

A New Consideration for Discrete-System Reduction via Impulse Response Gramian

Younseok Choo and Jaeho Choi

Abstract: Recently a method of model reduction for discrete systems has been proposed in the literature based on a new impulse response Gramian. In this method, the system matrix A_r of a reduced model is computed by approximating the reduced-order impulse response Gramian. The remaining matrices b_r and c_r are obtained so that various initial Markov parameters and time-moments of the original system are preserved in the reduced model. In this paper a different approach is presented based on the recursive relationship among the impulse response Gramians.

Keywords: Impulse response gramian, discrete-system, model reduction, Lyapunov equation.

1. INTRODUCTION

A method of model reduction based on the impulse response Gramian (IRG) was first suggested by Sreeram and Athoklis for linear continuous [1] and discrete [2] systems. In [1], a reduced model was obtained by matching the reduced-order IRG and some initial Markov parameters of the original continuous system, whereas the method of [2] preserves the states corresponding to the dominant eigenvalues of the discrete IRG in the reduced model. In [3], the approach of [1] was applied to the reciprocal system of the original system to determine the reduced model retaining the reduced-order Gram matrix and some initial time-moments. In [4], it was shown that more general reduced models can be obtained using the methods of [1,3]. The reduced models obtained in [5] for discrete systems possess the same properties as those of [1] for continuous systems. It was also shown that reduced models obtained in [5] are related by similarity transforms to those derived by the well-known q -cover method [6].

Recently a new discrete IRG was introduced in [7] and applied to the model reduction of discrete systems.

In [7], the system matrix A_r of a reduced model is obtained by approximating the reduced-order IRG. The remaining matrices b_r and c_r are determined so that the reduced model matches certain initial Markov parameters and time-moments of the original system.

A different approach is presented in this paper based on the recursive relationship among the IRGs. It is shown that the system matrix can also be obtained from the recursive relationship. Then the result is applied to the model reduction problem. The reduced model derived also approximates the reduced-order IRG, and matches various initial Markov parameters and time-moments of the original model as in [7]. However the method used in this paper leads to a different reduced model from that of [7].

This paper is organized as follows. In Section 2, the IRG introduced in [7] and its applications to model reduction are briefly reviewed. The main results of this paper are contained in Section 3, and two numerical examples are presented in Section 4. Finally, the paper is concluded in Section 5.

2. BRIEF REVIEW OF EXISTING RESULT

2.1. Discrete impulse response Gramian

Consider an n th-order stable single-input, single-output linear time-invariant discrete system described by the minimal state-space realization (A, b, c) with the impulse response $h(k) = cA^{k-1}b$. For the system, the n th-order IRG introduced in [7] is defined by

$$W_{q,n} = \sum_{k=1}^{\infty} \begin{bmatrix} h_q^2(k) & h_q(k)h_{q+1}(k) & \cdots & h_q(k)h_{q+n-1}(k) \\ h_q(k)h_{q+1}(k) & h_{q+1}^2(k) & \cdots & h_{q+1}(k)h_{q+n-1}(k) \\ \vdots & \vdots & \ddots & \vdots \\ h_q(k)h_{q+n-1}(k) & h_{q+1}(k)h_{q+n-1}(k) & \cdots & h_{q+n-1}^2(k) \end{bmatrix} \quad (1)$$

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where $-(n-1) \leq q \leq 0$ and

$$h_{l+1}(k) = h_l(k+1) - h_l(k), \quad l \geq 0 \quad (2)$$

$$h_{l-1}(k) = -\sum_{m=k}^{\infty} h_l(m), \quad l \leq 0 \quad (3)$$

with $h_0(k) = h(k)$. The $(n+1)$ th-order IRG $W_{q,n+1}$ is defined similarly.

For each q , the n th-order IRG $W_{q,n}$ is the unique positive definite solution to the Lyapunov equation [7]

$$\hat{A}^T W_{q,n} \hat{A} - W_{q,n} = -\hat{c}_q^T \hat{c}_q, \quad (4)$$

where the realization $(\hat{A}, \hat{b}_q, \hat{c}_q)$ is obtained from (A, b, c) by the similarity transformation as

$$\hat{A} = C_q^{-1} A C_q, \quad \hat{b}_q = C_q^{-1} b, \quad \hat{c}_q = c C_q \quad (5)$$

$$C_q = \begin{bmatrix} (A - I_n)^q b & \cdots & (A - I_n)^{q+n-1} b \end{bmatrix} \quad (6)$$

and takes the following form

$$\hat{A} = \begin{bmatrix} 1 & 0 & \cdots & 0 & -\hat{a}_n \\ 1 & 1 & \ddots & \vdots & -\hat{a}_{n-1} \\ 0 & 1 & \ddots & 0 & \vdots \\ \vdots & \ddots & \ddots & 1 & -\hat{a}_2 \\ 0 & \cdots & 0 & 1 & 1 - \hat{a}_1 \end{bmatrix}, \quad (7)$$

$$\hat{b}_q = [0 \quad \cdots \quad 0 \quad 1 \quad 0 \quad \cdots \quad 0]^T, \quad (8)$$

$$\hat{c}_q = [t_{-q} \quad \cdots \quad t_1 \quad \hat{m}_1 \quad \cdots \quad \hat{m}_{q+n}]. \quad (9)$$

In (8), \hat{b}_q has only 1 in the $(1-q)$ th position. In (9), $t_i = c(A - I_n)^{-i} b$ denotes the i th time-moment of the system and

$$\hat{m}_i = c(A - I_n)^{i-1} b = \sum_{j=0}^{i-1} (-1)^j \binom{i-1}{j} m_{i-j}, \quad (10)$$

where $m_i = cA^{i-1} b$ is the i th Markov parameter of the system.

Since $\hat{A} - I_n$ is in controllability canonical form [8], the \hat{a}_i s in (7) constitute the coefficients of the characteristic polynomial for $\hat{A} - I_n$. Consequently, the characteristic polynomial of the system can be determined once the \hat{a}_i s are obtained. It was shown in [7] that the $(n+1)$ th-order IRG $W_{q,n+1}$ contains information on the characteristic polynomial of the

system in the following sense: Partition $W_{q,n+1}$ as

$$W_{q,n+1} = \begin{bmatrix} W_{q,n} & \mathbf{w}_{q,n+1} \\ \mathbf{w}_{q,n+1}^T & w_{q,n+1} \end{bmatrix} \quad (11)$$

then the \hat{a}_i s in (7) can be computed by

$$\hat{\mathbf{a}} = -W_{q,n}^{-1} \mathbf{w}_{q,n+1}, \quad (12)$$

where $\hat{\mathbf{a}} = [\hat{a}_n \quad \hat{a}_{n-1} \quad \cdots \quad \hat{a}_1]^T$.

2.2. Model reduction

Based on the above results, Azou et al. [7] derived the r th-order reduced models such that, for each q , the reduced model approximates the r th-order IRG $W_{q,r}$ and retains various initial Markov parameters and time-moments of the original model. Let $(\tilde{A}_q, \tilde{b}_q, \tilde{c}_q)$ be the r th-order realization with the structure given in (7), (8) and (9). The coefficients $\{\tilde{a}_i\}_{1 \leq i \leq r}$ for \tilde{A}_q are computed from the $(r+1)$ th order IRG $W_{q,r+1}$ as in (12): Let

$$W_{q,r+1} = \begin{bmatrix} W_{q,r} & \mathbf{w}_{q,r+1} \\ \mathbf{w}_{q,r+1}^T & w_{q,r+1} \end{bmatrix} \quad (13)$$

then

$$\tilde{\mathbf{a}} = -W_{q,r}^{-1} \mathbf{w}_{q,r+1}, \quad (14)$$

where $\tilde{\mathbf{a}} = [\tilde{a}_r \quad \tilde{a}_{r-1} \quad \cdots \quad \tilde{a}_1]^T$. \tilde{b}_q and \tilde{c}_q consist of the first r elements of \hat{b}_q and \hat{c}_q , respectively.

The structure of $(\tilde{A}_q, \tilde{b}_q, \tilde{c}_q)$ indicates that the reduced model obtained for each q retains the first $-q$ time-moments and the first $(r+q)$ Markov parameters of the original model. On the other hand, the original r th-order IRG $W_{q,r}$ solves the following equation [7]

$$\tilde{A}_q^T W_{q,r} \tilde{A}_q - W_{q,r} = -\tilde{c}_q^T \tilde{c}_q - \delta Q_q, \quad (15)$$

where δQ is the $r \times r$ matrix with all entries equal to zero except the (r,r) th position. Hence the original r th-order IRG $W_{q,r}$ is approximately preserved in the reduced model.

3. MAIN RESULTS

It was illustrated in [7] that the \hat{a}_i s in (7) can be obtained from the $(n+1)$ th-order IRG by (12). Using

the result, the system matrix of a reduced model was determined by (14). In this section a different approach is presented based on the recursive relationship among the IRGs.

Theorem 1: Let $\bar{A} = \hat{A} - I_n$. Then, for each q , we have

$$W_{q+1,n} = \bar{A}^T W_{q,n} \bar{A}. \tag{16}$$

Proof: Premultiplying \bar{A}^T and postmultiplying \bar{A} on both sides of (4) lead to

$$\hat{A}^T \bar{A}^T W_{q,n} \bar{A} \hat{A} - \bar{A}^T W_{q,n} \bar{A} = -\bar{A}^T \hat{c}_q^T \hat{c}_q \bar{A}. \tag{17}$$

First we show that $\hat{c}_q \bar{A} = \hat{c}_{q+1}$. From (9),

$$\begin{aligned} \hat{c}_q \bar{A} &= \begin{bmatrix} t_{-q} & \cdots & t_1 & \hat{m}_1 & \cdots & \hat{m}_{q+n} \\ t_{-q-1} & \cdots & t_1 & \hat{m}_1 & \cdots & \hat{m}_{q+n} \end{bmatrix} \bar{A} \\ &= \begin{bmatrix} \hat{a}_n & \hat{a}_{n-1} & \cdots & \hat{a}_1 \\ \hat{a}_n & \hat{a}_{n-1} & \cdots & \hat{a}_1 \end{bmatrix} \bar{A} - \hat{c}_q \hat{\mathbf{a}} \end{aligned} \tag{18}$$

where $\hat{\mathbf{a}} = [\hat{a}_n \ \hat{a}_{n-1} \ \cdots \ \hat{a}_1]^T$. Since $(A - I_n)$ and $\hat{A} - I_n$ have the same characteristic equations, we have

$$|zI_n - (A - I_n)| = z^n + \hat{a}_1 z^{n-1} + \cdots + \hat{a}_{n-1} z + \hat{a}_n. \tag{19}$$

Then, by the Cayley-Hamilton theorem, we obtain

$$\begin{aligned} c(A - I_n)^{q+n} b &= -\hat{a}_1 c(A - I_n)^{q+n-1} b - \cdots \\ &\quad - \hat{a}_n c(A - I_n)^q b. \end{aligned} \tag{20}$$

From the definitions of t_i s and \hat{m}_i s, the last element in (18) is given by

$$\begin{aligned} -\hat{c}_q \hat{\mathbf{a}} &= -\left[c(A - I_n)^q b \ \cdots \ c(A - I_n)^{q+n-1} b \right] \hat{\mathbf{a}} \\ &= -\hat{a}_n c(A - I_n)^q b - \cdots - \hat{a}_1 c(A - I_n)^{q+n-1} b \\ &= c(A - I_n)^{q+n} b. \end{aligned} \tag{21}$$

Since the last term in (21) equals \hat{m}_{q+n+1} , we have $\hat{c}_q \bar{A} = \hat{c}_{q+1}$, and (17) becomes

$$\hat{A}^T \bar{A}^T W_{q,n} \bar{A} \hat{A} - \bar{A}^T W_{q,n} \bar{A} = -\hat{c}_{q+1}^T \hat{c}_{q+1}. \tag{22}$$

Then (16) easily follows from the fact that, for each q , $W_{q,n}$ is the unique solution to the Lyapunov equation given in (4). \square

The recursive relationship given in (16) provides an efficient way of computing IRGs. Once $W_{-(n-1),n}$ is computed by solving (4), then $W_{q,n}$, for each $-(n-2) \leq q \leq 0$, can be obtained by (16) without solving (4). Conversely, a formulation different from (12) can be derived from (16) for the computation of \hat{a}_i s in (7). For ease of presentation, let w_{ij} denote the (i,j) th element of the $(n+1)$ th-order IRG $W_{q,n+1}$, and let

$$W_{q,n+1}[k,l] = \begin{bmatrix} w_{kk} & w_{k,k+1} & \cdots & w_{kl} \\ w_{k,k+1} & w_{k+1,k+1} & \cdots & w_{k+1,l} \\ \vdots & \vdots & \ddots & \vdots \\ w_{kl} & w_{k+1,l} & \cdots & w_{ll} \end{bmatrix}. \tag{23}$$

Then we have $W_{q,n} = W_{q,n+1}[1,n]$ and $W_{q+1,n} = W_{q,n+1}[2,n+1]$.

Theorem 2: The \hat{a}_i s in (7) are computed by

$$\hat{a}_n = \sqrt{\frac{|W_{q,n+1}[2,n+1]|}{|W_{q,n+1}[1,n]|}}, \tag{24}$$

$$\hat{\mathbf{a}}_{n-1} = -W_{q,n+1}^{-1}[2,n](\mathbf{w}_{1,n-1} \hat{a}_n + \mathbf{w}_{2,n-1}), \tag{25}$$

where

$$\hat{\mathbf{a}}_{n-1} = [\hat{a}_{n-1} \ \hat{a}_{n-2} \ \cdots \ \hat{a}_1]^T,$$

$$\mathbf{w}_{1,n-1} = [w_{12} \ w_{13} \ \cdots \ w_{1n}]^T,$$

$$\mathbf{w}_{2,n-1} = [w_{2,n+1} \ w_{3,n+1} \ \cdots \ w_{n,n+1}]^T.$$

Proof: From (16), we obtain n equations as follows:

$$\mathbf{w}_{1,n-1} \hat{a}_n + W_{q,n+1}[2,n] \hat{\mathbf{a}}_{n-1} + \mathbf{w}_{2,n-1} = \mathbf{0}, \tag{26}$$

$$w_{11} \hat{a}_n^2 + \mathbf{w}_{1,n-1}^T \hat{\mathbf{a}}_{n-1} \hat{a}_n - \mathbf{w}_{2,n-1}^T \hat{\mathbf{a}}_{n-1} - w_{n+1,n+1} = 0. \tag{27}$$

Since $W_{q,n+1}[2,n+1]$ is positive definite, $W_{q,n+1}^{-1}[2,n]$ exists and (25) follows from (26). Substituting (25) into (27), we obtain the quadratic equation of the form

$$d_1 \hat{a}_n^2 + d_2 = 0, \tag{28}$$

where

$$\begin{aligned} d_1 &= w_{11} - \mathbf{w}_{1,n-1}^T W_{q,n+1}^{-1}[2,n] \mathbf{w}_{1,n-1} \\ &= \frac{|W_{q,n+1}[1,n]|}{|W_{q,n+1}[2,n]|}, \end{aligned} \tag{29}$$

$$\begin{aligned} d_2 &= \mathbf{w}_{2,n-1}^T W_{q,n+1}^{-1} [2, n] \mathbf{w}_{2,n-1} - w_{n+1,n+1} \\ &= -\frac{|W_{q,n+1}[2, n+1]|}{|W_{q,n+1}[2, n]|}. \end{aligned} \quad (30)$$

Then (24) follows from the fact the \hat{a}_i s are all positive since the given system is stable. \square

Now, for each $q = -(r-1), \dots, 0$, the r th-order reduced realization $(\tilde{A}_q, \tilde{b}_q, \tilde{c}_q)$ with the structure given in (7)-(9) can be obtained as follows: The coefficients $\{\tilde{a}_i\}_{1 \leq i \leq r}$ for \tilde{A}_q are computed from the $(r+1)$ th order IRG $W_{q,r+1}$ by

$$\tilde{a}_r = \sqrt{\frac{|W_{q,r+1}[2, r+1]|}{|W_{q,r+1}[1, r]|}}, \quad (31)$$

$$\tilde{\mathbf{a}}_{r-1} = -W_{q,r+1}^{-1} [2, r] (\mathbf{w}_{1,r-1} \tilde{a}_r + \mathbf{w}_{2,r-1}), \quad (32)$$

where

$$\tilde{\mathbf{a}}_{r-1} = [\tilde{a}_{r-1} \quad \tilde{a}_{r-2} \quad \dots \quad \tilde{a}_1]^T,$$

$$\mathbf{w}_{1,r-1} = [w_{12} \quad w_{13} \quad \dots \quad w_{1r}]^T,$$

$$\mathbf{w}_{2,r-1} = [w_{2,r+1} \quad w_{3,r+1} \quad \dots \quad w_{r,r+1}]^T,$$

\tilde{b}_q and \tilde{c}_q consist of the first r elements of \hat{b}_q and \hat{c}_q , respectively.

The r th-order reduced model obtained above preserves the first $-q$ time-moments and the first $(r+q)$ Markov parameters of the original model as in [7]. On the other hand, it can be shown that the original r th-order IRG $W_{q,r}$ satisfies

$$\tilde{A}_q^T W_{q,r} \tilde{A}_q - W_{q,r} = -\tilde{c}_q^T \tilde{c}_q - \delta \tilde{Q}_q, \quad (33)$$

where

$$\delta \tilde{Q}_q = \begin{bmatrix} 0 & \dots & 0 & \alpha \\ \vdots & \vdots & 0 & 0 \\ 0 & 0 & \dots & \vdots \\ \alpha & 0 & \dots & 0 \end{bmatrix}$$

with $\alpha = w_{1,r+1} + \tilde{a}_r w_{11} + \tilde{a}_{r-1} w_{12} + \dots + \tilde{a}_1 w_{1r}$.

Hence the r th-order reduced model approximates the original r th-order IRG.

4. EXAMPLES

In this section two numerical examples are presented to illustrate the method chosen for this paper. The reduced models obtained by the proposed method

are compared to those of [7] and the well-known bilinear Routh approximation method [9] in terms of time and frequency responses.

Example 1: Consider the fifth-order system described by the transfer function

$$H(z) = \frac{z^4 - 1.0616z^3 + 0.7545z^2 + 0.0015z - 0.0349}{z^5 - 0.3z^4 - 0.87z^3 + 0.307z^2 + 0.082z - 0.022} \quad (34)$$

which is to be approximated by the second-order system. The method used in this paper leads to the reduced models $\{H_{2,q}(z), q = -1, 0\}$ given by

$$H_{2,-1}(z) = \frac{z - 0.052}{z^2 + 0.1027z - 0.8195}, \quad (35)$$

$$H_{2,0}(z) = \frac{z - 0.466}{z^2 + 0.2956z - 0.602}. \quad (36)$$

Second-order reduced models $\{\tilde{H}_{2,q}(z), q = -1, 0\}$ derived by the method of [7] are respectively given by

$$\tilde{H}_{2,-1}(z) = \frac{z - 0.1481}{z^2 + 0.0687z - 0.8142}, \quad (37)$$

$$\tilde{H}_{2,0}(z) = \frac{z - 0.755}{z^2 + 0.0066z - 0.871}. \quad (38)$$

Alternatively, the bilinear approximation method [9] yields

$$\hat{H}_2(z) = \frac{0.5611z - 0.2842}{1.3132z^2 - 1.9586z + 0.7281}. \quad (39)$$

As noted earlier, $H_{2,-1}(z)$ and $\tilde{H}_{2,-1}(z)$ preserve the first time-moment and the first Markov parameter of the original system, while approximating the second-order IRG, $W_{-1,2}$ in (3). Similarly, $H_{2,0}(z)$ and $\tilde{H}_{2,0}(z)$ exactly match the first two Markov parameters of the original system, and approximately retain the second-order IRG, $W_{0,2}$. $\hat{H}_2(z)$ in (39) is guaranteed to be stable but fits the first two time-moments only.

Table 1 compares the approximation performance of five reduced models in terms of the time responses. $\tilde{H}_{2,0}(z)$ derived by the method of [7] is the best reduced model for the impulse response error, while $H_{2,-1}(z)$ obtained by the approach of this paper shows the best approximation performance for the unit-step response error.

In Fig. 1, the frequency responses of $H_{2,-1}(z)$,

$\tilde{H}_{2,0}(z)$ and $\hat{H}_2(z)$ are compared with that of the original system. Since $H_{2,-1}(z)$ preserves the first time-moment and the first Markov parameter, the frequency response of $H_{2,-1}(z)$ is close to that of the original system at both low and high frequencies. The reduced model $\tilde{H}_{2,0}(z)$ demonstrates a better frequency response than $H_{2,-1}(z)$ at high frequencies since more Markov parameters are retained in $\tilde{H}_{2,0}(z)$. The reduced model $\hat{H}_2(z)$ obtained by the bilinear Routh approximation method is very accurate at low frequencies, but possesses completely different characteristics at high frequencies.

Example 2: For the six-order system represented by the transfer function

$$H(z) = \frac{1.0996z^5 - 0.9875z^4 - 1.0777z^3 + 0.9852z^2 + 0.2886z - 0.2285}{z^6 - 4z^5 + 6.645z^4 - 5.863z^3 + 2.9008z^2 - 0.7648z + 0.0842} \quad (40)$$

we compute the third-order reduced model. For this example, the methods of [7] and this paper give rise to very similar results. Hence only the proposed method will be compared with the bilinear approximation method of [9]. The following three third-order reduced models are obtained by the proposed method:

$$H_{3,-2}(z) = \frac{1.0996z^2 + 1.5698z + 0.5897}{z^3 - 1.674z^2 + 0.941z - 0.1771} \quad (41)$$

$$H_{3,-1}(z) = \frac{1.0996z^2 + 1.5599z + 0.5696}{z^3 - 1.6834z^2 + 0.9552z - 0.1827} \quad (42)$$

$$H_{3,0}(z) = \frac{1.0996z^2 + 1.5441z + 0.5446}{z^3 - 1.6977z^2 + 0.9788z - 0.1934} \quad (43)$$

The method of [9] yields

$$\hat{H}_3(z) = \frac{6.1079z^2 - 10.1343z + 4.3915}{1.2543z^3 - 3.2007z^2 + 2.7507z - 0.7942} \quad (44)$$

In Table 2, the ISE of impulse and unit-step responses are compared for four reduced models. Among them, $H_{3,-1}(z)$ shows the best approximation performance in time responses. Fig. 2 compares the frequency responses of $H_{3,-2}(z)$ and $\hat{H}_3(z)$ with that of the original system. It can be seen that the frequency response of $H_{3,-2}(z)$ almost coincides with that of the original system while $\hat{H}_3(z)$ fails to reproduce the original frequency response.

Table 1. Comparison of integral-squared-error (ISE).

Reduced model	ISE of impulse response	ISE of unit-step response
$H_{2,-1}(z)$	1.5244	0.8554
$H_{2,0}(z)$	0.9507	∞
$\tilde{H}_{2,-1}(z)$	1.2825	1.0844
$\tilde{H}_{2,0}(z)$	0.7302	∞
$\hat{H}_2(z)$	7.2533	1.9176

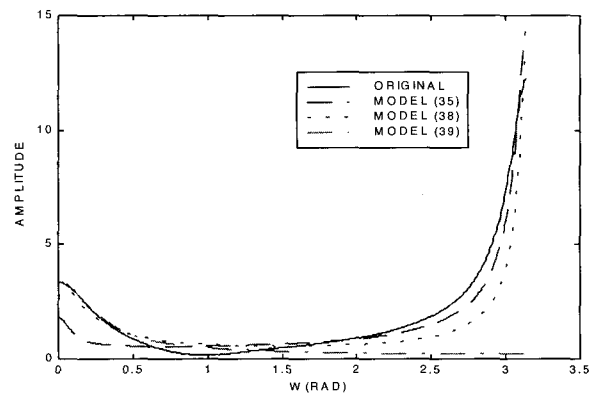


Fig. 1. Comparison of frequency responses.

Table 2. Comparison of integral-squared-error (ISE).

Reduced Model	ISE of impulse response	ISE of unit-step response
$H_{3,-2}(z)$	0.0014	0.0140
$H_{2,0}(z)$	0.0015	0.0185
$H_{3,0}(z)$	0.0021	∞
$\hat{H}_3(z)$	32.6749	120.4323

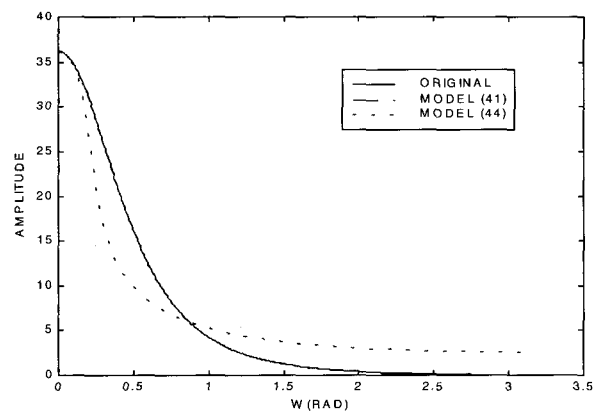


Fig. 2. Comparison of frequency responses.

5. CONCLUSIONS

In this paper an alternative approach was presented for a recently proposed method of model reduction for discrete systems. A recursive relationship among the IRGs was established. It was shown that the transformed system matrix of the original system can be obtained from the recursive relationship. The result was applied to the model reduction problem. Numerical examples indicate that mixed use of the methods of [7] and this paper may raise the possibility of obtaining superior reduced models.

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