Comparison of Small Signal Stability Analysis Methods in Complex Systems with Switching Elements

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Abstract - A new small signal stability analysis method for eigenvalue analysis is presented. This method utilizes the Resistive Companion Form (RCF) for the computation of the transition matrix over a specified time interval, which corresponds to a single cycle operation of the system. This method is applicable to any system, with or without switching element. An illustrative example of the method is presented and the eigenvalues are compared with those of the conventional state space method (analog) in order to demonstrate the accuracy of the proposed eigenvalue analysis method. Also, the variations of oscillation modes that are caused by the switching operation can be precisely analyzed using this method.

Keywords: Eigenvalue, RCF methods, Small Signal Stability Analysis, Switching Element

1. Introduction

FACTS technology utilizes high power electronic devices to enhance controllability of electric power flow in modern power systems. This has the potential to increase power transfer capability and provide economic loading of existing transmission facilities. These attractive features of FACTS elements also cause unwanted effects, which are mainly waveform distortion and additional oscillation modes. These effects are generated by the switching operation of power electronic devices.

In conventional small signal stability analysis, a system is assumed to be invariant and state space equations are used to calculate the eigenvalues of the state matrix. However, when a system contains switching elements such as FACTS devices, it becomes a non-continuous system. In this case, a mathematically rigorous approach to a system's small signal stability analysis is through eigenvalue analysis of the system periodic transition matrix based on the discrete system analysis method. In this paper, the Resistive Companion Form (RCF) method is used to analyze the small signal stability of a non-continuous system including switching elements. To demonstrate the relative merits of the proposed method, a comparison of the conventional state space method and RCF method is presented for application systems both with and without switching elements.

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2. Resistive Companion Form (Rcf) Method

For small signal stability analysis, any power system device is described with a set of algebraic differential integral equations. These equations can be arranged in the following general form:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = \begin{bmatrix} f_1(v(t), \dot{y}(t), v(t), y(t), u(t)) \\ f_2(v(t), \dot{y}(t), v(t), y(t), u(t)) \end{bmatrix} \quad (1)$$

where,

$i(t)$: vector of terminal currents,

$v(t)$: vector of terminal voltages,

$y(t)$: vector of device internal state variables,

$u(t)$: vector of independent controls

This form includes two sets of equations, which are named external equations and internal equations, respectively. The terminal currents appear only in the external equations and the device state variables consist of two sets: external states (i.e. $v(t)$) and internal states (i.e. $y(t)$).

An example of the above modeling is a switching device represented with linear elements. Between switchings, the model is described with a linear differential equation of the form:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} \quad (2)$$

Equation (1) is integrated using a suitable numerical integration method such as the trapezoidal method. Assuming an integration time step h, the result of the integration is manipulated to be in the following form:

$$\begin{bmatrix} i(t) \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} v(t-h) \\ y(t-h) \end{bmatrix} - \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} i(t-h) \\ 0 \end{bmatrix} \quad (3)$$

To consider the connectivity constraints among the devices of the system, Kirchoff's current law is applied to each node of the system. The application of KCL at each node will result in elimination of all device terminal currents. The overall network equation has the form:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{s11} & Y_{s12} \\ Y_{s21} & Y_{s22} \end{bmatrix} \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} - \begin{bmatrix} P_{s11} & P_{s12} \\ P_{s21} & P_{s22} \end{bmatrix} \begin{bmatrix} v(t-h) \\ y(t-h) \end{bmatrix} - \begin{bmatrix} Q_1(t-h) \\ Q_2(t-h) \end{bmatrix} \quad (4)$$

or the equivalent:

$$\begin{bmatrix} v(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} Y_{s11} & Y_{s12} \\ Y_{s21} & Y_{s22} \end{bmatrix}^{+} \begin{bmatrix} P_{s11} & P_{s12} \\ P_{s21} & P_{s22} \end{bmatrix} \begin{bmatrix} v(t-h) \\ y(t-h) \end{bmatrix} + \begin{bmatrix} Y_{s11} & Y_{s12} \\ Y_{s21} & Y_{s22} \end{bmatrix}^{+} \begin{bmatrix} Q_1(t-h) \\ Q_2(t-h) \end{bmatrix} \quad (5)$$

where the superscript + indicates a generalized inverse matrix. Note that the above equation represents the state transition equation for the entire system from time t-h to time t. The above linear equation form is the resistive companion form that results from the trapezoidal integration method. The transition matrix is:

$$\begin{bmatrix} Y_{s11} & Y_{s12} \\ Y_{s21} & Y_{s22} \end{bmatrix}^{+} \begin{bmatrix} P_{s11} & P_{s12} \\ P_{s21} & P_{s22} \end{bmatrix} \quad (6)$$

Eigenvalue analysis of the transition matrix provides the small signal stability of the system.

In general, we are interested in the transition matrix over at least one period of system operation. The proposed method provides an algorithm for the recursive computation of the transition matrix over a desired time period and around the operating conditions of the system. The entire transition matrix over a preferred time period can be performed by sequential substitution of the transition matrix state variables in each time step. The overall transition matrix has the form:

$$\Phi_T(t_n,t_0) = \Phi_n(t_n,t_{n-1}) \cdot \Phi_{n-1}(t_{n-1},t_{n-2}) \cdots \Phi_2(t_2,t_1) \cdot \Phi_1(t_1,t_0) \quad (7)$$

where, $\Phi_i(t_i,t_{i-1})$ is the transition matrix of the specified time step.

The location of an eigenvalue of the transition matrix indicates the nature of the mode. In order to interpret the eigenvalues in terms of modal damping factors and natural frequencies, we can use eigenvalue mapping between the transition matrix eigenvalue and state space eigenvalue. It is known that:

$$\lambda_d = e^{\lambda_c T} = e^{-\alpha T} e^{-\beta T} = re^{j\theta} \quad (8)$$

where, λ_d and λ_c are the eigenvalues of the transition matrix (i.e. discrete system) and state space matrix (i.e. continuous system), respectively, while T is the 60Hz period, and $\lambda_c = -\alpha + j\beta$.

The magnitude r is related to the damping factor α and the angle θ is related to the angular frequency β. Therefore, the eigenvalues of transition matrix have the effect of mapping those of state space matrix to unit circle. It implies that highly damped modes are identified with eigenvalues near the center of the unit circle, stable oscillatory modes are identified with eigenvalues within the unit circle and unstable modes are identified with eigenvalues outside the unit circle.

3. Application Examples

To compare the eigenvalues of the transition matrix and the state space matrix, two complex systems with and without switching elements are used.

3.1 Complex system without switching element

The application system without switching element is shown in Fig. 1. The parameters of the application system are:

$$R_1 = 10.0[\Omega], R_2 = 50.0[\Omega], R_3 = 30.0[\Omega], R_4 = 20.0[\Omega]$$
$$L_1 = 0.05[H], L_2 = 0.1[H], L_3 = 0.2[H],$$
$$C_1 = 0.002[F], C_2 = 0.0005[F], V = 110.0[V]$$

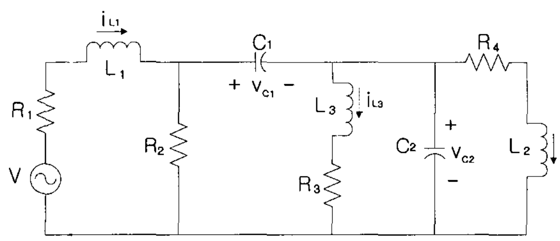

Fig. 1 Complex system without switching element

State space method

The state space equations of Fig. 1 are:

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{i}_{L3} \\ \dot{V}_{C1} \\ \dot{V}_{C2} \end{bmatrix} = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & 0 & \frac{-1}{L_1} & \frac{-1}{L_1} \\ 0 & \frac{-R_4}{L_2} & 0 & 0 & \frac{1}{L_2} \\ 0 & 0 & \frac{-R_3}{L_3} & 0 & \frac{1}{L_3} \\ \frac{1}{C_1} & 0 & 0 & \frac{-1}{C_1R_2} & \frac{-1}{C_1R_2} \\ \frac{1}{C_2} & \frac{-1}{C_2} & \frac{-1}{C_2} & \frac{-1}{C_2R_2} & \frac{-1}{C_2R_2} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ i_{L3} \\ V_{C1} \\ V_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1}V \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

RCF method

From the circuit diagram, the state transition equations are:

$$\begin{bmatrix} L_1+\frac{hR_1}{2} & 0 & 0 & \frac{h}{2} & \frac{h}{2} \\ 0 & L_2+\frac{hR_4}{2} & 0 & 0 & \frac{-h}{2} \\ 0 & 0 & L_3+\frac{hR_3}{2} & 0 & \frac{-h}{2} \\ \frac{-h}{2} & 0 & 0 & C_1+\frac{h}{2R_2} & \frac{h}{2R_2} \\ \frac{-h}{2} & \frac{h}{2} & \frac{h}{2} & \frac{h}{2R_2} & C_2+\frac{h}{2R_2} \end{bmatrix} \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \\ i_{L3}(t) \\ V_{C1}(t) \\ V_{C2}(t) \end{bmatrix} + \begin{bmatrix} \frac{-h}{2}V(t) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} L_1-\frac{hR_1}{2} & 0 & 0 & \frac{-h}{2} & \frac{-h}{2} \\ 0 & L_2-\frac{hR_4}{2} & 0 & 0 & \frac{h}{2} \\ 0 & 0 & L_3-\frac{hR_3}{2} & 0 & \frac{h}{2} \\ \frac{h}{2} & 0 & 0 & C_1-\frac{h}{2R_2} & \frac{-h}{2R_2} \\ \frac{h}{2} & \frac{-h}{2} & \frac{-h}{2} & \frac{-h}{2R_2} & C_2-\frac{h}{2R_2} \end{bmatrix} \begin{bmatrix} i_{L1}(t-h) \\ i_{L2}(t-h) \\ i_{L3}(t-h) \\ V_{C1}(t-h) \\ V_{C2}(t-h) \end{bmatrix} + \begin{bmatrix} \frac{h}{2}V(t-h) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From the above equations, the transition matrix can be calculated as (6).

Comparison of eigenvalues

The eigenvalues of the state space method and transition matrix are compared from Table 1 to Table 3. In this example, the time step h is defined as .0001 sec and the eigenvalues of the state space method are transformed into a unit circle.

From the above tables, the largest error ratio in Table 3 is 0.005584%. Therefore, as in the complex system without switching elements, the eigenvalues of the state space method and RCF method are almost the same.

Table 1 Comparison of eigenvalues, t=.0001 sec

Mode	RCF Method	State Space Method	Error Ratio (%)
1	.987609 + j .026216	987609 + j .026216	.000208
2	.987609 - j .026216	.987609 - j .026216	.000208
3	.981543	.981544	.000055
4	.997399	.997399	.000005
5	.985573	.985573	.000023

Table 2 Comparison of eigenvalues, t= 0.0002 sec

Mode	RCF Method	State Space Method	Error Ratio (%)
1	.974697 +j .051779	.974681 + j .051783	.001654
2	.974697 - j .051779	.974681 - j .051783	.001654
3	.963425	.963425	.000428
4	.994806	.994806	.000001
5	.971352	.971352	.000206

Table 3 Comparison of eigenvalues, t= 0.0003 sec

Mode	RCF Method	State Space Method	Error Ratio (%)
1	.961297 +j .076683	.961245 + j .076694	.005584
2	.961297 - j .076683	.961245 - j .076694	.005584
3	.945634	.945648	.001454
4	.992219	.992219	.0
5	.957334	.957341	.000692

3.2 Complex system with switching elements

The application system with switching elements is shown in Fig. 2. The parameters of the application system are:

$$R_1 = 20[\Omega], R_2 = 40[\Omega], R_3 = 30[\Omega], L_1 = 0.05[H],$$
$$L_2 = 0.1[H], C_1 = 0.2[F], V_s = 110[V]$$

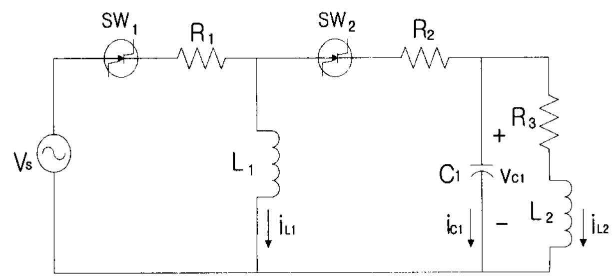

Fig. 2 Complex system with switching elements

3.2.1 Case 1 (SW 1 : close, SW 2 : open)

State space method

The state space equations of Fig. 2 are:

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{V}_{c1} \end{bmatrix} = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & 0 \\ 0 & \frac{-R_3}{L_2} & \frac{1}{L_2} \\ 0 & \frac{-1}{C_1} & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \end{bmatrix} + \begin{bmatrix} \frac{V_s}{L_1} \\ 0 \\ 0 \end{bmatrix}$$

RCF method

From the circuit diagram, the state transition equations are:

$$\begin{bmatrix} 1+\frac{hR_1}{2L_1} & 0 & 0 \\ 0 & 1+\frac{hR_3}{2L_2} & \frac{-h}{2L_2} \\ 0 & \frac{h}{2C_1} & 1 \end{bmatrix} \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \\ V_{C1}(t) \end{bmatrix} + \begin{bmatrix} \frac{-hV_s(t)}{2L_1} \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{hR_1}{2L_1} & 0 & 0 \\ 0 & 1-\frac{hR_3}{2L_2} & \frac{h}{2L_2} \\ 0 & \frac{-h}{2C_1} & 1 \end{bmatrix} \begin{bmatrix} i_{L1}(t-h) \\ i_{L2}(t-h) \\ V_{C1}(t-h) \end{bmatrix} + \begin{bmatrix} \frac{h}{2L_1}V_s(t-h) \\ 0 \\ 0 \end{bmatrix}$$

3.2.2 Case 2 (SW 1: open, SW 2: close)

State space method

The state space equations of Fig. 2 are:

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{V}_{c1} \end{bmatrix} = \begin{bmatrix} \frac{-R_2}{L_1} & 0 & \frac{1}{L_1} \\ 0 & \frac{-R_3}{L_2} & \frac{1}{L_2} \\ \frac{-1}{C_1} & \frac{-1}{C_1} & 0 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_{C1} \end{bmatrix}$$

RCF method

From the circuit diagram, the state transition equations are:

$$\begin{bmatrix} 1+\frac{hR_2}{2L_1} & 0 & \frac{-h}{2L_1} \\ 0 & 1+\frac{hR_3}{2L_2} & \frac{-h}{2L_2} \\ \frac{h}{2C_1} & \frac{h}{2C_1} & 1 \end{bmatrix} \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \\ V_{C1}(t) \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{hR_2}{2L_1} & 0 & \frac{h}{2L_1} \\ 0 & 1-\frac{hR_3}{2L_2} & \frac{h}{2L_2} \\ \frac{-h}{2C_1} & \frac{-h}{2C_1} & 1 \end{bmatrix} \begin{bmatrix} i_{L1}(t-h) \\ i_{L2}(t-h) \\ V_{C1}(t-h) \end{bmatrix}$$

From the above equations, the transition matrix can be calculated as (6).

Comparison of eigenvalues

The eigenvalues of the state space method and transition matrix are compared from Table 4 to Table 7. In this example, the time step h is also defined as .0001 sec and all eigenvalues of the state space method are transformed into a unit circle in Table 4 and Table 5, while all the eigenvalues of the RCF method are transformed into a S-plane in Table 6 and Table 7.

Table 4 Comparison of eigenvalues, t=0.0006 sec in Case 1

Mode	RCF Method	State Space Method	Error Ratio (%)
1	.99989	.99989	.0
2	.83534	.83535	.00134
3	.78660	.78662	.00318

In Table 4, the eigenvalues are calculated at t=0.0006 sec in Case 1 (SW1 is closed and SW2 is opened), which are from 0 to 0.0006 sec with a time step of 0.0001 sec. It is assumed that the errors of the transition matrix by sequential substitution of state variables in each time step will be the largest at 0.0006 sec. In this case, the largest error ratio between the state space method and RCF method is 0.00318%, which means that the eigenvalues from the two methods are almost the same.

Table 5 Comparison of eigenvalues, t = 0.001 sec in Case 2

Mode	RCF Method	State Space Method	Error Ratio (%)
1	.99988	.99988	.00089
2	.88697	.88697	.0
3	.72606	.72618	.01706

In Table 5, the eigenvalues are calculated at t=0.001 sec in Case 2 (SW1 is opened and SW2 is closed), which are from 0.0006 to 0.001 sec with a time step of 0.0001 sec. It is also assumed that the errors of the transition matrix by sequential substitution of state variables in each time step will be the largest at 0.001 sec. In this case the largest error ratio between the state space method and RCF method is 0.01706%, which means that the eigenvalues from the two methods are almost the same.

Table 6 Eigenvalues of Case 1 and Case 2 by state space method (S-plane)

Mode	τ=.0006	τ=.001
1	−.16675	−.29187
2	−299.83323	−299.83312
3	−399.99999	−799.87498

In Table 6, all the eigenvalues of the state space method in Case 1 and Case 2 are shown in a S-plane. It is clear that there is no relation between the eigenvalues of Case 1 and those of Case 2 caused by the switching action in the state space method.

Table 7 Eigenvalues of RCF method including switching effect (S-plane)

Mode	τ=.0006	τ=.0007	τ=.0008	τ=.0009	τ=.001
1	-.16675	-.1498011	-.1792794	-.1876583	-.1954392
2	-299.8332	-262.3736	-299.8555	-299.8555	-299.8555
3	-399.9999	-457.2448	-500.1340	-533.4900	-560.1738

The variation of eigenvalues by the RCF method in each time step is shown in Table 7. All the eigenvalues are transformed into a S-plane. To compare the eigenvalues of Tables 6 and 7, the loci of eigenvalues in Table 7 by the RCR method in Case 2 start from those of 0.0006 sec and become closer to those of 0.001 sec in Table 6. It is clear that the variation of eigenvalues in each time step in Table 7 is caused by the switching operation. These results are impossible to analyze by the state space method. Upon comparing the eigenvalues of the state space method and RCF method at 0.001 sec, mode 2 is almost the same. However, modes 1 and 3 of the state space method and RCF method are different and the errors are somewhat significant.

4. Conclusion

The eigenvalues from the conventional state space method and RCF method are compared in small signal stability analysis. Those are almost the same in continuous system analysis. But, in non-continuous systems including switching elements, the eigenvalues from the state space method and RCF method are different. The RCF method can calculate exactly the variation of oscillation modes after switching action. Therefore, RCF method is a useful one to analyze a non-continuous systems including switching elements in small signal stability analysis.

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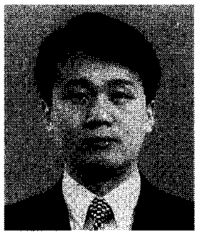

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