# An Analysis on the Echo Cancellation Algorithm Reducing the Computational Quantities

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***Abstract***—An adaptive algorithm for reducing the hardware complexity is presented. This paper proposes a modified LMS algorithm for the adaptive system and analyzes its convergence characteristics mathematically. An objective of the proposed algorithm is to reduce the hardware complexity. In order to test the performances, it is applied to the echo canceller, and a program is described. The results from simulations show that the echo canceller adopting the proposed algorithm achieves almost the same performances as one adopting the NLMS algorithm. If an echo canceller is implemented with this algorithm, its computation quantities are reduced to the one third as many as the one that is implemented with the NLMS algorithm, without so much degradation of performances.

***Index Terms***—Adaptive algorithm, Echo cancellation, LMS algorithm

## I. INTRODUCTION

Since the LMS(Least Mean Square) algorithm[1] for the adaptive system was presented, it has been used widely in the various applications. Its reasons are because it is simple and stable. However, to improve its performance, many attempts have been done until the recent. Among the works, there are to increase the convergence speed or to reduce computation quantities. The sign algorithm[2] as a way for reducing computation quantities lessens dramatically the hardware complexity, but slows down the convergence speed. In this paper, a new adaptive algorithm which reduces computation quantities without the degradation of the convergence speed, is proposed. This algorithm is applied to the echo cancellation system and its performances are testified by simulations.

This paper includes the proposed adaptive algorithm in section II, its convergence characteristics in section III, the simulations and results in section IV. And the conclusion is described in section V.

## II. PROPOSED ADAPTIVE ALGORITHM

Each The echo cancellation problem of the unknown system as showed in Figure 1 is solved repeatedly by using adaptive algorithm. The characteristics of the unknown system are adaptively evaluated from the information of the error. We use usually the LMS algorithm as the adaptive algorithm. This algorithm is simple and robust.

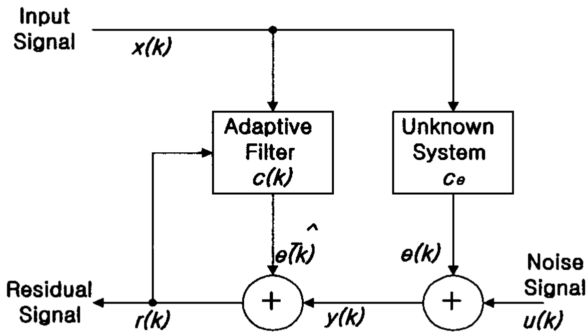


Fig. 1 Echo cancellation system

In Figure 1, $x(k)$ is the input signal to the system and $e(k)$ is the echo signal from the unknown system. Its estimate is generated by convoluting the input signals with the coefficients of the filter.

$$\hat{e}(k) = c(k)\,x(k) \tag{1}$$

where the number of taps of the filter is $N$. The residual $r(k)$ is the value which subtracts the estimate signal from the echo and noise signal. The LMS algorithm for updating the coefficients of the adaptive filter is

$$c(k+1) = c(k) + 2\mu r(k)x(k) \tag{2}$$

where a constant μ is the step size which adjusts the coefficients.

In the equation (2), the right update step is changed by the magnitude of the input signal, Therefore, to avoid it, the input signal is normalized by the mean input signal.

$$c(k+1) = c(k) + 2\mu\, r(k)\frac{x(k)}{x_m(k)} \tag{3}$$

It is the NLMS(normalized LMS) algorithm. This operation requires computations which multiplys the input signal by the residual signal and divides the input signal by its mean value for updates of all coefficients. If the number of taps is large, its computation quantities also are increased. Therefore, it is difficult that we implement the hardware for the large number of taps. To resolve this problem, the sign algorithm was presented.

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$$c(k+1) = c(k) + 2\mu\,\mathrm{sign}(r(k))\mathrm{sign}(x(k)) \tag{4}$$

In the above equation, there is no need to multiply and to divide. The sign algorithm is simple, but its convergence speed is slow. Then we propose a new adaptive algorithm for reducing its complexity with the fast convergence.

$$c(k+1) = c(k) + 2\mu r(k)\mathrm{sign}(x(k)) \tag{5}$$

where $2\mu = 2^{-14}$ is the step value of adaptations. Though there are the multiplicative arithmetics for the updates of the coefficients in the proposed algorithm, it is needed a XOR gate only. That is, the operation which multiply the sign of residual by the sign of inputs is processed by a XOR gate. Since there are no multiplications and divisions, we can dramatically reduce computation quantities. Its performances after this modification are almost the same levels as the ones of the NLMS algorithm. The convergence time and the ERLE(Echo Return Loss Enhancement) for both algorithm are showed as the results of simulations in section IV.

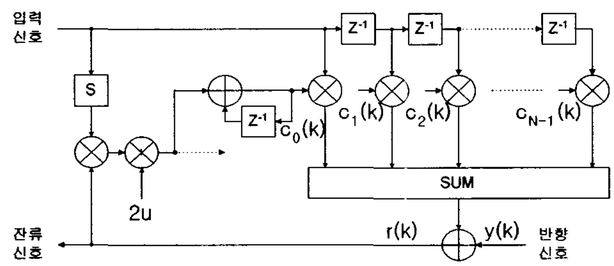


Fig. 2 Schematic of the proposed algorithm

In case that an echo canceller is implemented with $N$-tap, the required computation quantities are summarized at Table 1. It shows that the number of multiplication in the sign and the proposed algorithm, which burden the hardware with heavy computations, is half as many as the one of the NLMS algorithm without divisions.

<Table 1> Comparisons of the hardware complexity

| Items | NLMS | Sign | Proposed |
|---|---|---|---|
| Multiplication | $2N$ | $N$ | $N$ |
| Division | $N$ | 0 | 0 |
| Addition | $2N$ | $2N$ | $2N$ |

## III. CONVERGENCE CHARACTERISTICS OF THE PROPOSED ALGORITHM

We are trying to analyze convergences of the proposed algorithm. In order to be possible it, the following assumptions are introduced.[3][4]

i) $x(k)$ and $u(k)$ are zero-mean wide-sense stationary, mutually independent stochastic signal.

$$E\{x(k)\} = E\{u(k)\} = 0 \tag{6}$$

ii) $x(k)$ is white signal.

$$E\{x(k)x(k+n)\} = \sigma_x^2\delta(k) \tag{7}$$

iii) $x(k)$ and $d(k)$ are statistically independent.
iv) $err(k)$ and $u(k)$ are statistically independent.

The error vector between the echo path and the filter is

$$d(k) = c_e - c(k) \tag{8}$$

The residual signal after echo compensations is

$$\begin{aligned} r(k) &= d^T(k)\cdot x(k) + u(k) \\ &= err(k) + u(k) \end{aligned} \tag{9}$$

The variance of the input signal is

$$E\{\| x(k)\|^2\} = N\sigma_x^2 \tag{10}$$

The variance of the error signal is

$$\sigma_e^2(k) = E\{err^2(k)\} = \sigma_x^2 E\{\| d(k)\|^2\} \tag{11}$$

From (5) and (8)

$$\begin{aligned} d(k+1) &= c_e - c(k+1) \\ &= d(k) - 2\mu r(k)S(k) \end{aligned} \tag{12}$$

With $\|S(k)\|^2=N$ for the sign vector of the input signal

$$\begin{aligned} \| d(k+1)\|^2 = {} & \| d(k)\|^2 - 4\mu r(k)\| S^T(k)\cdot d(k)\| \\ & + 4N\mu^2 r^2(k) \end{aligned} \tag{13}$$

Multiplying both sides by $\|x(k)\|^2$ and taking the expected values

$$\begin{aligned} & E\{\| d(k+1)\|^2\}E\{\| x(k)\|^2\} \\ & = E\{\| d(k)\|^2\}E\{\| x(k)\|^2\} \\ & \quad - 4N\mu[E\{err(k)\} + E\{err(k)\}E\{u(k)\}]E\{| x(k)|\} \\ & \quad + 4N\mu^2[E\{err^2(k)\} + E\{err(k)\}E\{u(k)\} \\ & \quad + E\{u^2(k)\}]E\{\| x(k)\|^2\} \end{aligned} \tag{14}$$

Using the previous assumptions and arranging the upper equation

$$\begin{aligned} \sigma_e^2(k+1) = {} & \sigma_e^2(k)[1 - 4\mu\sigma_x(k)[1 - N\mu\sigma_x(k)]] \\ & + 4N\mu^2\sigma_u^2(k)\sigma_x^2(k) \end{aligned} \tag{15}$$

The ratio of the variance of error signal to the variance of noise signal is defined by

$$R^2(k) = \frac{\sigma_e^2(k)}{\sigma_u^2(k)} \tag{16}$$

From (15)

$$R^2(k+1) = \rho R^2(k) + 4N\mu^2\sigma_x^2(k) \qquad (17)$$

where

$$\rho = 1 - 4\mu\sigma_x(k)[1 - N\mu\sigma_x(k)] \qquad (18)$$

Then, the curve of convergences is

$$R^2(k) = \rho^k R^2(0) + 4N\mu^2\sigma_x^2(k)\frac{1-\rho^k}{1-\rho} \qquad (19)$$

From |ρ|<1, the condition of convergence is

$$0 < \mu < \frac{1}{N\sigma_x(k)} \qquad (20)$$

After convergence, the final ratio of variances is

$$R^2(\infty) = \frac{N\mu\sigma_x(k)}{1 - N\mu\sigma_x(k)} \qquad (21)$$

Therefore, using (21)

$$\rho = 1 - \frac{4}{N}\frac{R^2(\infty)}{(1+R^2(\infty))^2} \qquad (22)$$

$$R^2(k) = \rho^k[R^2(0) - R^2(\infty)] + R^2(\infty) \qquad (23)$$

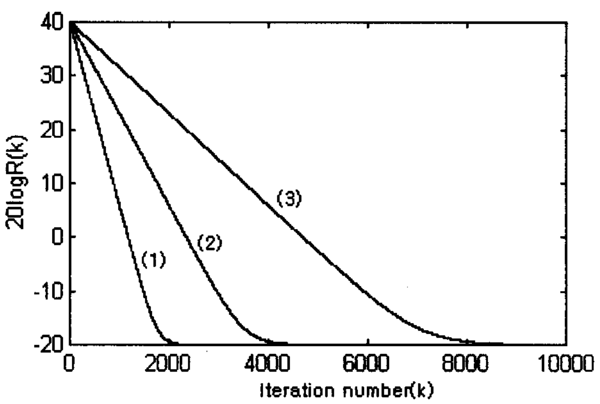


Fig. 3 convergence curves of (22) with a variable of $\sigma_m^2/\sigma_x^2$ : (1) 0.5, (2) 1.0, (3) 2.0

Due to $R^2(\infty) \ll 1$, the number of iterations required to get R(k) is

$$k = \frac{\log\left(\frac{R^2(k) - R^2(\infty)}{R^2(0) - R^2(\infty)}\right)}{\log\left(1 - \frac{4}{N}R^2(\infty)\right)} \qquad (24)$$

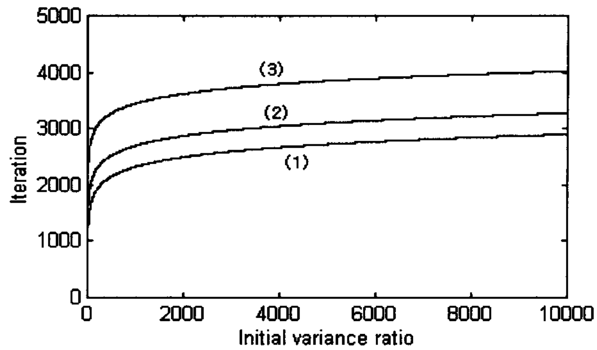


Fig. 4 The number of iterations, k, required for (1) $R^2(k)$=0.1, (2) 0.03, (3) 0.01

The speed of the convergence is defined by the equation

$$\lambda(R(k)) = -10\log\left(\frac{R^2(k+1)}{R^2(k)}\right)$$
$$= -10\log\left(\rho + \frac{4N\mu^2\sigma_x^2(k)}{R^2(k)}\right) \qquad (25)$$

By using (21)

$$\lambda(R(k)) = -10\log$$
$$\left(1 - \frac{4}{N}\frac{R^2(\infty)}{1+R^2(\infty)}\left[1 - \frac{R^2(\infty)}{1+R^2(\infty)}\left(1 + \frac{1}{R^2(k)}\right)\right]\right) \qquad (26)$$

In case of $R^2(\infty) \ll 1$ and the large $R^2(k)$

$$\lambda(R(k)) \cong -10\log\left(1 - \frac{4}{N}R^2(\infty)\right) \qquad (27)$$

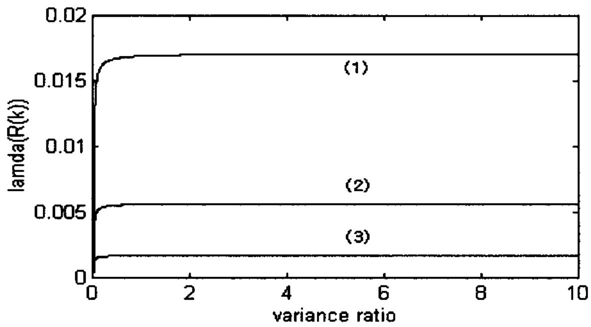


Fig. 5 The speed of convergences with (1) $N$=10, (2) 30, (3) 100

## IV. SIMULATIONS AND RESULTS

To check the performances of the proposed algorithm, a C-program for the echo cancellation system is described. Its performances are represented by the ERLE which is defined as the ratio of the echo and noise signal to the residual signal after the echo compensation.

$$ERLE(k) = 10\log\left(\frac{E\{y^2(k)\}}{E\{r^2(k)\}}\right)[\text{dB}] \qquad (28)$$

The echo canceller is formed of the FIR filter. The ERLE and the convergence time are gotten from the simulations. The simulations are proceeded with three algorithm and its convergence curves are showed in Fig. 6. In this Fig., the curve (1) indicates the ERLE by the proposed algorithm and the curve (2) by the NLMS algorithm, and the curve (3) by the Sign algorithm.

As what is viewed from the above curves, the echo canceller adopting the proposed algorithm displays almost the same result as one adopting the NLMS algorithm in terms of the convergence speed and the ERLE. And the proposed algorithm converges faster than the Sign algorithm with the same computation quantity.

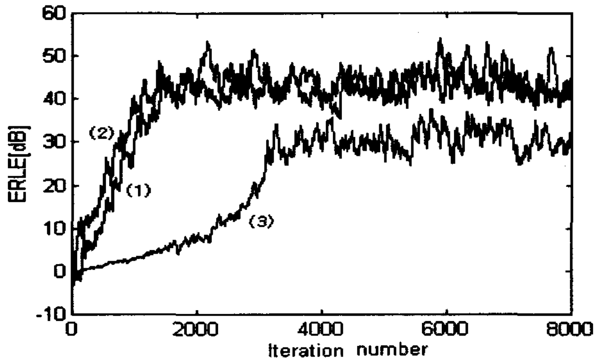


Fig. 6 Convergence curves

## V. CONCLUSIONS

In this paper, an adaptive algorithm for reducing the hardware complexity is proposed. Analyses on performances of the proposed algorithm are proceeded by the mathematical and simulational method. The results of simulations show that the echo canceller using the proposed algorithm gets almost the same results as one using the NLMS algorithm. But, the computation quantities needed by this algorithm are almost the same as the one needed by the Sign algorithm.

## REFERENCES

[1] B. Widrow, J. R. Glover, J. M. McCool, J. Kaunitz, C. S. Williams, R. H. Hearn, J. R. Zeidler, E. Dong, R. C. Goodlin, "Adaptive noise cancelling : Principles and applications," Proc. IEEE, vol.63, pp. 1692-1716, Dec. 1975.

[2] Niek A. M. Verhoeckx, Harry C. van den Elzen, Fred A. M. Snijders, Piet J. van Gerwen, "Digital Echo Cancellation for Baseband Data Transmission," IEEE Trans. on Acoustics, Speech, and Signal Processing, vol. ASSP-27, No.6, pp.768-781, Jun. 1981.

[3] Theo A. C. M. Claasen, Wolfgang F. G. Mecklenbrauker, "Comparison of the Convergence of Two Algorithms for Adaptive FIR Digital Filters," IEEE Trans. on Acoustics, Speech, and Signal Processing, vol. ASSP-29, No.3, pp.670-678, Jun. 1981.

[4] Niek A. M. Verhoeckx, Theo A. C. M. Claasen, "Some Considerations on the Design of Adaptive Digital Filters Equipped with the Sign Algorithm," IEEE Trans. on Comm. vol. COM-32, No.3, pp.258-266, Mar. 1984.

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