Focal Plane Irradiance from MCF in Millimeter Wave Systems

이종길*

Jong Gil Lee*

본 연구는 정보통신부 지원 정보통신기초기술연구지원사업에 의하여 수행되었음

ABSTRACT

Millimeter waves are potentially useful for high resolution ranging and imaging in low optical visibility conditions such as fog and smoke. Also, They are used for wide band communications. However, it is necessary to develop a theoretical and experimental understanding of millimeter wave propagation to assess the performance of millimeter wave systems. The intensity fluctuations and mutual coherence function (MCF) describe atmospheric effects on the millimeter wave propagation. Using the quasi-optical method (QOM), an efficient and practical method was suggested to obtain the intensity distribution of the antenna focal plane from MCF which can be determined using meteorological data.

요 약

밀리미터파는 안개 또는 연기등에 의한 시계불량 환경에서의 고해상도 거리 및 영상정보 획득에 사용될수 있으며 광대역 통신에서도 활용되고 있다. 그러나 밀리미터파 시스템의 성능평가를 위해서는 밀리미터파 전파에 관한 이론 및 실험적인 고찰을 필요로 한다. 밀리미터파의 강도변화 및 MCF는 대기현상이 밀리미터파 전파에 미치는 영향을 표시한다. 본 논문에서는 QOM 기법을 이용하여 기상자료에 의하여 얻어질수 있는 MCF 로부터 안테나 초점평면에서의 강도분포를 효율적으로 구할수 있는 실질적인 방법을 제안하였다.

키워드

MCF, millimeter wave, intensity, propagation

I. Introduction

Millimeter waves are being used for wide band communication and other special purposes. However, it is necessary to develop a theoretical and experimental understanding of electromagnetic wave propagation at millimeter wavelengths through the atmosphere in order to assess the performance of millimeter wave systems. A general theory of beam wave propagation through an atmosphere was presented in [1]. This theory accounts for the finite aperture size of the source and describes the propagation of five different wave field configurations through

*인천대학교 정보통신공학과
접수일자 : 2004. 3. 30

an atmosphere taken to have mean and fluctuating dispersive and absorbing components. In [2] a general model of beam wave propagation was extended and related to common experimental quantities capable of being measured via a parabolic reflector antenna. These experimentally measurable quantities i.e., intensity covariance angle of arrival and the mutual coherence function are very important because atmospheric effects on the propagation of electromagnetic waves are usually described in terms of these quantities. Of particular interest here is the *mutual coherence function* defined as the cross-correlation function of the complex fields in a direction transverse to the direction of propagation.

Following the Rytov method [3], the MCF, Γ, is given by

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2) = E_0(\mathbf{r}_1) E_0^*(\mathbf{r}_2) \times \exp[-\frac{1}{2} D(\mathbf{r}_1, \mathbf{r}_2)] \quad (1)$$

where E_0 and D are an unperturbed field component and a wave structure function respectively. Here $\mathbf{r}$ is the position vector and E represents the electric field. From Eq.(1), we see that the MCF describes the loss of coherence of an initially coherent wave propagating in a turbulent medium. As a result, the MCF is important for a number of practical applications. It determines the S/N ratio of an optical heterodyne detector, the limiting resolution obtainable along an atmospheric path and the mean irradiance distribution from an initially coherent wave emanating from a finite aperture. This MCF can be obtained from measured temperature and humidity structure constants using the wave structure function. However, it is important to compute the focal plane irradiance distribution from the given MCF to investigate all weather performance of millimeter wave systems. Therefore, an efficient method was suggested to obtain the focal plane irradiance distribution from the measured MCF.

II. Quasi-Optical Method(QOM)

The MCF is related to fluctuating inhomogenities in the index of refraction of the propagation medium, so meteorological information is an important factor in assessing all-weather performance of millimeter wavelength systems. The transverse coherence length (defined as the transverse separation at which the MCF is reduced by e^{-1}) can be significantly reduced by fog, smoke, etc. If we define ρ_0 as the transverse separation at which the atmospheric MCF is reduced by e^{-1}, the minimum resolvable length at distance Z from an observer is well known to be $Z/(k\rho_0)$. Thus, a decrease of coherence length means a decrease in the resolution and an increase of beam wave spreading. The QOM can be used here to get instantaneous focal plane irradiance distribution from the MCF, that is, the spatial Fourier transform of the product of the electromagnetic fields at the aperture plane.

The intensity I(q) at a point q on the focal plane is given by

$$I(q) = [k/(2\pi f)]^2 \times \int\int E(\mathbf{r}) E^*(\mathbf{r}') W(\mathbf{r}) W^*(\mathbf{r}') \times \exp[-ik\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')/f]\, d^2\mathbf{r}\, d^2\mathbf{r}' \quad (2)$$

The electric field E is, however, a random function due to the statistical nature of the atmosphere through which it propagated. Thus only the ensemble averaged intensity can be considered. Ensemble averaged equation of (2) yields

$$\langle I(q)\rangle = [k/(2\pi f)]^2 \int\int \Gamma(\mathbf{r}, \mathbf{r}') W(\mathbf{r}) \times W^*(\mathbf{r}') \exp[-ik\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')/f]\, d^2\mathbf{r}\, d^2\mathbf{r}' \quad (3)$$

where $\Gamma = \langle E(\mathbf{r}) E^*(\mathbf{r}')\rangle$ is the mutual coherence function of the electric field at the aperture plane. It is the mutual coherence function that describes the effects of the fluctuating electrical parameters of the atmosphere on the propagating electromagnetic wave. Thus, if one can solve the integral equation (3), the focal plane intensity can be obtained from the measured MCF.

Changing variables in Eq.(3) to $\boldsymbol{\rho}=\boldsymbol{r}-\boldsymbol{r}'$, Eq.(3) becomes

$$\langle I(q)\rangle = [k/(2\pi f)]^2 \iint \Gamma(\boldsymbol{r}, \boldsymbol{r}-\boldsymbol{\rho}) W(\boldsymbol{r}) \times W^*(\boldsymbol{r}-\boldsymbol{\rho}) \exp[-ik\boldsymbol{q}\cdot\boldsymbol{\rho}/f]\, d^2\boldsymbol{\rho}\, d^2\boldsymbol{r} \quad (4)$$

Here, the antenna transfer function, $H_A(\boldsymbol{\rho})$ simply represents the area of overlap of two circles of diameter d, one having a center at the origin of the $\boldsymbol{r}$ coordinate system and the other having its origin at $\boldsymbol{\rho}$ in the same system. Thus the r integration yields the area of overlap between the two circles given by

$$H_A(\boldsymbol{\rho}) = \int W(\boldsymbol{r})\, W^*(\boldsymbol{r}-\boldsymbol{\rho})\, d^2\boldsymbol{r} = (d^2/2)[\cos^{-1}(\rho/d) - (\rho/d)(1-(\rho/d)^2)^{1/2}] \quad (5)$$

while the value of $H_A(\boldsymbol{\rho})$ is 0 for $\rho > d$.

In the case that $\Gamma(\boldsymbol{r}, \boldsymbol{r}-\boldsymbol{\rho})$ is independent of the aperture coordinate, that is, the statistical inhomogenity is negligible, Eq.(4) decouples and a simplified form can be derived to be

$$\langle I(q)\rangle = k^2/(2\pi f^2) \times \int_0^\infty \Gamma(\rho)\, H_A(\rho)\, J_0(k\rho q/f)\rho d\rho \quad (6)$$

III. A Special Method for the Integration of Rapidly Oscillatory Functions

$\langle I(q)\rangle$ in Eq. (6) is an integral of a rapidly osciilatory function; i.e., a Bessel function that gives numerous local maxima and minima over the range of integration. As ρ goes to infinity, the integrand function looks less and less like a polynomial of low degrees, and suggesting that special methods should be used.

If Eq. (6) can be solved numerically, one can theoretically predict the intensity distribution of the focal plane. What one wants in computing Eq. (6) is not methods that have small absolute error, but methods that have small relative error. Only a few methods are available and applicable in this case. They are Longmans method [4], Piessens quadrature formula [5] and Linzs method based on Abel transform [6]. All these methods have weak points when applied to Eq. (6). Longmans method uses Gauss-Legendre formulas, but this method cannot be used for the integration involving Bessel functions of large arguments. the Linzs method is inefficient because it requires Abel integration. The use of Piessens quadrature formula is acceptable but it needs the storage of a large number of abscissae and weights. Piessens method splits the integral into a sum of subintegrals. For each subintegral the Gaussian quadrature formula is used to evaluate the subintegral from the given abscissae and weights. This method is very useful if the integration range is not large.

Even though the aperture plane is not large, one can see that Piessens method is inefficient because the argument of Bessel function is a constant determined by variables k, q, f, ρ and usually the values of (kq/f) is very large. This problem cannot be solved by Piessens method without storing a huge amount of numbers. However, if one can approximate the Bessel function as a rational function for large arguments, the usual methods such as the Gauss-Legendre formula will be available. Thus a compromise must be made to get accurate results over the whole range, which means each method must be used as the argument changes.

To reduce Eq. (6) to a numerically simpler task, some changes and approximations must be made. Let L=kq/f and $f(\rho)=\Gamma(\rho)\, H_A(\rho)$, then Eq. (6) becomes

$$\langle I(L)\rangle = k^2(2\pi f^2)\int_0^D f(\rho)\, J_0(L\rho)\rho d\rho \quad (7)$$

where D is the diameter of aperture plane, i.e, the actual integration interval. The asymptotic behaviour of the Bessel function, $J_0(x)$ suggests an approximation of the form,

$$J_0(x)=\sqrt{2/(\pi x)}\ \psi_0(x)\sin[\ \phi_0(x)] \tag{8}$$

where $\psi_0(x)=1+R(x)$ and $\phi_0(x)=x+\pi/4+Q(x)$. It is difficult to determine Q(x) and R(x) so that Eq. (6) is a good approximation over the whole range interval. For this reason, we limit the approximation interval of this method to $[\ j_{0,5},\infty)$ where $j_{0,5}$ is the fifth zero of zero order Bessel function. This means that Eq. (7) must be divided into two subintegrals as shown below,

$$\langle I(L)\rangle = k^2(2\pi f^2)\left[\int_0^{j_{0,5}/L} f(\rho)\ J_0(L\rho)\rho d\rho + \int_{j_{0,5}/L}^{D} f(\rho)\ J_0(L\rho)\rho d\rho\right] \tag{9}$$

where the second integral can be rewritten as

$$\int_{j_{0,5}/L}^{D} f(\rho)\ J_0(L\rho)\rho d\rho = \int_0^{DL-j_{0,5}} S_L(\rho)\times \sin(\rho)d\rho + \int_0^{DL-j_{0,5}} C_L(\rho)\cos(\rho)d\rho \tag{10}$$

where the integrands are described in the following:

$$S_L(\rho)=F_L(\rho)\cos[G(\rho)],\quad C_L(\rho)= F_L(\rho)\sin[G(\rho)],\ G(\rho)=j_{0,5}+(\pi/4)+ Q(\rho+j_{0,5}),\ F_L(\rho)=\sqrt{2\pi(\rho+j_{0,5})}\times f[(\rho+j_{0,5})/L]\ \psi_0(\rho+j_{0,5})/(\pi L^2) \tag{11}$$

The function, R(x) and Q(x) are the rational functions represented by

$$Q(x)=\frac{\sum_{i=0}^{k} a_i x^{2i}}{x+\sum_{i=1}^{k} b_i x^{2i+1}},$$

$$R(x)=\frac{\sum_{i=0}^{k} c_i x^{2i}}{x+\sum_{i=1}^{k} d_i x^{2i+1}} \tag{12}$$

where a, b, c, and d are the coefficients [7]. The first part of Eq. (9) can be solved using Piessen's method. The integral can be written as a sum of subintegrals; i.e.,

$$1/\ L^2\int_0^{j_{0,5}} f(\rho/L)\ \rho J_0(\rho)\rho d\rho = 1/\ L^2\sum_{s=1}^{5}(-1)^{s+1} \int_{j_{0,s-1}}^{j_{0,s}} f(\rho/L)\ \rho J_0(\rho)\rho d\rho \tag{13}$$

where each subintegral is approximated by Gaussian quadrature formula.

IV. Results and Error Analysis

If the index of refraction of the medium can be calculated from meteorological parameters such as temperature and humidity structure constants, the wave structure functions explained for the different types of waves in ref. [2] will be clearly determined. The index of refraction structure parameter, C_n^2 can be defined as a mean square statistical average of the difference in the index of refraction between two points separated by a distance r, given by

$$C_n^2=\langle\ (\ n_1-\ n_2)^2\rangle/\ r_{12}^{2/3} \tag{14}$$

where the angle bracket denotes an ensemble average. Since C_n^2 is difficult to measure directly at millimeter wavelengths, a convenient alternative is to measure the temperature structure parameter, C_T^2 which is related by

$$C_n^2=(7.9\times 10^{-6}P/\ T^2)C_T^2 \tag{15}$$

where P is the atmospheric pressure in millibars and T is the atmospheric temperature in degree Kelvin. Using the MCF for a plane wave propagating through the atmosphere, i.e.,

$$\Gamma(\rho)=A\exp(-b\,\rho^{5/3}) \tag{16}$$

where $b = 1.456\ k^2 R C_n^2$ and R and k represents the total propagation distance and the wave number respectively.

For the given MCF, the intensity distribution in the focal plane was obtained as bvaries. The results in Figure 1 and Figure 2 show that the intensity is blurred by atmospheric effects. This blurring parameter is denoted as b.

One can expect that received intensity pattern will spread out more for a larger b. If b equals zero, the intensity is not affected by atmospheric conditions and just describes the diffraction pattern of a circular aperture which is given by

$$I(q) = \frac{(2kD)^2 J_1^2(0.5kDq/f)}{(4kDq)^2} \qquad (17)$$

Using Eq. (17), numerical errors of the suggested method to obtain intensity were checked against the exact solution available for b=0.0 . Results are given in Figure 3 and it shows that the suggested method is accurate enough except the vicinity of null regions where the percentage errors are inherently exaggerated as having large values.

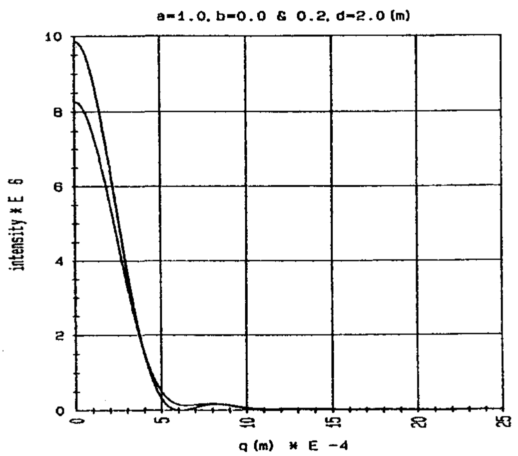

Fig. 1 The intensity distribution of a focal plane with the aperture diameter of 2.0 m for b=0.0, 0.2

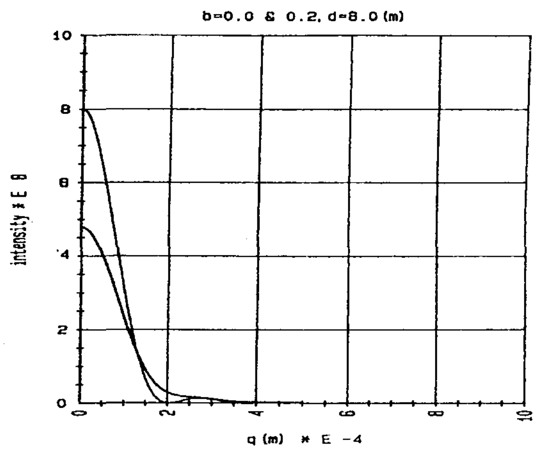

Fig. 2 The intensity distribution of a focal plane with the aperture diameter of 6.0 m for b=0.0, 0.2

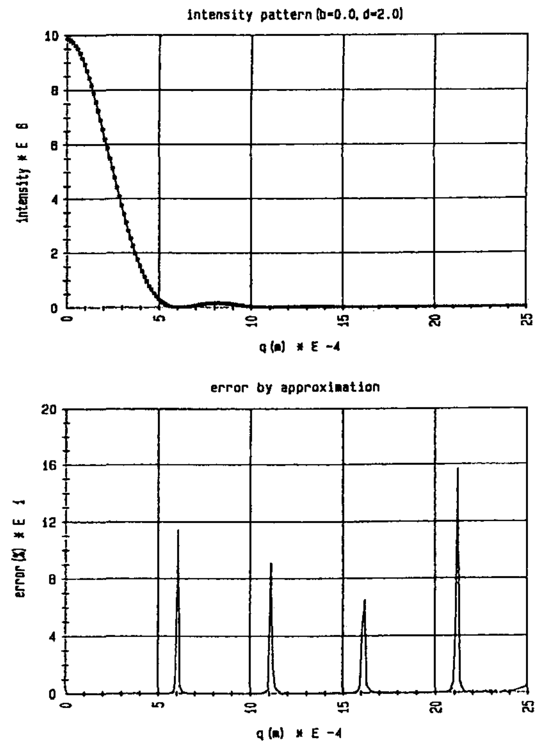

Fig. 3 The percentage error between the exact solution and the suggested approximation method

Ⅴ. Concluding Remarks

In this paper, a practical method is derived and suggested to obtain the intensity distribution of a focal plane from the experimentally measured MCF. These results can be used to confirm whether the experimentally measured MCF is correct. The suggested method is very convenient and simple to predict the intensity distribution from the MCF measurements using weather data. This will enable to check the accuracy of existing weather turbulence model in wave propagation by comparing with the intensity from the flux measurements. Also, it is shown that errors of this approximation method can be considered to be negligible practically.

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저자소개

이종길(Jong Gil Lee)

1982년 2월 연세대학교 전자공학과(공학사)
1985년 1월 미국 Case Western Reserve University, Electrical Eng. (M.S.)
1990년 5월 미국 Clemson University, Electrical Eng.(Ph.D.)
1991년 1월 ~ 1993년 2월 삼성전자 특수연구소
1993년 3월 ~ 현재 인천대학교 정보통신공학과 교수
※관심 분야: 무선통신, 레이다신호처리, 통신시스템