Vibration of Initially Stressed Beam with Discretely Spaced Multiple Elastic Supports

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Vibration behavior of an initially stressed beam on discretely spaced multiple elastic supports has been studied and a theoretical formulation of the system is derived using the variational principle. Unlike beams on an elastic foundation, discretely spaced supports can distort the beam mode shapes when the supports have rather large stiffness, i.e. usually expected beam modes cannot be obtained, but rather irregular mode shapes are observed. Conversely, irregular modes can be recovered by changing initial stress. Since support location is closely associated with the dynamic characteristics, this work also discusses eigenvalue sensitivity with respect to the support position and some numerical examples are investigated to illustrate the above findings.

Key Words : Vibration, Initial Stress, Multiple Elastic Supports

1. Introduction

A beam is a very practical mechanical component as its utility can be seen in various fields of industry. A fuel rod in a nuclear fuel assembly, obviously, is understood as a beam since its length is much bigger than its cross sectional dimensions (Chen et al., 1972). The fuel rods in reactor are also initially stressed due to coolant pressure. It is known that the initial stress gives rise to stiffening or weakening of a structure. The traction which causes initial stress on a structure can be the result of gravity loads, as on a long drill pipe, or rotational loads, as on a whirling propeller (Blevins, 1979). The initial stress or strain problem is classical, and the theoretical explanations are found in the reference (Washizu, 1983). It is also known that bending stiffness in a beam or a plate is affected by membrane forces (Cook et al., 1989). For example, in the case of a beam structure, tensile force acting axially on the ends of the structure increase buckling resistance and natural frequencies, while axially compressive forces decrease both buckling resistance and natural frequencies. Since initial stress affects a structure's behavior seriously, for control or design it is necessary to predict the behavior or dynamic characteristics of a structure with initial stress.

Since the structure system with initial stress is an important engineering problem, numerous reports have been published. Raju et al. investigated the effects of shear deformation and rotary inertia on the vibration characteristics of a pre-stressed simply supported beam (Raju et al., 1986). Chen et al. studied the relationship between the angle of a curved beam with initial stress and the natural frequency (Chen et al., 1997). Yang et al. solved the stability of initially stressed thick laminated plates, and also studied the effects of twisting and the global material constants on the buckling (Yang et al., 1993). Finite element results for the buckling loads and moderately thick annular plates subjected to in-

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plane compressive loads at the outer edge were presented (Nayar et al., 1994). Carrera researched the effect of transverse normal stress on vibration of multi-layered structures (Carrera, 1999).

The work studied dynamics of a fuel rod structure in a fuel assembly with multiple supports in the initial stress state. The effects of initial stress and discretely spaced supporters on its vibration characteristics are discussed. Unlike the studies about a beam on an elastic foundation, the situation wherein a beam is on discretely spaced elastic supports is analyzed in this paper.

2. Equations of Motion

2.1 General formulation

If a structure is pre-stressed and in equilibrium state, the equilibrium equation, neglecting body force, can be expressed as (Chen et al., 1997; Bolotin, 1963)

$$[\sigma_{ij}^0(\delta_{kj}+u_{k,j}^0)]_{,i}=0 \tag{1}$$

where σ_{ij}^0, δ_{jk}, and μ_j^0 are initial stress tensor, Kronecker delta and initial deformation respectively. And the traction boundary condition is

$$\sigma_{ij}^0(\delta_{kj}+u_{k,j}^0)n_i=p_k^0 \tag{2}$$

where n_i is surface normal vector and p_k^0 is applied axial load. After perturbation these quantities become

$$\begin{aligned}\sigma_{ij}^t&=\sigma_{ij}^0+\sigma_{ij}\\ u_k^t&=u_k^0+u_k\\ p_k^t&=p_k^0+p_k\end{aligned} \tag{3}$$

where the second terms on the right hand sides of the last Eq. (3) are caused by load perturbation.

The potential energy of the perturbed system, which is shown in Fig. 1, on discretely spaced elastic supports, assuming that initial deformation gradient ($u_{i,j}^0$) is small, is

$$\begin{aligned}\Pi=&\frac{1}{2}\int_0^t\int_V(\sigma_{ij}^0+\sigma_{ij})\varepsilon_{ij}dVdt\\ &+\frac{1}{2}\int_0^t\int_V\sum_{i=1}^r k_i u^2\delta(\vec{x}-\vec{x}_i)dVdt\\ &-\int_0^t\int_S(p_i^0+p_i)u_i ds dt\end{aligned} \tag{4}$$

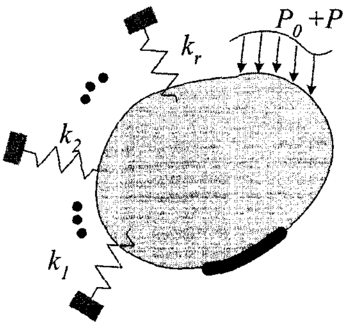


Fig. 1 Elastic body with discretely spaced elastic supports

where k_i is i-th elastic spring constant, r is the number of elastic spring, and δ is Dirac delta function. Note that summation rule is not applicable to the second term in the right hand side of Eq. (4). S denotes the surface where traction is specified. ε_{ij} in the first term of the equation is strain tensor denoted by

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}+u_{s,j}u_{s,j}) \tag{5}$$

Also the kinetic energy of the system, ignoring the influence of rotatory inertia, is written as

$$K=\frac{1}{2}\int_0^t\int_V\sum_{i=1}^3\rho\dot{u}_i^2dVdt \tag{6}$$

where ρ means density. Equation of motion of the system can be derived by applying variation principle. First the variation of potential energy, considering Eqs. (1) and (2), is reduced to

$$\begin{aligned}\delta\Pi=&\int_0^t\int_V(\sigma_{ij}\delta\varepsilon_{ij}+\sigma_{ij}^0u_{s,i}\delta u_{s,j})dVdt\\ &+\int_0^t\sum_{i=1}^r k_i u(t,\vec{x}_i)dt-\int_0^t\int_S p_i\delta u_i dVdt\end{aligned} \tag{7}$$

Neglecting higher order terms to construct linear equation in Eq. (5), it can be written as

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{8}$$

And finally the linearized formulation is obtained as

$$\begin{aligned}\delta\Pi=&\int_0^t\int_V(\sigma_{ij}\delta e_{ij}+\sigma_{ij}^0u_{s,i}\delta u_{s,j})dVdt\\ &+\int_0^t\sum_{i=1}^r k_i u(t,\vec{x}_i)dt-\int_0^t\int_S p_i\delta u_i dVdt\end{aligned} \tag{9}$$

Similarly the variation of the kinetic energy can be derived, and Euler equation of the system is expressed as Eq. (10).

$$[\sigma_{ij}+\sigma^0_{sj}u_{s,i}]_{,j}-\sum_{i=1}^{r}k_i u\delta(\vec{x}-\vec{x}_i)=\rho\ddot{u}_i \quad (10)$$

And the boundary condition can be written as

$$\sigma_{ij}n_j+\sigma^0_{sj}u_{s,j}n_j=p_i \quad (11)$$

2.2 Beams on discretely spaced elastic supports

Figure 2 shows geometry of a beam structure with several supports. It assumes that section properties are not varying along the length and the forces acting on the ends are not changing in magnitude and direction. It is also presumed that shear strains are zero. Displacement field, considering in-plane motion, can be expressed as

$$\begin{aligned} U&=u(x,t)-zw_{,x}(x,t)\\ V&=0\\ W&=w(x,t)\end{aligned} \quad (12)$$

where u, w are the axial and transverse displacements, respectively. z denotes the distance from the neutral axis. Then the stress-strain and strain-displacement relationships are given by

$$\sigma_{xx}=Ee_{xx} \quad (13)$$

$$e_{xx}=u_{,x}-zw_{,xx} \quad (14)$$

where E is Young's modulus. Substituting Eqs. (13), (14) into Eq. (9) leads to

$$\begin{aligned}\delta\Pi=&\int_0^t\int_V(Ee_{xx}\delta e_{xx}+\sigma^0_{xx}u_{s,x}\delta u_{s,x})\,dVdt\\ &+\int_0^t\sum_{i=1}^{r}k_i u(t,x_i)\,dt\end{aligned} \quad (15)$$

Further processing the above equation, the final formulation of the variation of the potential energy can be expressed as

$$\begin{aligned}\delta\Pi=&\int_0^t\int\Big((E+\sigma^0_{xx})Iw_{,xxx}-\sigma^0_{xx}Aw_{,xx}+\sum_{i=1}^{r}k_i w\delta(x-x_i)\Big)\delta w dxdt\\ &-\int_0^t\int(E+\sigma^0_{xx})Au_{,xx}\delta u dxdt\\ &+[(EAu_{,x}+\sigma^0_{xx}Au_{,x})\delta u]_0^L+[(E+\sigma^0_{xx})Iw_{,xx}\delta w_{,x}]_0^L\\ &-[((E+\sigma^0_{xx})Iw_{,xxx}-\sigma^0_{xx}Aw_{,x})\delta w]_0^L\end{aligned} \quad (16)$$

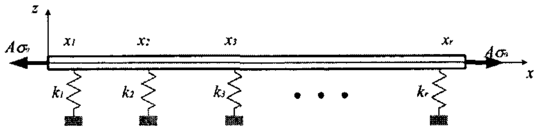


Fig. 2 An initially stressed beam with multiple elastic supports

Considering the kinetic energy, the following equations of motion can be derived:

$$\begin{aligned}&(E+\sigma^0_{xx})Iw_{,xxxx}-\sigma^0_{xx}Aw_{,xx}\\ &+\sum_{i=1}^{r}k_i w\delta(x-x_i)+\rho A\ddot{w}=0\end{aligned} \quad (17)$$

$$(E+\sigma^0_{xx})Au_{,xx}=\rho A\ddot{u} \quad (18)$$

And boundary conditions are

$$\begin{aligned}&A(E+\sigma^0_{xx})u_{,x}|_0=0,\ A(E+\sigma^0_{xx})u_{,x}|_L=0\\ &(E+\sigma^0_{xx})Iw_{,xx}|_0,\ (E+\sigma^0_{xx})Iw_{,xx}|_L=0\\ &((E+\sigma^0_{xx})Iw_{,xxx}-\sigma^0_{xx}Aw_{,x})_0=0,\\ &((E+\sigma^0_{xx})Iw_{,xxx}-\sigma^0_{xx}Aw_{,x})|_L=0\end{aligned} \quad (19)$$

Note that the term, $E+\sigma^0_{xx}$, which was introduced from the initial stress consideration is involved in Eqs. (17) through (19), and it is no better than Eq. (10) and Eq. (11). It can be seen that the initial stress affects the coefficient of the fourth derivative. In most cases the magnitude of the initial stress, $|\sigma^0_{xx}|$, is far below that of the elastic modulus, E. Thus it can be written that

$$E+\sigma^0_{xx}\approx E \quad (20)$$

If external force is perturbed after static equilibrium, the first traction boundary condition in Eq. (19) comes to

$$A(E+\sigma^0_{xx})u_{,x}=\Delta p \quad (21)$$

Obviously the second condition of Eq. (19) is associated with moment and slope. Description of the last condition of Eq. (19) is about shear force and transverse displacement. The traction boundary condition is delineated in Fig. 3, and it is understood that the shear force acting on the boundary is also generated by the initial stress. Note that this work is only interested in transverse motion, therefore axial motion will be ignored.

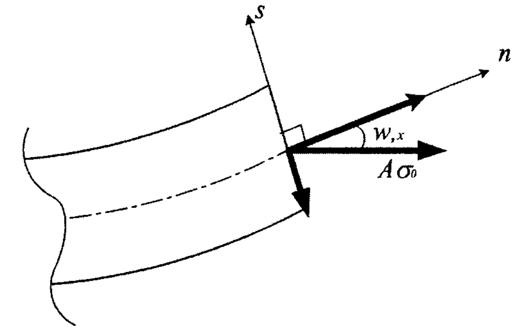


Fig. 3 Traction boundary condition when the applied load is constant ; it can be thought that vertical force is equal to $A\sigma^0 w_{,x}$ when transverse displacement is small

3. Effect of Support Position over Eigenvalue

In the case when the initial stress is present and is not small, design performance should be examined. Eigenvalue is an important dynamic characteristic in vibration and it is dependent on several design parameters. If conventional design parameters such as thickness is not allowed to be changed, it is reasonable to consider shape, boundary conditions, or support locations as its design variables. The concept of shape design method was developed to account for the effects of variation of these configuration design parameters (Haug etc., 1980 ; Lee etc., 1992). This paper focused on the study of eigenvalue sensitivity with respect to support locations. Since the location of each support in a structure is considered as its configuration, shape design methodology can be applied.

Note that, for simplified expression, σ^0_{xx} is replaced with σ^0 from now on. Also note that, presuming the applied initial stress is not large, the approximation of Eq. (20) is adopted. Assuming harmonic motion, $u(x,\ t)=u(x)e^{j\omega t}$, and introducing operators D, and M, Eq. (17) results in the eigenvalue equation given by

$$Du=\lambda Mu \tag{22}$$

It is easy to show that $D=EI\dfrac{d^4}{dx^4}-\sigma^0\dfrac{d^2}{dx^2}+\sum_{i=1}^{r}k_i\delta(x-x_i)$, $M=\rho A$, $\lambda=\omega^2$. If we define functional a and d, which is defined on inner product space, Eq. (22) can be cast in a bilinear form (Hughes, 1987)

$$a(y,\ \bar{y})=\lambda d(y,\ \bar{y}) \tag{23}$$

where y denotes an eigenfunction and $\bar{y}$ is an arbitrary displacement field that satisfies the boundary condition. Introducing inner product symbol ($\langle\bullet,\ \bullet\rangle$), $a(y,\ \bar{y})$ and $d(y,\ \bar{y})$ are defined as

$$\begin{aligned} a(y,\bar{y}) &\equiv \langle Dy,\ \bar{y}\rangle \\ &= \int_0^L\left[EIy_{,xx}\bar{y}_{,xx}-\sigma^0 y_{,x}\bar{y}_{,x}+\sum_{i=1}^{r}k_i y\bar{y}\delta(x-x_i)\right]dx \end{aligned} \tag{24}$$

$$d(y,\ \bar{y})\equiv\langle My,\ \bar{y}\rangle=\int_0^L\rho Ay\bar{y}dx \tag{25}$$

As mentioned before, support locations are chosen as configuration design parameters. Design velocity field, which is the design trajectory of a particle at the beginning, characterizes configuration design. Thus it is necessary to define design velocity field to investigate eigenvalue sensitivity with respect to each support location. Introducing the design velocity, $V(x)$, the design sensitivity for simple eigenvalue can be found as (Haug etc., 1980)

$$\begin{aligned} \lambda' =& 2\iint_\Omega[-c(y,\ \nabla y^TV)+\lambda e(y,\ \nabla y^TV)]d\Omega \\ &+\int_\Gamma[c(y,\ \nabla y^TV)-\lambda e(y,\ \nabla y^TV)](V^Tn)d\Gamma \end{aligned} \tag{26}$$

where c, e are the kernel of a, d respectively in Eq. (24) and Eq. (25). It is understood that Eq. (26) is composed of two parts, domain integral in Ω and boundary integral on Γ. Finally, considering Eqs. (24) ~ (26), the sensitivity of the system can be written as

$$\begin{aligned} \lambda' =& 2\int_0^L\Big[-EIy_{,xx}(\nabla y^TV)_{,xx}+\sigma^0 y_{,x}(\nabla y^TV)_{,x} \\ &-\sum_{i=1}^{r}k_i y(\nabla y^TV)\delta(x-x_i)+\lambda\rho Ay(\nabla y^TV)\Big]dx \\ &+\Big[EIy_{,xx}(\nabla y^TV)_{,xx}-\sigma^0 y_{,x}(\nabla y^TV)_{,x} \\ &+\sum_{i=1}^{r}k_i y(\nabla y^TV)\delta(x-x_i)-\lambda\rho Ay(\nabla y^TV)\Big](V^Tn)\Big|_0^L \end{aligned} \tag{27}$$

It can be seen that the design velocity must have C^1 regularity since second derivatives of the velocity field is involved in Eq. (27). Note that when a velocity field with C^0 regularity is used, Eq. (27) contains singularity. To satisfy the regularity condition, an exponential function as a design velocity is introduced, and it can be rewritten as

$$V(x) = \exp\left(-\frac{(x-\mu_0)^2}{q_0^2}\right)\Delta x \tag{28}$$

where μ_0, q_0 are constants and Δx means infinitesimal movement. In fact the design velocity approaches 0 rapidly as q_0 becomes smaller. It is predicted, as can be seen in Eq. (27), that the boundary movement has no significant effect under the above design velocity condition. In this case, a more simple formulation is possible, and that reduces to

$$\lambda' = 2\int_0^L \Big[-EIy_{,xx}(\nabla y^T V)_{,xx} + \sigma^0 y_{,x}(\nabla y^T V)_{,x} - \sum_{i=1}^{r} k_i y(\nabla y^T V)\delta(x-x_i) + \lambda\rho A y(\nabla y^T V)\Big] dx \tag{29}$$

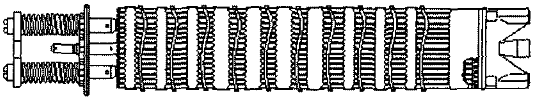


Fig. 4 A fuel assembly with 11 grids and fuel rods

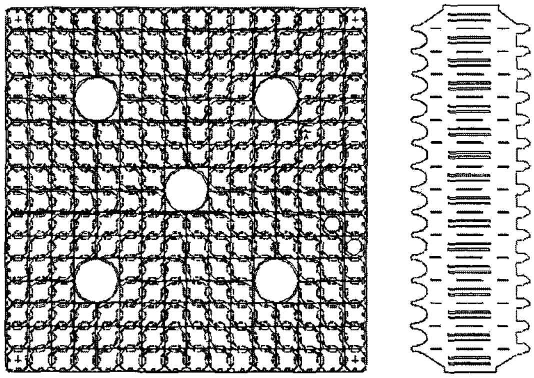


Fig. 5 Details of a grid assembly; plane view (left) and side view (right)

4. Numerical Example

4.1 Nuclear fuel assembly structure

A Nuclear reactor core consists of an array of square fuel assemblies, and the core is cooled and moderated by water at a pressure. Each fuel assembly is installed vertically in the reactor vessel and stands upright on the bottom of core. Each fuel assembly contains an array of fuel rods which are supported at intervals along their length by grid assemblies to maintain the lateral spacing between the rods.

The grid assembly consists of an arrangement of interlocked straps, containing springs and dimples formed within the straps, for gripping the fuel rods and holding them in the proper position within the assembly structure. Each fuel rod is given support at some contact points within each grid by a combination of support dimples and springs which are integral to the straps.

Figure 4 shows one of nuclear fuel assembly consisted of 11 grid assemblies and fuel rods. Details of a grid are plotted in Fig. 5 and rods are inserted into the cells. It is reasonable to simplify a grid assembly as a spring since the grid supports fuel rods elastically. Since it is very difficult to analyze dynamics of an entire fuel assembly, this paper focuses on dynamics of a rod supported by several grid structures.

4.2 Analysis of simplified fuel rod with supports

To simplify the modeling of a fuel rod supported by grids, grids were modeled as linear spring and it is assumed that elastic spring constants are equal and that they are symmetrically placed and equally spaced along the beam as is shown in Fig. 6. The spring constant of elastic support is set to 100 kN/m. The system, actually, can be understood as a simplified fuel rod supported by grids in nuclear fuel assembly which is placed in high pressurized reactor vessel. Pressure in reactor vessel is about -15.5 Mpa which is much smaller than elastic modulus of steel at normal temperature, 200×10^3 Mpa. Thus it is concluded that approximation of Eq. (20) has no significant errors.

Figure 7 shows the mode shape which is different from the usual mode shape of a beam. The irregular shapes are caused by the very stiff elastic supports, but it is an expected phenomenon if the extreme case, $k=\infty$, is considered. Based

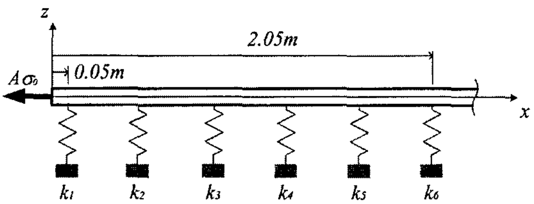
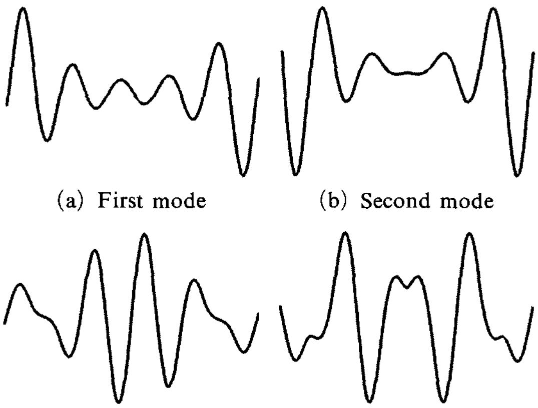


Fig. 6 A beam model for numerical simulation ; it is assumed that all supports are equally spaced and the beam section properties are not varying



(a) First mode (b) Second mode

(c) Third mode (d) Fourth mode

Fig. 7 Irregular beam mode shapes when every spring constant is 100 kN/m and initial stress is free

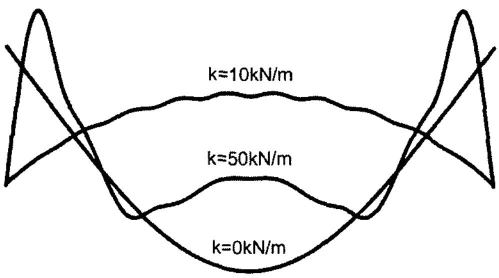
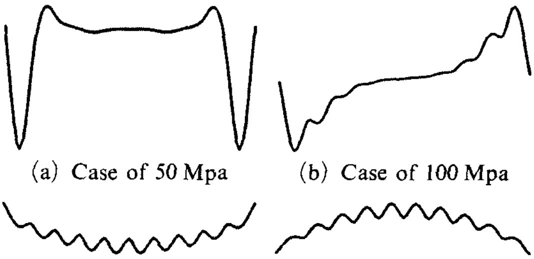


Fig. 8 First modes of beam with different spring constant under zero initial stress

Table 1 First five natural frequencies (Hz) when applied initial stress is increasing

Mode	−30 Mpa	−20 Mpa	−10 Mpa	0 Mpa
1st	134.9	142.7	148.8	155.0
2nd	135.3	142.4	149.0	155.1
3rd	148.1	149.3	155.2	160.9
4th	150.6	155.6	160.4	165.0
5th	158.9	162.7	166.4	169.9



(a) Case of 50 Mpa (b) Case of 100 Mpa

(c) Case of 150 Mpa (d) Case of 200 Mpa

Fig. 9 First modes vs. different initial stresses ; when every spring constant is 100 kN/m, it can be seen that as the initial stress positively increasies, irregular first mode shape becomes ordinary shape with fluctuation

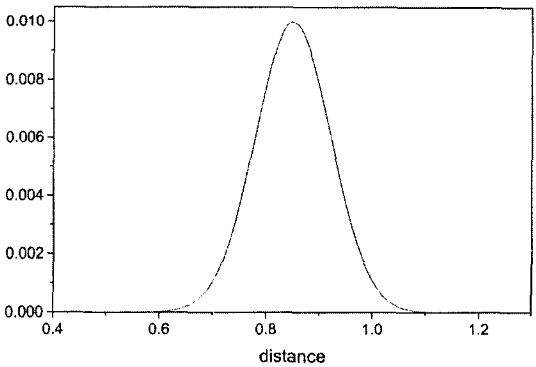


Fig. 10 One of the defined design velocity fields ; a case when center position is located at 0.85 m ; μ_0=0.85, q_0=0.1, Δx=0.01

on several numerical simulations, without initial load, unusual modes appear when stiffness comes to about 10kN/m in the beam structure as shown in Fig. 8. It can be found that, if applied initial stress is positively increased, the irregular modes follow usual shape as shown in Fig. 9. It is interesting to note that global shape is similar to half sine curve but fluctuation occurs around each elastic support in Fig. 9. Table 1 shows some natural frequencies of the structure when applied pressure is −30, −20, −10, and 0 Mpa. It can be seen that eigenvalue becomes larger as applied pressure approaches more positive values.

To identify sensitivity with respect to support position, Eq. (29) is evaluated when initial stress

Table 2 Sensitivity of the first five eigenvalues with respect to each support position

Initial Stress	Mode	D1	D2	D3	D3	D5	D6
−30 Mpa	1	1795.1	−28762.1	−12596.2	−4774.2	−1713.7	−540.8
	2	24142.5	−2080.4	−1908.5	−1488.9	−770.1	49.8
	3	4796.9	−3893.5	2345.5	7535.9	3277.0	−6651.8
	4	4589.3	−3192.2	11125.5	−2139.0	−14808.6	1537.6
	5	3611.8	1328.3	10402.5	−14682.6	11358.9	−3858.3
−20 Mpa	1	1395.6	−32690.5	−14244.4	−5351.6	−1853.6	−572.3
	2	27924.9	−1563.7	−1465.5	−1202.4	−664.5	53.5
	3	5110.1	−3317.3	2631.3	7458.8	2049.8	−8172.2
	4	4931.5	−2578.8	11237.3	−3614.6	−15112.4	1808.2
	5	3877.1	1782.7	10099.2	−14442.5	11591.4	−4378.6
−10 Mpa	1	927.7	−37619.8	−15983.2	−5808.8	−1884.2	−535.9
	2	32202.8	−1000.8	−957.3	−826.6	−497.3	53.8
	3	4999.3	−2481.3	2982.0	7160.2	11.4	−10214.6
	4	5014.0	−1705.6	11299.3	−5819.6	−15163.3	2154.8
	5	3981.0	2395.5	9449.2	−13881.1	11839.3	−4994.7
0 Mpa	1	482.0	−43438.2	−17535.9	−5915.3	−1715.3	−415.6
	2	37252.5	−501.6	−488.5	−442.4	−293.9	48.3
	3	4344.0	−1474.6	3296.8	6421.5	−3389.8	−13025.4
	4	4746.7	−597.2	11229.7	−9061.4	−14648.7	2604.3
	5	3876.0	3148.5	8189.5	−12791.3	12081.8	−5706.8

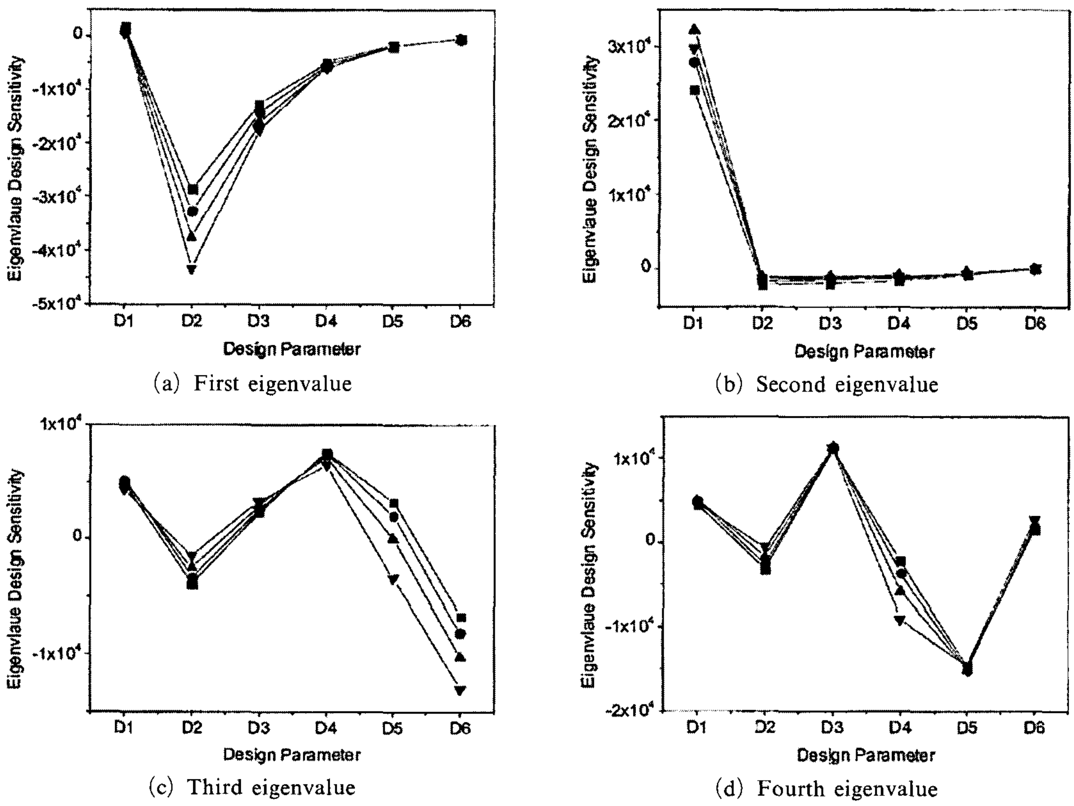


Fig. 11 The first four eigenvalue sensitivity when initial stresses are different (■ : −30 Mpa, ● : −20 Mpa, ▲ : −10 Mpa, ▼ : 0 Mpa)

is set to −30, −20, −10, and 0 Mpa, respectively. Since the finite element model used for previous example is too coarse to compute sensitivity with respect to support location, a finer model with 410 elements for the analysis was constructed. In this case the first five eigenvalues are considered and the first six support locations are chosen as design parameters. One of design velocity field, shown in Fig. 10 is defined to allow only one support location to move using the previously mentioned exponential function. Sensitivity results are given with 1% uniform change in shape design parameter in Table 2. Negative sign means that eigenvalue will be decreased when support location moves to the right slightly. Variation of grid support location causes to eigenvalue change, but its propensity can be estimated based on the sensitivity analysis. For example, since the first natural frequency is the most sensitive to the second support location, D2, effective design of the first natural frequency is expected by controlling the second support location. Figure 11 shows eigenvalue sensitivity with respect to design parameters when initial stresses are different. In this case, based on Table 2 and Fig. 11, the sensitivity behavior does not change largely despite different initial stresses.

5. Conclusions

After deriving the dynamic equation of an initially stressed elastic body with discretely spaced elastic supports, vibration of the beam with supports was studied. This type of beam structure is common in nuclear fuel assembly. Thus, the simulated model geometry are chosen to be similar to those of real structure in nuclear reactors.

When initial stress is involved, it causes the potential energy change. Although the initial stress effect is negligible compared to modulus of elasticity for most industrial environments, the coefficient of the fourth derivative in the dynamic equation is clearly under the influence of initial stress in this case. It is also noted that the tensile stress affects mode shapes of the system with stiff supports. Although irregular modes are produced in a fuel rod with stiff supports, unlike ordinary beam modes, the irregular modes change to ordinary beam modes as tensile stress increases. Concerning eigenvalue sensitivity, it is assumed that the modification of the system dimensions is restricted, hence support locations are considered as design parameters. Introducing the shape design concept, rate of eigenvalue change to support location is evaluated to consider effect of elastic supports. Although the sensitivity is varying with different initial stresses, it is less susceptible to the initial stress in this case.

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