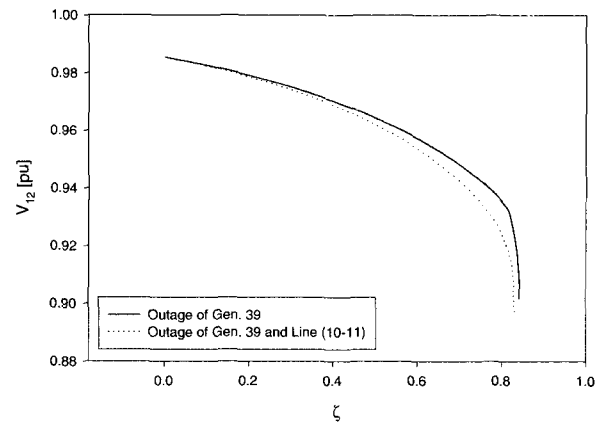


Fig. 5 ζ -V curves of the two contingencies

Next, ζ enhancement sensitivity with respect to the load shedding of each bus is calculated with (5). Table 1 shows the top 10 most sensitive buses and their sensitivity for load shedding of each contingency.

Table 1 Top 10 most sensitive buses and their sensitivity of each contingency

	Contingency 1 (Gen. 39)		Contingency 2 (Gen. 39 & Lines 10-11)	
	Bus	Sensitivity	Bus	Sensitivity
1	12	0.2915	12	0.2782
2	39	0.1087	39	0.1026
3	8	0.02887	8	0.02794
4	7	0.02650	7	0.02571
5	4	0.02028	4	0.01853
6	3	0.01851	3	0.01660
7	15	0.01356	15	0.01165
8	18	0.01097	18	0.00939
9	16	0.00928	16	0.00770
10	27	0.00764	27	0.00724

As shown in Table 1, the ranks of the two contingencies are exactly the same, and the reason for this is that outage of the generating unit at bus 39 is most severe from N-1 contingencies and dominates the insolvability of the system.

Then, control locations should be selected using the above sensitivity information. Load shedding of bus 12 is most effective in Table 1, but bus 12 has a small active power load, 8.50 [MW]; thus, bus 12 is excluded in the set of control locations. In this paper, buses 39, 8, and 7 are arbitrarily chosen for the set of each contingency.

Lastly, the minimum load shedding amount of the selected locations is determined with SOPF. It is noted that voltage magnitude of load buses should not be constrained because voltage levels of load buses are generally quite low at the margin boundary. In the simulation, lower and upper limits of voltages at generating buses are all set to 0.95 and 1.05 respectively; thus, within the limits, reactive generation is re-dispatched. Table 2 shows the load shedding amount of the selected buses at each contingency case. In the earlier results, the difference of ζ at the maximum points between contingencies 1 and 2 is just 0.0128, but that of total amounts of load shedding for restoring is 82.22 [MW], which is about 42% of the required amount of contingency 2.

Table 2 Amount of load shedding at each case for restoring solvability

Bus #	Contingency 1 (Gen. 39)	Contingency 2 (Gen. 39 & Lines 10-11)
39	64.19 [MW]	106.62 [MW]
8	26.73 [MW]	44.96 [MW]
7	25.57 [MW]	43.14 [MW]
Total	116.5 [MW]	194.72 [MW]

After shedding the amount of load according to the above results, each case is tested with continuation power flow to check if the operating point is on the margin boundary. The resulting operating point of each case is near the boundary.

With sensitivity information in Table 1 and the control amount in Table 2, the ζ enhancement at each case is calculated. For this purpose, the following equation is used.

$$\Delta\zeta_{\text{exp}} = \sum_{i \in SB, m} S_i P_{LS, i} \quad (6)$$

where SB, m denotes the set of control locations at contingency m , S_i is the sensitivity of bus i , and $P_{LS, i}$ represents the amount of load shedding of bus i . Table 3 shows the results of the expected and actual ζ enhancements.

Table 3 Expected and real ζ enhancements

Bus #	Expected	Real
Cont. 1	0.084235	0.1567
Cont. 2	0.133045	0.1695

From Table 3, it is known that the actual ζ enhancement of contingency 1 is quite different from the expected one, but that of contingency 2 is closer to the expected one. Thus, it is also known that the ζ enhancement is quite nonlinear. One possible reason is that reactive generation dispatch is not considered in the expectation even though the dispatch is actually performed during SOPF execution. However, the sensitivity is useful for determination of control location because ranking information is utilized.

5. Conclusion

This paper presents a framework for determining minimum load shedding in order to restore power flow solvability in unsolvable cases. The framework uses OCPF and SOPF. OCPF is used to trace the path of operating points with respect to a parameter representing a given outage, and SOPF is utilized to determine the proper amount of load shedding to restore solvability at control locations, which are chosen considering the sensitivity of ζ enhancement obtained at the maximum point.

The goal of this paper is not only to minimize the amount of load shedding but also to reduce the number of control locations. Unsolvable cases in which locations are most effective to control are difficult to analyze, because those cases don't have power flow solutions. Thus, this paper proposes using OCPF as it can deal with severe multiple outages as well as a single equipment outage. In the future, the framework would be extended to be applicable to actual systems.

Acknowledgements

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