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Three Dimensional Dynamic Added Variable Plots

Han Son Seo[1)]

Abstract

Graphical methods for the specification of the curvature as a function of two predictors are animated to see the effect of an added variable to the model. Through a 3D animated plot it might be difficult to find a sequence of interpretable plots. But examples demonstrate that useful information can be obtained by using rotation technique in 3D plot. Besides 3D plots, an example of 2D animated plot applied to the case of high correlation between predictors and an added predictor is also given. It implies that speed of the convergence to a certain image in a dynamic plot may be understood as an influence of collinearity.

Keywords : ARES plot, ARC, Augmented partial residual plots, Dynamic graphics, Partial residual plots, Xlisp-stat.

1. Introduction

Consider a usual linear regression model :

$$Y=\beta_0+X\beta_1+\varepsilon \tag{1.1}$$

where β_0 is an intercept, X is an $n\times(p-1)$ full rank matrix, β_1 is a $(p-1)\times 1$ vector of unknown parameters, $E(\varepsilon)=0$ and $Var(\epsilon)=\sigma^2 I$. Assume that a new predictor W is added to the model (1.1):

$$Y=\beta_0+X\beta_1+\psi W+\varepsilon \tag{1.2}$$

where ψ is a scalar.

To identify the effects of adding a new variable to the regression model Cook and Weisberg(1989, 1994) proposed ARES plot, an acronym for "Adding REgressors Smoothly", which is controled by a control parameter $\lambda\in[0,1]$. As λ increases from 0 to 1 ARES plot represents a smooth transition of model so that λ=0 corresponds to fitting (1.1) and at λ=1

1) Professor, Department of Applied Statistics, Konkuk University
1 Hwayang-Dong, Kwangjin-Ku, Seoul 133-701, Korea
Email : hsseo@konkuk.ac.kr

the full model (1.2) is fit. An animated plot of { $\hat{y}_{\lambda,} e_{\lambda}$} gives a dynamic view of the effects of adding W to model (1.1) where $\hat{y}_{\lambda,} e_{\lambda}$ are, respectively, the fitted values and residuals obtained when the control parameter is equal to λ.

As in the model (1.1) a standard linear regression is based on the linearity assumption of the regression function. However in many cases, instead of using model (1.1), a model allowing a nonlinear relationship between a response and an explanatory variable, Z, is generally considered :

$$Y=\beta_0+X\beta_1+f(Z)+\varepsilon \tag{1.3}$$

where f is an unknown function.

For visualizing f in (1.3), besides added variable plot, partial residual plot (Larsen and McLeary, 1972; Weisberg, 1985), augmented partial residual plot (Mallows, 1986) and CERES plot, an abbreviated acronym for "Combining Conditional Expectations and RESiduals", (Cook, 1993) are suggested and compared (Johnson and McCulloch, 1987: Cook, 1996 : Berk and Booth, 1995)

Now consider adding a variable to the model (1.3), which is represented by the following model:

$$Y=\alpha_0+X\alpha_1+g(Z)+\gamma W+\varepsilon. \tag{1.4}$$

Parameters of eq. (1.3) and (1.4) are different as W is included in the model. But if W is independent of X given Z then $\beta_0=\alpha_0$, $\beta_1=\alpha_1$ and $f(Z)=g(Z)+\gamma E(W|Z)$. The impact of adding a variable on function f is expressed in terms of $E(W|Z)$. Appling ARES plot idea to this problem the dynamic plot of CERES plots, partial residual plots and augmented partial residual plots were constructed. (Seo, 1999).

In this article, 3D animated added variable plots are suggested with two dimensional variable of Z in (1.4). A couple of examples are given demonstrating the usefulness of the 3D animated plots. Also an example of applying 2D animated added variable plots of CERES plot, partial residual plot and augmented partial residual plot to the highly collinearity situation is provided. In section 2, the procedure for obtaining 2D animated added variable plot is reviewed and an example is given. In section 3 3D animated partial residual plot and augmented partial residual plot are presented with examples. Section 4 contains concluding remarks and discussions.

2. Two Dimensional Animated Plots

The procedure of constructing animated added variable plots is briefly reviewed. For details see Seo (1999). Animated added variable plots can be developed by using ARES idea. Consider the following model :

$$Y = X^* a + bZ + \gamma W + \varepsilon \tag{2.1}$$

where $X^* = (1_n : X)$, Z and W are $n \times p$, $n \times 1$ and $n \times 1$ known matrices respectively, and a, b , γ are corresponding parameter vector or scalar. Let $U = (X^* : Z)$, $\delta = (a^T : b)^T$, $\delta^* = \delta + \gamma (U^T U)^{-1} U^T W$, $\gamma^* = \| Q_u W \| \gamma$ and $\tilde{W} = Q_u W / \| Q_u W \|$ where Q_u is the projection operator on the orthogonal complement of the space spanned by the columns of U. Then eq. (2.1) can be modified and rewritten as

$$Y = U\delta^* + \gamma^* \tilde{W} + \varepsilon. \tag{2.2}$$

For each $0 < \lambda \leq 1$ $\alpha = (\delta^* : \gamma^*)^T$ is estimated by

$$\hat{\alpha}_\lambda = (V^T V + \frac{1-\lambda}{\lambda} cc^T)^{-1} V^T Y \tag{2.3}$$

where c is a $2p$ by 1 vector of zeros except for a single 1 corresponding to W, and $V = (U : \tilde{W})$. And for each λ and $\hat{\alpha}_\lambda$, other estimators are denoted as $\hat{\delta}_\lambda$, $\hat{\gamma}_\lambda$, $\hat{a}_\lambda$ and $\hat{b}_\lambda$.

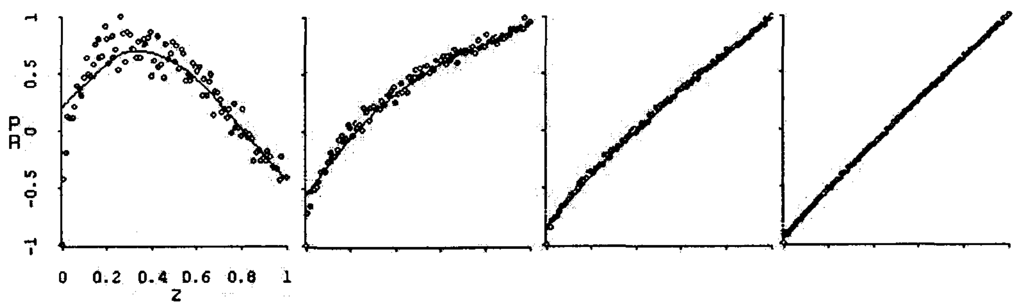
Animated partial residual plot is constructed as $e_\lambda + \hat{b}_\lambda Z$ versus Z, where $e_\lambda = Y - \hat{Y}_\lambda$. Animated augmented partial residual plot uses $Y = \rho_0 + X\rho_1 + \phi_1 Z + \phi_2 Z^2 + \gamma W + \varepsilon$ for some parameters, ρ_0, ρ_1, ϕ_1, ϕ_2, γ instead of using model (2.1) and is defined as $e_\lambda + \hat{\phi}_{1\lambda} Z + \hat{\phi}_{2\lambda} Z^2$ versus Z. Animated CERES plots are also constructed by letting $Y = \rho_0 + X\rho_1 + (E(X \mid Z) - E(Z))b + \varepsilon$ in the model (2.1) and defined as $e_\lambda + (E(X \mid Z) - E(Z)) \hat{b}_\lambda$ versus Z.

When $\lambda = 0$ estimators correspond to those in the fit of the regression model of Y on X and Z. As λ increases from 0 to 1, estimators become a sequence of estimators that represents the effect of adding W smoothly to the smaller model. Animated plots give a dynamic view of the effects of adding W to the model which already includes X and Z.

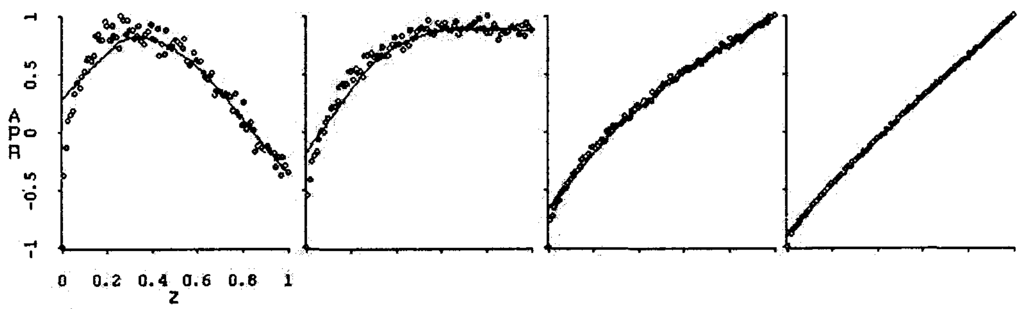
Example 1:

In the highly collinearity situation, as discussed by Berk and Booth (1995), the regression coefficients can be pushed to outsize levels, which causes the vertical range of partial residual plots, augmented partial residual plots and CERES plots to be extreme sufficiently that the curve is difficult to perceive. Berk and Booth (1995) recommended to use the plot of residual against the predictor for showing the curve when collinearity among predictors is very severe. Now we apply animated plots to a highly collinear predictors case and see how to get a valuable information from the animated display.

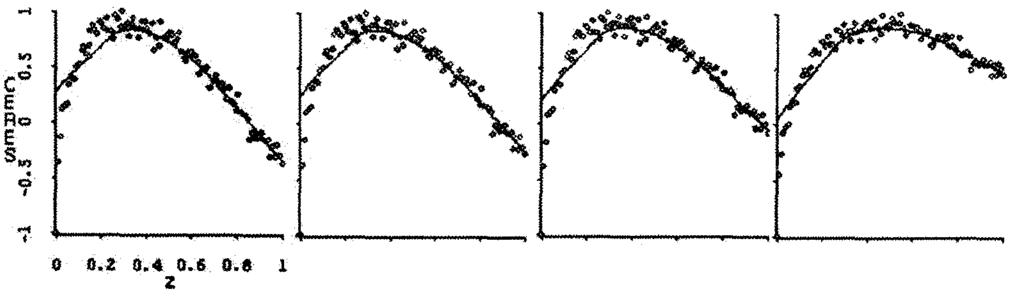
A data of 100 observations was generated similarly as in Berk and Booth(1995). X_1 and X_2 are independent normal random variables with mean 0 and variance 0.1^2. Z is consist of evenly spaced values between 0 and 1. W was constructed to have a high correlation of r =0.99 with Z . The response variable Y was defined by $Y=X_1+X_2+\sqrt{Z}-0.7Z-0.3W+0.01\varepsilon$ where ε is a standard normal random variable. Figure 1 contains four frames of three kinds of animated added plots of Z for adding W after X_1 , X_2 and Z (λ = 0, 0.1, 0.3, 1). From dynamic display of plots in figure 1 we find that when a ruinous collinearity exists animated plots, except CERES plots, converge very quickly to a certain image. The first frame suggests a quadratic form of the function. But as soon as λ moves from 0 plots converge instantly to a linear form and change a little ever after. When λ equal to 1 , that is W is fully included, as argued by Berk and Booth(1995), the partial residual plot and the augmented partial residual plot are stretched vertically by the collinearity, but the CERES plot is not.



(a) Animated Partial Residual Plots



(b) Animated Augmented Partial Residual Plots



(c) Animated CERES Plots

Figure 1. Animated plots for example 1. λ = 0, 0.1, 0.3, 1.

3. Three Dimensional Animated Plots

3D animated plot is constructed by extending the dimension of function f and g in model (1.3) and (1.4). Consider models :

$$Y = a_0 + Xa_1 + f(Z_1, Z_2) + \varepsilon \quad (3.1)$$

and

$$Y = b_0 + Xb_1 + g(Z_1, Z_2) + \rho W + \varepsilon. \quad (3.2)$$

Changes of g are observed through an animation displaying smooth transition between the fits of (3.1) and (3.2). For the estimation of the model we consider the following model

$$\begin{aligned} Y &= \rho_0 + X\rho_1 + \phi_1 Z_1 + \phi_2 Z_2 + \gamma W + \varepsilon \quad (3.3) \\ &= U\delta^* + \gamma^* W + \varepsilon \end{aligned}$$

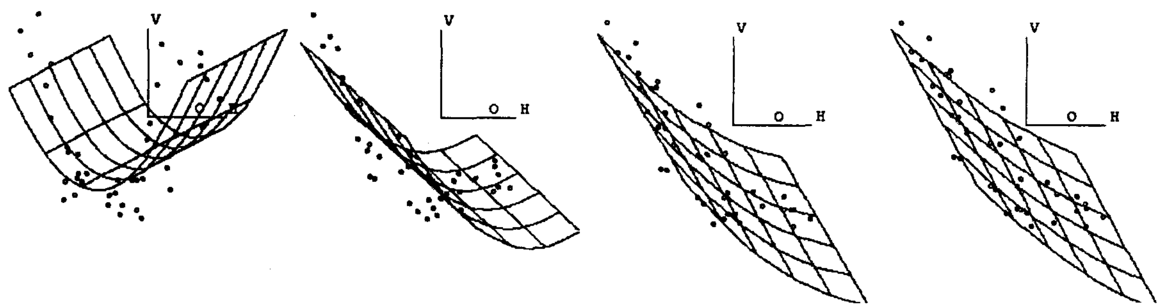
where $U = (1_n : X : Z_1 : Z_2)$, $\delta = (\rho_0,\ \rho_1^T,\ \phi_1,\ \phi_2)^T$, $\overline{W} = Q_u W / \|Q_u W\|$, $\delta^* = \delta + \gamma (U^T U)^{-1} U^T W$. With estimator calculated by the formula of (2.3), 3D animated partial residual plot is defined as the plot of $e_\lambda + \hat{\Phi}_{1\lambda} Z_1 + \hat{\Phi}_{2\lambda} Z_2$ versus (Z_1, Z_2). 3D augmented partial residual plot is constructed as the plot of $e_\lambda + \hat{\Phi}_{1\lambda} Z_1 + \hat{\Phi}_{2\lambda} Z_2 + \hat{\Phi}_{3\lambda} Z_1^2 + \hat{\Phi}_{4\lambda} Z_2^2$ versus (Z_1, Z_2), in which estimators come from the model

$$Y = \rho_0 + X\rho_1 + \phi_1 Z_1 + \phi_2 Z_2 + \phi_3 Z_1^2 + \phi_4 Z_2^2 + \gamma W + \varepsilon. \quad (3.4)$$

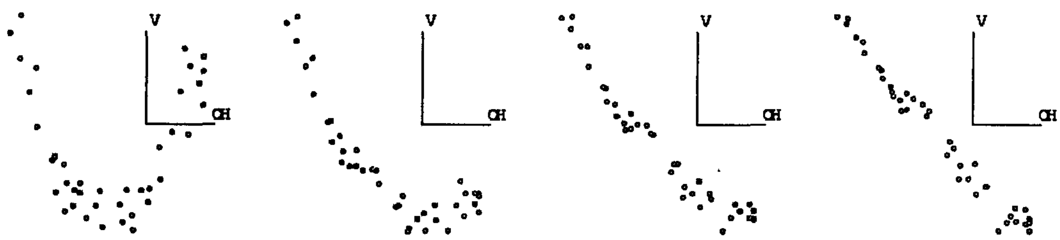
Models which include interaction terms can also be considered for the estimation of coefficients of the model (3.1). Throughout the examples 3D augmented partial residual plot based on (3.4) is used. Programs are coded under the environment of the package ARC (Cook and Weisberg, 1999) which is made by using Xlisp-stat (Tierney, 1990). We display $\hat{g}(Z_1, Z_2)$ on the vertical axis (V axis), and Z_1 and Z_2 on the two horizontal axis (H axis and O axis). 3D plot can be rotated about each of three axes. 3D rotation about the vertical axis is nothing more than rapidly updating the 2D plot y versus a liner combination of Z_1 and Z_2 in small steps. We usually stop rotation about vertical axis when 2D plot shows the most evident trend. We call this plot as *the best view plot*. 3D plot has a slider bar reflecting the change of λ. As holding down the mouse button on the slider, scroll bar is moving and the value of λ is changing. Accordingly the linear combination of Z_1 and Z_2 is calculated and updated plot is drawn instantly. By observing 3D plot as λ varies information about the function of Z_1 and Z_2 can be obtained.

Example 2:

An artificially created sample of 40 observations was used. Three variables X_1, X_2 and Z are independent uniform random variables with ranges (40, 80), (10, 60) and (10, 60) respectively. Z is randomly divided into two parts and assigned as Z_1 and Z_2. Y was generated as $0.31X_1+0.71X_2+(Z-10)(Z-60)+\varepsilon_1$ where ε_1 is a standard normal random variable. Added variable W was generated as $(Z^2+600)/100+\varepsilon_2$, where ε_2 is another independent standard normal random variable. Under this construction of variables the function of a linear combination of Z_1 and Z_2 is quadratic when W is not included in the model, and becomes linear as W is included in the model. Figure 2 (a) shows four frames of an animated plot of $\{ \hat{g}(z_1, z_2), (z_1, z_2) \}$ for adding W after X_1, X_2, Z_1, Z_2 . Vertical axis, $\hat{g}(z_1, z_2)$ has been scaled to have values between -1 and 1. The axis scales are identical for all frames. The horizontal axes stand for Z_1 and Z_2 . The first frame is for λ =0 and the next three frames in Figure 2 (a) correspond to λ = 0.3, 0.6, and 1 respectively. The first frame indicates a substantial nonlinearity. As λ moves from 0 to 1 the points give a clear message to warrant linear model. Thus the effect of adding W to the model is to make a linear fit of $\hat{g}(z_1, z_2)$ possible. Figure 2 (b) is the best view plot for each λ= 0, 0,3, 0,6, 1. Adding W makes curvature $\hat{g}(z_1, z_2)$ linear for the specific linear combination of Z_1 and Z_2, represented by the horizontal axis in the plot.



(a) 3D image of the animated plots. OLS smooth is superimposed.

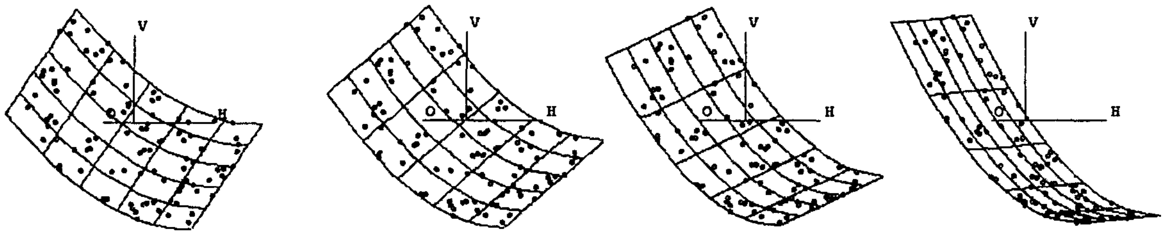


(b) Best view plots

Figure 2. Animated plots for example 2. λ = 0, 0.3, 0.6, 1.

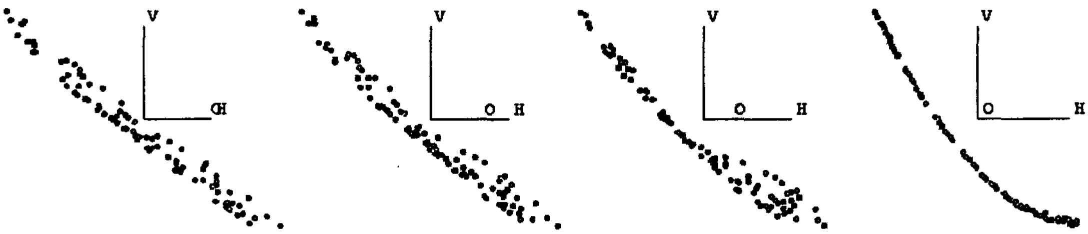
Example 3:

This example deals with the case that an added variable W is highly correlated with one of Z variables. A sample of 100 observations was generated by the model $Y = X_1 + X_2 + Z_1^2 - 0.7Z_2 - 0.3W + 0.1\varepsilon$ where X_1 and X_2 are independent normal random variables with mean 0 and variance 0.1^2, Z_1 is uniform random variable with range (-1, 0), Z_2 contains equally spaced values between -1 and 0 , W is constructed to be highly correlated, r =0.92, with Z_2 and ε is a standard normal random variable.



(a) 3D image of the animated plots. OLS smooth is superimposed.

(b) Views without rotation



(c) Best view plots

Figure 3. Animated plots for example 3. λ = 0, 0.3, 0.6, 1.

In figure 3 animated plots of augmented partial residual plots (V: $\hat{g}(Z_1, Z_2)$ H: Z_1 O: Z_2) are shown. Four frames of animated plots in figure 3 (a), (b) and (c) correspond to λ = 0, 0.3 0.6 1 respectively. As we can see in the figure (a) the slope of estimated surfaces become steep as λ moves from 0 to 1. Figure 3 (b) shows 3D augmented partial residual plots without rotation about vertical axis, which are equivalent to the 2D plot of $\hat{g}(Z_1, Z_2)$ vs. Z_1. Figure 3 (b) suggests that as λ moves from 0 to 1 g depends on only Z_1 and g is a quadratic function. As a result it implies that the addition of W makes it possible to remove variable Z_2 from the model. Four flames of figure 3 (c) are the best view of 3D augmented partial residual plot for each value of λ. Their corresponding horizontal linear combinations are $1.44Z_1+1.40Z_2$, $1.71Z_1+1.06Z_2$, $1.93Z_1+1.58Z_2, 2.01Z_1+0.04Z_2$ respectively. As λ moves from 0 to 1 best view's linear combination of Z_1 and Z_2 approaches to Z_1 and best views become more substantial.

4. Concluding Remarks

Dynamic added variable plots of partial residual plot, augmented partial residual plot and CERES plots are suggested to get additional information about the model. As noticed in the example, speed of the convergence to a certain image in a dynamic plot may be understood as influence of collinearity. 3D dynamic added variable plots can be used to grasp the influence of added variable on the model. But the problem of finding a short path of rotation to reduce exploration time still remains to be solved.

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