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Control Scheme Using Forward Slip for a Multi-stand Hot Strip Rolling Mill

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Forward slip is an important parameter often used in rolling-speed control models for tandem hot strip rolling mills. In a hot strip mill, on-line measurement of strip speed is inherently very difficult. Therefore, for the set-up of the finishing mill, a forward slip model is used to calculate the strip speed from roll circumferential velocity at each mill stand. Due to its complexity, most previous researches have used semi-empirical methods in determining values for the forward slip. Although these investigations may be useful in process design and control, they do not have a theoretical basis. In the present study, a better forward slip model has been developed, which provides for a better set-up and more precise control of the mill. Factors such as neutral point, friction coefficient, width spread, shape of deformation zone in the roll bite are incorporated into the model. Implementation of the new forward slip model for the control of a 7-stand hot strip tandem rolling mill shows significant improvement in roll speed set-up accuracy.

Key Words: Finishing Mill, Forward Slip, Hot Strip Mill, Neutral Point, Coefficient of Friction

Nomenclature

F_i : Shear force between roll and strip of the i^th^ roll stand
f_i : Forward slip of the i^th^ roll stand
f_i^* : Plane-strain forward slip of the i^th^ roll stand
$\bar{f}$: Improved value for forward slip with adjustment for width variation
h_i : Exit strip thickness at the i^th^ roll stand
Δh_i : Draft of the i^th^ roll stand
h_{N_i} : Strip thickness at the neutral point of the i^th^ roll stand
L_p : Projected contact length between the roll and the strip
K_i : i^th^ roll adjustment factor for determining the friction coefficient
K_i^* : Factor for the i^th^ roll stand used in the determination of K_i.
K_m : Mill deformation resistance
N : Number of coils
P_m : Mill load
P_i : Roll force normal to roll surface for the i^th^ roll stand
p_i : Roll pressure for the i^th^ roll stand
R_i : Radius of roll in the i^th^ roll stand
T_m : Mill torque
$V_{i,actual}$: Actual strip speed (measured at steady-state conditions) for the i^th^ roll stand
$V_{i,error}$: Strip speed prediction error for the i^th^ roll stand
$V_{i,predict}$: Predicted strip speed for the i^th^ roll stand

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v_i : Exit strip velocity at the ith roll stand

v_i^0 : Circumferential velocity of the ith roll stand

v_{N_i} : Strip velocity at the neutral point of the ith roll stand

w_i : Exit strip width at the ith roll stand

w_{N_i} : Strip width at the neutral point of the ith roll stand

x : Horizontal position in deformation region

α_i : Bite angle of the ith roll stand

β_i : Neutral angle of the ith roll stand

γ_i : Plane-strain prediction accuracy parameter for the ith roll stand

θ : Angular position in deformation region

μ_i : Friction coefficient between the roll and the strip of the ith roll stand

μ_i^* : Parameter based on torque, load and roll radius which reflects friction conditions in plane strain

Φ_i : Parameter to account for width changes during deformation in the ith roll stand

1. Introduction

In a finishing mill, the roll gap and the roll speed for each stand must be preset in advance before the front end of strip reaches the entry side of finishing mill. Hence, the accuracy of the set-up values directly influences quality of the front end of the hot strip and the subsequent performance of the mill control system. A roll speed model is used to control the mass balance between adjacent mill stands. So the roll speed control is responsible for attaining stability of the strip velocity and accuracy of the strip thickness. In general, a forward slip model (Koncewicz, 1991 ; Bakhtinov, 1988 ; Zhang, 1995 ; Lenard, 1997 ; Lee, 2002 ; Zhang, 1989 ; Hum, 1996 ; Seregin, 1989) is used for the roll speed because it converts roll circumferential velocity into strip speed at each mill stand. In a tandem hot strip rolling mill, forward slip is difficult to measure due to high temperature and the harsh environment surrounding the mill. Hence, the forward slip in hot strip mill is often obtained from a prediction model but high accuracy is difficult to achieve due to many process variables. The purpose of the present study is to develop a better model for forward slip.

Since, in practice, the velocity of the strip is calculated from the circumferential velocity of the roll, it is important to have a reliable roll velocity model. The roll velocity model influences not only the thickness of the strip at the front and back ends but also the thickness profile of the entire coil and the operational stability of the mill. Accuracy of the roll velocity model is directly dependent on the accurate prediction of forward slip.

2. Basic Model for Forward Slip Parameter

A schematic of the roll bite geometry for the ith roll stand is shown in Fig. 1. The relative slip between the strip and the roll surface is one of the important factors in the control models for the rolling process. In general, forward slip (f_i) is considered as a measure of the relative slip and defined as the difference between strip velocity (v_i) and roll circumferential velocity (v_i^0) divided by the roll circumferential velocity, or

$$f_i = \frac{v_i - v_i^0}{v_i^0}. \tag{1}$$

From the volume flow balance between adjacent mill stands,

$$h_{i-1} v_{i-1} w_{i-1} = h_i v_i w_i \tag{2}$$

where h_i and w_i are the strip thickness and width, respectively, on exit from the ith stand.

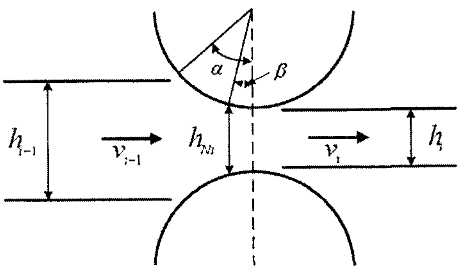


Fig. 1 Schematic drawing of roll bite geometry

From Eq. (1), the exit strip velocity can be expressed as

$$v_i = v_i^0(1+f_i). \tag{3}$$

Substitution of Eq. (3) into the volume flow balance equation yields

$$h_{i-1}v_i^0 w_{i-1}(1+f_{i-1}) = h_i v_i^0 w_i(1+f_i). \tag{4}$$

The volume balance is satisfied at any point within the roll bite. For the neutral point of the ith stand (defined by the subscript N_i),

$$h_{i-1}v_{i-1}w_{i-1} = h_i v_i w_i = h_{N_i} v_{N_i} w_{N_i}. \tag{5}$$

Rearrangement of Eq. (5) gives

$$v_i = \frac{h_{N_i} v_{N_i} w_{N_i}}{h_i w_i}. \tag{6}$$

Substitution of Eq. (3) into Eq. (6) yields

$$f_i = \frac{h_{N_i} v_{N_i} w_{N_i}}{h_i v_i^0 w_i} - 1 \tag{7}$$

From the geometry, as shown in Fig. 1, one can show that

$$h_{N_i} = h_i + 2R_i(1-\cos\beta_i) \tag{8}$$

where R_i is the roll radius of the ith roll and β_i is the neutral point angle.

There is no slip at the neutral point. Hence,

$$v_{N_i} = v_i^0 \cos\beta_i \tag{9}$$

and substituting into Eq. (7), the basic equation for forward slip can be obtained as

$$f_i = \frac{h_{N_i} w_{N_i}}{h_i w_i}\cos\beta_i - 1. \tag{10}$$

The strip thickness at the neutral point (h_{N_i}) can be calculated from Eq. (8) if $\cos\beta_i$ is known. Likewise, if the neutral angle (β_i) and strip width at the neutral point (w_{N_i}) are known, the forward slip can be calculated from Eq. (10).

From a force balance in the roll bite region, as shown in Fig. 2, one can find

$$\int_0^{\alpha_i} p_i w_i R_i \sin\theta\, d\theta + \int_0^{\beta_i} p_i w_i \mu_i R_i \cos\theta\, d\theta - \int_{\beta_i}^{\alpha_i} p_i w_i \mu_i R_i \cos\theta\, d\theta = 0 \tag{11}$$

where α_i is the bite angle, p_i is the roll pressure and μ_i is the coefficient of friction for the ith stand.

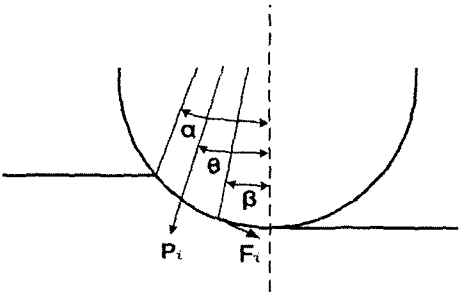


Fig. 2 Forces in the roll bite

If the friction coefficient, the strip width and the roll pressure are constant in the roll bite region, then Eq. (11) can be solved (Koncewicz, 1991; Bakhtinov, 1988) for the neutral angle. This gives

$$\sin\beta_i = \frac{1}{2}\left(\sin\alpha_i - \frac{1-\cos\alpha_i}{\mu_i}\right). \tag{12}$$

The bite angle (α_i) can be derived from geometrical relationships (see Fig. 1) as

$$R_i(1-\cos\alpha_i) = \frac{1}{2}(h_{i-1}-h_i). \tag{13}$$

From Eq. (10) and assuming a constant strip width ($w_i = w_{N_i}$) during rolling, the forward slip for this plane-strain condition is calculated as

$$f_i^* = \frac{h_{N_i}}{h_i}\cos\beta_i - 1 \tag{14}$$

where, $h_{N_i} = h_i + 2R_i(1-\cos\beta_i)$, and $\cos\beta_i = \sqrt{1-(\sin\beta_i)^2}$. The value for β_i can be determined from Eq. (12) if the friction coefficient is known.

3. Determination of Friction Coefficient in the Forward Slip Equation

To calculate forward slip from Eq. (14), the friction coefficient must be known. In a production strip rolling process, the friction coefficient depends on a large number of variables, which may vary during the operation, such as bite angle, rolling speed, lubrication, strip surface condition, etc. The selection of a unique value for the friction coefficient is very difficult task.

One possible approach to selecting a numerical value for the friction coefficient is as follows. It can be shown that the more the strip width expands during rolling, the forward slip becomes less. It was assumed in derivation of Eq. (13) that the strip width does not expand. It is asserted that $\sin\beta_i$ is at a maximum value for this condition. Hence, as the friction coefficient approaches a minimum value, the value for $\sin\beta_i$, as calculated from Eq. (12) increases and approaches a more realistic value because it can compensate width variation during rolling.

For the strip to enter the roll bite,

$$F_i\cos\alpha_i \geq P_i\sin\alpha_i \tag{15}$$

where F_i is the shear force due to friction at the interface between the roll and the strip and P_i is the normal force (see Fig. 2). Rearrangement yields

$$\frac{F_i}{P_i} \geq \frac{\sin\alpha_i}{\cos\alpha_i}. \tag{16}$$

From the definition of the friction coefficient $F_i=\mu_i P_i$, one finds

$$\mu_i \geq \tan\alpha_i \approx \left(\frac{\Delta h_i}{R_i}\right)^{\frac{1}{2}}. \tag{17}$$

The minimum friction coefficient is the value satisfying the equality of Eq. (17). The minimum friction coefficient is

$$\mu_{i,\min} = \left(\frac{\Delta h_i}{R_i}\right)^{\frac{1}{2}}. \tag{18}$$

The minimum friction coefficient of Eq. (18) can be easily used for the determination of forward slip and hence for the roll-speed control model, but it should be refined in order to compensate for the characteristics of individual mill stands. The refinement can be done through the use of mill processing data. The rolling torque is strongly correlated with the friction coefficient. Hence, a refinement based on the roll torque is developed to obtain more realistic value for the friction coefficient in the roll-speed control model.

The mill load $(P_{m,i})$ for each stand can be obtained from the mill deformation resistance $(K_{m,i})$ and the projected contact length $(L_{p,i})$ between the roll and the strip by

$$\frac{P_{m,i}}{w_i} = \int_0^{L_{p,i}} K_{m,i}dx \tag{19}$$

The roll torque $(T_{m,i})$ can be calculated as

$$\begin{aligned}\frac{T_{m,i}}{w_i} &= \int_0^{L_{p,i}} (\mu_i^* K_{m,i}dx)\,R_i \\ &= \mu_i^* R_i \int_0^{L_{p,i}} K_{m,i}dx\end{aligned} \tag{20}$$

By substituting Eq. (19) into Eq. (20), one can obtain

$$\frac{T_{m,i}}{w_i} = \mu_i^* R_i \frac{P_{m,i}}{w_i}. \tag{21}$$

From Eq. (21), a new parameter that reflects the frictional conditions is determined as

$$\mu_i^* = \frac{T_{m,i}}{R_i P_{m,i}}. \tag{22}$$

Instead of using the minimum friction coefficient of Eq. (18) in the roll-speed model, one can select a friction coefficient based on the operating parameters as

$$\mu_i = K_i \mu_i^*. \tag{23}$$

The value of μ_i^* can be obtained from Eq. (22) and the value of K_i can be determined from averaging across all the mill stands because the exact value for each stand is difficult to obtain in actual tandem mill. For a seven-stand tandem mill, K_i is

$$\begin{aligned}&K_i = K_i^* \Big/ \left(\frac{1}{7}\sum_{i=1}^{7} K_i^*\right) \\ &\text{with } K_i^* = \frac{1}{N}\sum_{j=1}^{N} T_{m,j}/(P_{m,j}R_j)\end{aligned} \tag{24}$$

where N is the number of coils that have been rolled and K_i^* is found by averaging over the number of coils that have been rolled. Therefore, forward slip for the plane-strain condition can be obtained from Eq. (14) if the value of the friction coefficient from Eq. (23) is used to determine the neutral point angle in Eq. (12).

4. Consideration of Strip-width Variation

During production rolling, plane-strain conditions are not maintained within the roll bite and the characteristics of each roll are different from one another. Therefore, the forward slip value calculated under the plane-strain condition should be revised according to the width variation by using operational rolling data.

The basic forward slip model, given in Eq. (10), can be re-written using the parameter Φ, which compensates for width variation as

$$f_i=\frac{h_{N_i}}{h_i}\Phi\cos\beta_i-1 \tag{25}$$

If one uses a predicted friction coefficient, which determines the most accurate value for forward slip, $\bar{f}$, then the most desirable situation would be that the predicted friction coefficient is equal to real friction coefficient, leading to

$$f_i=\bar{f}_i \tag{26}$$

From Eqs. (14), (25) and (26) one can obtain,

$$\bar{f}_i=(f_i^*+1)\Phi_i-1 \tag{27}$$

Therefore, the parameter that compensates for width variation can be defined as

$$\Phi_i=\frac{\bar{f}_i+1}{f_i^*+1}. \tag{28}$$

Hence if the predicted friction coefficient, which allows calculation of the most accurate value for forward slip $(\bar{f})$, can be determined, then the average value of parameter Φ_i shown in Eq. (28) can be obtained at each stand, and the optimal forward slip can be obtained from Eq. (25).

5. Determination of $\bar{f}$

To determine the friction coefficient that produces most accurate value for forward slip $(\bar{f})$, the following procedure has been used.

For a plane-strain condition, the circumferential velocity of the i^{th} roll can be determined from Eqs. (4) and (14) as

$$v_{i,NEW1}^0=\frac{v_{7,NEW1}^0(1+f_{7,NEW1}^*)h_7}{(1+f_{i,NEW1}^*)h_i} \tag{29}$$

which uses the exit velocity, exit thickness and forward slip from the last mill stand (i=7). The subscript 'NEW 1' notes the calculations are based on the plane-strain forward slip model.

For the revised situation where width variation in the roll bite can occur, the forward slip based upon the refined model $(\bar{f})$, yields

$$v_{i,NEW2}^0=\frac{v_{7,NEW2}^0(1+\bar{f}_{i,NEW2})h_7}{(1+\bar{f}_{7,NEW2})h_i} \tag{30}$$

where the subscript '$NEW2$' denotes the forward slip model after refinement $(\bar{f})$.

To estimate the prediction accuracy of the plane-strain forward slip model, the parameter (γ_i) is defined as

$$\gamma_i=\frac{v_{i,TARGET}^0-v_{i,NEW1}^0}{v_{i,NEW1}^0} \tag{31}$$

where the target circumferential speed $(v_{i,TARGET}^0)$ is the actual measured speed after strip stability has been achieved in the mill. Therefore, when the predicted circumferential speed $(v_{i,NEW1}^0)$ is the same as target circumferential speed $(v_{i,TARGET}^0)$, γ_i would be zero.

For the refined model, the following must be satisfied.

$$v_{i,NEW2}^0=v_{i,NEW1}^0(1+\gamma_i). \tag{32}$$

Eq. (32) can be expressed as

$$\frac{v_{7,NEW2}^0(1+\bar{f}_{7,NEW2}^0)}{(1+\bar{f}_{i,NEW2}^0)}=\frac{v_{7,NEW1}^0(1+f_{7,NEW1}^*)}{(1+f_{i,NEW1}^*)}(1+\gamma_i). \tag{33}$$

In the mill set-up, speed for the final (7^{th}) stand does not change significantly, Hence $v_{7,NEW1}^0$ is nearly the same as $v_{7,NEW2}^0$. Therefore,

$$\bar{f}_{i,NEW2}=\frac{(1+f_{i,NEW1}^*)(1+\bar{f}_{7,NEW2})}{(1+f_{7,NEW1}^*)(1+\gamma_i)}-1. \tag{34}$$

To calculate $\bar{f}_{i,NEW2}$ from Eq. (34), $\bar{f}_{7,NEW2}$ must be known.

The value of $\bar{f}^{0}_{7,NEW2}$ in the right side of Eq. (34) can be initially approximated by $f^{*}_{7,NEW2}$. Then, Eq. (34) becomes

$$\begin{aligned}\bar{f}_{i,NEW2}&=\frac{(1+f^{*}_{i,NEW1})(1+f^{*}_{7,NEW1})}{(1+f^{*}_{7,NEW1})(1+\gamma_i)}-1\\&=\frac{f^{*}_{i,NEW1}-\gamma_i}{(1+\gamma_i)}.\end{aligned}\tag{35}$$

From Eq. (35), $\bar{f}_{7,NEW2}$ can be obtained as

$$\bar{f}_{7,NEW2}=\frac{f^{*}_{7,NEW1}-\gamma_7}{(1+\gamma_7)}\tag{36}$$

By substituting Eq. (36) into Eq. (34), the final expression for the $\bar{f}_i$ can be obtained as

$$\begin{aligned}\bar{f}_i&=\bar{f}_{i,NEW2}\\&=\frac{(1+f^{*}_{i,NEW1})\left(1+\frac{f^{*}_{7,NEW1}-\gamma_7}{1+\gamma_7}\right)}{(1+f^{*}_{7,NEW1})(1+\gamma_i)}-1.\end{aligned}\tag{37}$$

6. Calculation of Forward Slip

Figure 3 shows the predicted friction coefficient at each stand from operational data. The friction coefficients obtained by Eq. (18) show a steeply decreasing trend with increasing stand number. On the other hand, friction coefficients obtained by Eq. (23) do not show a steep decrease for the later stands.

Figure 4 shows the predicted forward slip at each stand from operational data. The forward slip obtained by Eq. (25) lies in the range of 0.03—0.09 and shows maximum at the 3rd and 4th mill stands.

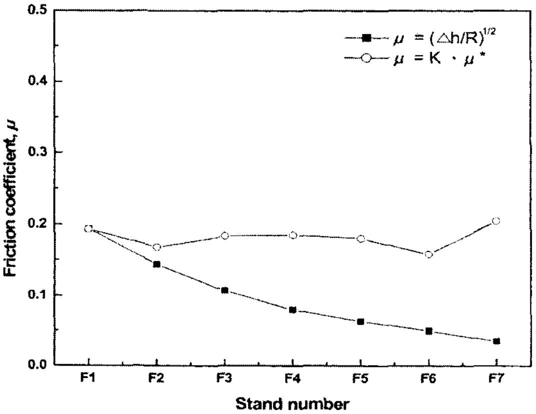


Fig. 3 Friction coefficient at each stand

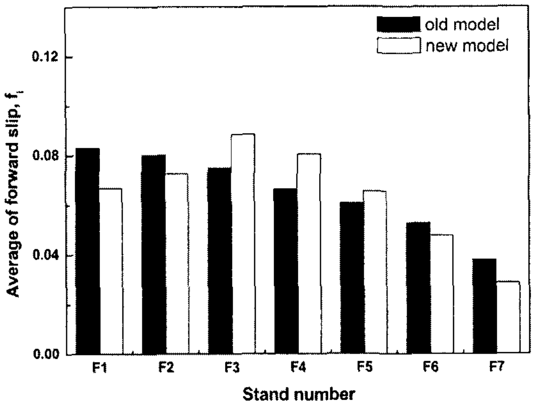
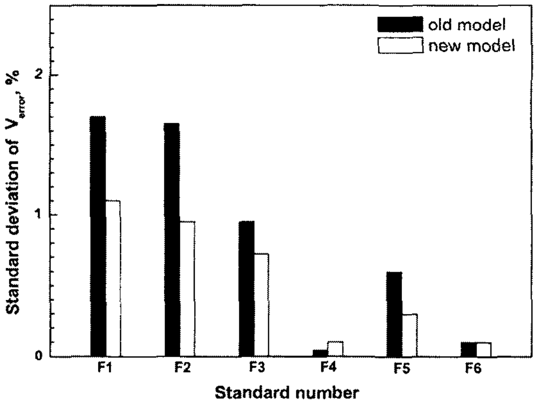
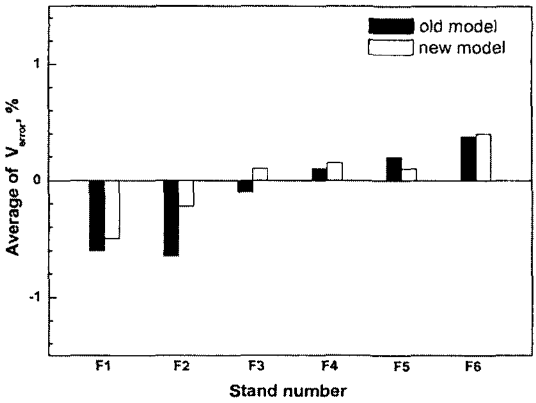


Fig. 4 Predicted forward slip at each stand



(a) Standard deviation of $V_{i,error}$



(b) Average of $V_{i,error}$

Fig. 5 Comparison of strip speed prediction error ($V_{i,error}$)

Figure 5 shows an implementation of the newly

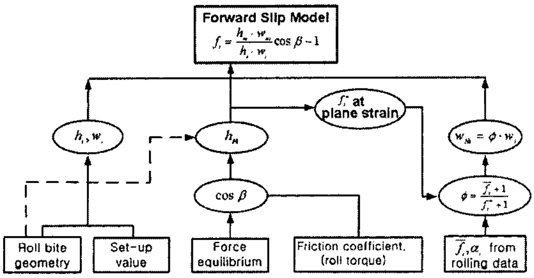


Fig. 6 Summary of forward slip calculation

developed model. The operational data obtained by existing forward slip model, designated as 'old model', had been compared with newly developed model. Although not specifically expressed in this paper, the semi-empirical old model had only considered reduction ratio and strip thickness at each stand in the calculation of forward slip and the prediction accuracy had not been so good.

As seen in the figure, the roll speed set-up accuracy is significantly increased when using the newly developed model. In Fig. 5, the strip speed prediction error, $V_{i,error}$ can be obtained by Eq. (38).

$$V_{i,error} = \frac{V_{i,actual} - V_{i,predict}}{V_{i,predict}} \tag{38}$$

where $V_{i,actual}$ is the actual strip speed measured by pulse-generating speedmeter after strip stability is obtained, and $V_{i,predict}$ is the predicted strip speed from the forward slip obtained from Eq. (25).

The newly developed forward slip model is schematically summarized in Fig. 6.

7. Conclusion

A new forward slip model based on strip geometry, roll diameter, neutral point, friction coefficient, volume fractions undergoing width spread, shape of deformation zone and roll torque has been developed. For the application of theoretical model, a methodology to convert some 'difficult-to-measure' parameters from operational mill data has also been developed. Implementation of newly developed model shows good improvement in roll speed set-up accuracy for a 7-stand hot strip finishing mill.

References

Bakhtinov, Yu. B., 1988, "Forward and Backward Slip during Rolling," *Steel in the USSR*, Vol. 18, pp. 364~367.

Hum, B., 1996, "Measurements of Friction during Hot Rolling of Aluminum Strip," *Journal of Materials Processing Technology*, Vol. 60, pp. 331~338.

Koncewicz, S., 1991, "Investigation on the Forward-slip and the Neutral Angle in Flat Rolling," *Archives of Metallurgy*, Vol. 36, pp. 115~130.

Lee, W. H., 2002, "Mathematical Model for Cold Rolling and Temper Rolling Process of Thin Steel Strip," *KSME International Journal*, Vol. 10, pp. 1296~1302.

Lenard, J. G., 1997," A Study of Friction during the Lubricated Cold Rolling of an Aluminum Alloy," *Journal of Materials Processing Technology*, Vol. 72, pp. 293~301.

Seregin, S. A., 1989, "Relation Between Forward Slip and Spread in Rolling," *Steel in the USSR*, Vol. 19, pp. 438~440.

Zhang, W., 1995, "Determination of Forward Slip in H-Beam Rolling," *Journal of Materials Processing Technology*, Vol. 54, pp. 114~119.

Zhang, Y., 1989, "Research on the Forward Slip Model in a Universal H-Beam Mill," *Steel Iron*, Vol. 11, pp. 39~41.