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준 최적 빔 형성과 페이딩 상관을 갖는 송신 안테나 다이버시티 시스템의 성능

(Performance of Closed-loop Transmit Antenna Diversity System with
Sub-optimal Beam-forming and Fading Correlation)

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요 약

주파수 비선택적 Rayleigh 페이딩 채널에서 준 최적 빔 형성과 페이딩 상관이 페루프 송신 안테나 다이버시티(CTD) 시스템에 주는 영향을 분석하였다. 이동국에서 채널을 완벽하게 예측하는 것은 빠르게 변하는 채널의 페이딩으로 매우 어렵기 때문에 각각의 송신 안테나에 불완전한 가중치를 곱하게 된다. 이 불완전한 가중치는 준 최적의 빔을 형성하게 되고, CTD 시스템의 성능을 저하시킨다. 무선 채널의 페이딩 상관 역시 다이버시티 이득을 감소시키는 하나의 요소이다. CTD 시스템의 비트 오류율을 채널 예측 에러, 채널 상관계수, 궤환지연, 그리고 페이딩 지수의 함수로 해석적으로 유도하였다. 해석 결과 채널 예측 에러가 채널상관보다 시스템성능에 더 많은 영향을 주는 것을 알 수 있었다.

Abstract

The effect of the sub-optimal beam-forming and the fading channel correlation on the closed loop transmit antenna diversity(CTD) system is investigated in frequency flat Rayleigh fading channels. The fast channel fading prevents the perfect channel estimation at a mobile station, hence the imperfect weight is applied to the antenna branch of transmitter. The weight causes sub-optimal beam-forming and aggravates the performance of CTD system. The fading correlation of a wireless channel also is one of the factors decreasing the diversity gain. A bit error rate expression for the CTD system is analytically derived as a function of the channel estimation error, the channel correlation coefficient, the feedback delay, and fading index. It is shown that the channel estimation error gives more severe effect to the system performance than the channel correlation.

Keywords: fading correlation, channel estimation, transmit antenna diversity, Rayleigh channel

I. INTRODUCTION

Recently, a considerable number of studies have been conducted on the multiple transmit/receive antenna systems that have huge capacity increase and good transmission quality without sacrificing spectral bandwidth [1]-[2]. Multiple antenna systems are classified into four schemes; Multiple-Input Multiple

Output (MIMO), Multiple-Input Single-Output (MISO), Single-Input Multiple-Output (SIMO), Single-Input Single-Output (SISO) systems. It is well known that the MIMO system has largest capacity, however, the physical limit prevents its adoption with multiple antennas at both the base station (BS) and the mobile station (MS) [3]. This limit leads to the MISO, also called as transmit antenna diversity, for the downlink channel, which has multiple transmit antennas at BS and single receive antenna at MS.

There are two kinds of transmit antenna diversity systems, open-loop transmit diversity (OTD) system

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and closed-loop transmit diversity (CTD) system [4]-[5]. The OTD system has spatial diversity gain in wireless fading channels, where the channel state information (CSI) is not required at the transmitter. However, the CTD system requires CSI at the BS in order to adjust the transmit beam to the desired MS, which leads to a beam-forming gain in fading channels. It is well known that the CTD system has better performance than the OTD system under the assumption of perfect CSI at the transmit side [5].

In general, the downlink and the uplink channel characteristics are different in frequency division duplex (FDD) systems. In such a system, the MS estimates CSI and sends the result to the BS through a feedback loop in order to provide the downlink CSI. Therefore, in practical FDD CTD systems, the assumption of perfect CSI is unrealistic. There are three main reasons to cause imperfect CSI in FDD systems; (a) CSI feedback delay, (b) channel estimation errors, and (c) channel errors in the feedback loop [1]. The effect of the imperfect channel estimation on Alamouti's transmit diversity scheme is investigated in [6].

While most of the previous studies of the CTD system are assumed the independent identically distributed fading channel which is not practical in most of the wireless channel. It is well known that the fading channel correlation degrades system performance and system capacity. The correlation is affected by the spacing between the antenna elements, the angle of incident wave, and the environment around transmitter and receiver [7]-[9].

Therefore, in this paper, the performance of a CTD system in the presence of channel estimation error and channel correlation is examined. The channel is assumed to be a frequency-flat Rayleigh fading channel. The effect of the symbol errors in the feedback path is not addressed since the data rate of the feedback loop is very low (i.e., large signal energy of the feedback information is achieved from the long symbol duration).

This paper is organized as follows: In Section II, the CTD system model is presented. In Section III,

the effect of the channel estimation error and correlation is addressed. The probability density function of a received envelope is derived in Section IV. Numerical examples and discussions are given in Section V. Finally, Section VI concludes the paper

II. SYSTEM MODEL

A CTD system with P transmit antennas to be considered in this paper is shown in Fig. 1. The input data is weighted for the beam-forming gain at each transmit branch. The weight is determined by the feedback information from a receiver. The baseband equivalent complex received signal $x(m)$ at time m can be described by

$$x(m) = \sqrt{\frac{E_b}{N_o}} H(m) W(m) s(m) + n(m) \quad (1)$$

where E_b/N_o is the received signal-to-noise ratio (SNR), and $H(m) = [h_1(m), h_2(m), \dots, h_p(m)]$ is the $1 \times P$ complex Gaussian channel gain vector. We assume that each element of $H(m)$ has zero mean with unit variance. $W(m) = [w_1(m), w_2(m), \dots, w_p(m)]^T$ is the $P \times 1$ weighting vector where the superscript T denotes the transpose. We assume the transmit power is constrained as $W(m)^2 = 1$ where \cdot denotes the norm. $s(m)$ is the transmitted antipodal signal, ± 1 with equal probability, and $n(m)$ is the zero mean complex WSS(wide sense stationary) additive Gaussian noise with unit variance. The total

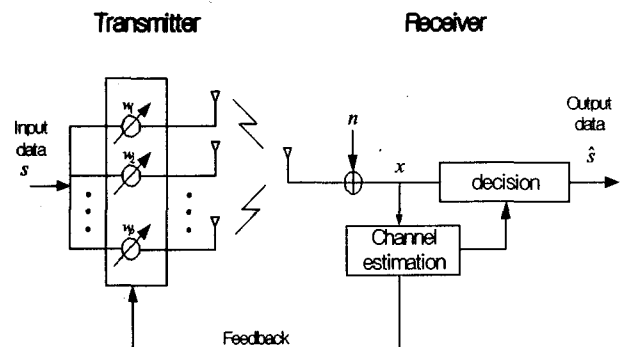


그림 1. 시스템 모델
Fig. 1. System model.

transmitted energy is constrained to E_b regardless to the number of the transmit antennas.

III. CHANNEL ESTIMATION ERROR AND FEEDBACK DELAY

The channel gain vector is estimated in a receiver. If the estimation is perfect, there are no estimation errors. In realistic channel, however, the fast fluctuation of the wireless channel prevents the perfect estimation, consequently the imperfect channel estimation is frequent.

a. Channel estimation error

The $1 \times P$ estimated channel gain vector $G(m) = [g_1(m), g_2(m), \dots, g_p(m)]$, and each element has zero mean complex Gaussian with variance σ_G^2 . can be represented by

$$G(m) = H(m) + Z(m) \quad (2)$$

where $Z(m) = [z_1(m), z_2(m), \dots, z_p(m)]$ is the $1 \times P$ estimation error vector whose element has zero mean complex Gaussian with variance σ_Z^2 . The variance σ_Z^2 is

$$\sigma_G^2 = \sigma_H^2 + \sigma_Z^2 \quad (3)$$

The estimated channel gain vector $G(m)$ is obtained from the given condition of the complex Gaussian channel gain vector $H(m)$, and they are jointly Gaussian [10]. We can write the channel gain vector

$$H(m) = \rho_e G(m) + D(m) \quad (4)$$

where $D(m) = [d_1(m), d_2(m), \dots, d_p(m)]$ is the $1 \times P$ vector, each element has independent zero mean complex Gaussian with variance σ_D^2 . And the variance of the element of channel gain vector can be written by

$$\sigma^2 H = \rho_e^2 \sigma_G^2 + \sigma_D^2 \quad (5)$$

where the coefficient ρ_e is given by [6], [10]

$$\rho_e = \rho_{GH} \frac{\sigma_H}{\sigma_G} \quad (6)$$

where the correlation coefficient ρ_{GH} between the elements of $G(m)$ and $H(m)$ is defined by

$$\rho_{GH} = \frac{\sigma_H}{\sigma_G} = \sqrt{\frac{1}{1 + \sigma_Z^2/\sigma_H^2}} = \sqrt{\frac{1}{1 + ESR}} \quad (7)$$

where ESR denotes estimation error-to-signal ratio,

$$ESR = \frac{\sigma_Z^2}{\sigma_H^2} \quad (8)$$

Replacing (6) and (7) into (5), the variance σ_D^2 can be written by [9]

$$\sigma_D^2 = \sigma_H^2 - \rho_e^2 \sigma_G^2 = (1 - \rho_{GH}^2) \sigma_H^2 \quad (9)$$

b. Feedback delay

Let us denote d as the feedback delay from a receiver to the transmitter. Then, the relationship between the current and the delayed estimated channel gain vector is given by [11]

$$G(m) = \rho_G G(m-d) + \epsilon(m) \quad (10)$$

where $G(m-d)$ is the delayed channel gain vector, and ρ_G is the time correlation coefficient. When using the generally known Jakes fading channel model [12], ρ_G is given by

$$\rho_G = J_0(2\pi f_d T d) \quad (11)$$

where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, f_d is the Doppler frequency shift, T is symbol duration, and d is time delay.

In (10), $\epsilon(m)$ is the $1 \times P$ delay error vector whose element is independent zero mean complex Gaussian distributed with variance $\sigma_\epsilon^2 = (1 - \rho_G^2) \sigma_G^2$ [10].

Substituting (4) and (10) into (1), the received signal in the presence of the feedback delay and the channel estimation error at time m can be rewritten

as

$$x(m) = \sqrt{\frac{E_b}{N_o}} [\rho_e \{\rho_G G(m-d) + \epsilon(m)\} + D(m)] \times W_{sub}(m) s(m) + n(m) \quad (12)$$

where $W_{sub}(m)$ is the sub-optimal antenna weighing in the presence of the feedback delay. In the case of perfect CSI at the transmitter, the optimum weighting is well known as [5]

$$W_o(m) = \frac{H^H(m)}{H(m)} \quad (13)$$

If the channel estimation is not perfect, then the channel gain vector $H(m)$ is replaced by the estimated channel gain vector $G(m)$ at time m . Also the feedback delay is present, the optimal weighting vector is modified to the sub-optimal weighting $W_{sub}(m)$ which is written by

$$W_{sub}(m) = \frac{G^H(m-d)}{G(m-d)} \quad (14)$$

When the transmit data $s(m) = 1$, the BER with the given imperfect CSI vector $G(m-d)$ at time m is

$$P_e = Q \left(\sqrt{\frac{2E_b}{N_o}} \text{Re} \left[\rho_e \rho_G G(m-d) + \rho_e \epsilon(m) + D(m) \right] \times \frac{|G^H(m-d)|}{|G(m-d)|} \right) \quad (15)$$

where $Q(x)$ equals $1/\sqrt{2\pi} \int_x^\infty \exp(-u^2/2) du$ and

$\text{Re}(\cdot)$ denotes Real.

If we define a random variable η as

$$\eta = \text{Re} [\rho_e \rho_G G(m-d) + \rho_e \epsilon(m) + D(m)] \times \frac{|G^H(m-d)|}{|G(m-d)|} \quad (16)$$

then η is a Gaussian random variable for the given $G(m-d)$; i.e.,

$$\eta \sim N \left(\rho_e \rho_G |G(m-d)|, \frac{1}{2} (\rho_e^2 \sigma_\epsilon^2 + \sigma_D^2) \right) \quad (17)$$

Thus, using (16) and (17), the instantaneous BER of (15) can be rewritten as

$$P_e = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-u^2/2) Q \left(\sqrt{\frac{2}{\rho_e^2 \sigma_\epsilon^2 + \sigma_D^2}} \times \left(\rho_e \rho_G |G(m-d)| - \frac{u}{\sqrt{2E_b/N_o}} \right) \right) du \quad (18)$$

IV. FADING CHANNEL CORRELATION

The WCDMA cellular system, a transmit diversity system with feedback, TxAA (Transmit adaptive array), adopts 2 transmit antenna element [2]. In this section we will consider two transmit antenna array system.

If $G(m)$ is wide sense stationary, then the pdf of $G(m)$ is equal to the pdf of $G(m-d)$. Assume the number of transmit antenna elements are two, then the estimated channel gain vector equals

$$G(m) = \sqrt{g_1(m)^2 + g_2(m)^2} \quad (19)$$

where $g_1(m)$ and $g_2(m)$ is the complex channel gain from transmit antenna 1 and transmit antenna 2 to receive antenna, respectively. If we define $p_i(m) = |g_i(m)|^2$, $i = 1, 2$, the joint power pdf of the two random variables $g_1(m)$ and $g_2(m)$ whose envelopes are Rayleigh is given by [7]

$$f_{p_1, p_2}(p_1, p_2) = \frac{1}{\bar{P}^2 (1 - \gamma_p^2)} I_0 \left(\frac{2\gamma_p \sqrt{p_1 p_2}}{\bar{P} (1 - \gamma_p^2)} \right) \times \exp \left(-\frac{p_1 + p_2}{\bar{p} (1 - \gamma_p^2)} \right) \quad (20)$$

where \bar{p} is an average power of $p_i(m)$, $i = 1, 2$ and γ_p is the channel correlation coefficient between $p_1(m)$ and $p_2(m)$. And where $I_0(\cdot)$ denotes the modified Bessel function of the first kind and zeroth order. For the notational convenience, we drop the sampling time m . The pdf of the random variable $\delta = g_1^2 + g_2^2$ is given as [13]

$$f_{\delta}(\delta) = \frac{1}{2\bar{p}\gamma_p} \exp\left(-\frac{\delta}{\bar{p}(1+\gamma_p)}\right) - \exp\left(-\frac{\delta}{\bar{p}(1-\gamma_p)}\right) \quad (21)$$

Define $\chi = \sqrt{\delta} = \overline{G}$. From the following relation

$$p_{\chi}(\chi) = p_{\delta}(\delta) \frac{d\delta}{d\chi} \quad (22)$$

the pdf of χ can be represented by

$$p_{\chi}(\chi) = 2\chi p_{\delta}(\delta), 0 \leq \chi \quad (23)$$

We can obtain the average BER by replacing (23) into (18),

$$P_s = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{u^2}{2}\right) \int_0^{\infty} Q \sqrt{\frac{2}{\rho_c^2 \sigma_c^2 + \sigma_D^2}} \times \left\{ \rho_c \rho_{c\chi} - \frac{u}{\sqrt{\frac{2E_b}{N_o}}} \right\} p_{\chi}(\chi) du d\chi \quad (24)$$

V. Numerical Examples and Discussions

To demonstrate the effect of the feedback delay and channel correlation, we assume the data rate for feedback is 1.6 Ksps ($T=0.625$ ns) as in WCDMA [2]. And the feedback delay is assumed 1 symbol. In Fig.2, the BER performance of the CTD system, which has two transmit antennas, is presented as a function of Doppler frequency and the correlation coefficient without channel estimation errors. It is noticed that the E_b/N_0 degradation with the correlation coefficient of 0.5 is negligible irrespective of the fading index. However, when the correlation coefficient increases to 1, the degradation increases to 3.3 dB and 9.8 dB at the BER of 1×10^{-2} with $f_D = 60$ Hz and 140 Hz, respectively.

Table 1 shows the E_b/N_0 loss versus correlation coefficient at the BER of 1×10^{-2} with ESR of 20 dB. The correlation coefficient whose E_b/N_0 loss is greater than 1 dB is $\rho = 0.8$ and $\rho = 0.6$ at

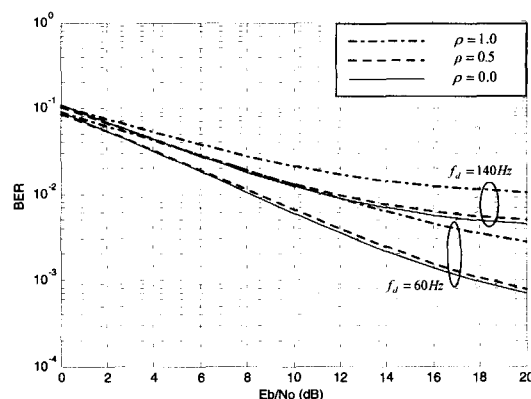


그림 2. 채널상관이 있는 CTD 시스템의 평균 BER 성능

Fig. 2. Average BER performance of CTD with channel correlation.

표 1. 상관계수와 E_b/N_0 손실

Table 1. E_b/N_0 loss versus correlation coefficient (BER of 1×10^{-2} , $ESR = -20$ dB).

Doppler freq. correlation coeff.	60 Hz	140 Hz
0.0	0.0 dB	0.0 dB
0.2	0.1 dB	0.1 dB
0.4	0.3 dB	0.7 dB
0.6	0.8 dB	1.1 dB
0.8	1.0 dB	3.3 dB
1.0	1.7 dB	Infinite

$f_D = 60$ Hz and 140 Hz, respectively. With Doppler frequency of 140 Hz and the correlation coefficient $\rho=1.0$, the BER of 1×10^{-2} can not be reached. We can conclude the effect of fading correlation is quite sensitive with Doppler frequency.

The effect of the channel estimation errors is shown in Fig. 3 ($P = 2$, $f_D = 60$ Hz, $\rho = 0$). It is noticed that the channel estimation error also gives the sensitive effect on the system performance. Under the given condition in Fig. 3, the E_b/N_0 loss is about 0.8 dB with 15 dB of ESR while the loss increases to 4.8 dB with 10 dB of ESR at the BER of 1×10^{-2} . The analytical results shows that the 4.8 dB loss caused by the channel estimation error ($f_D = 60$ Hz, $ESR = -10$ dB) is greater than the 3.3 dB loss caused by the fading correlation ($f_D = 60$

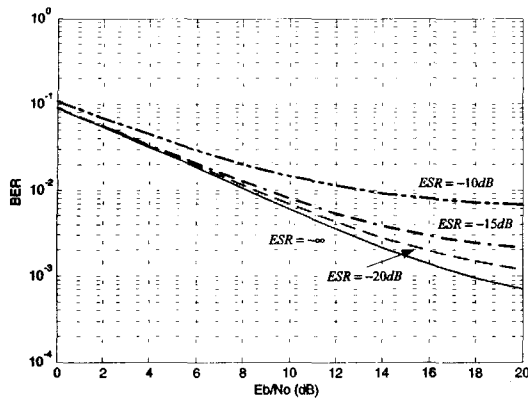


그림 3. ESR 값에 따른 CTD의 평균 BER
Fig. 3. Average BER performance of CTD for different ESRs ($f_D = 60 \text{ Hz}$, $\rho = 0$).

표 2. ESR 과 E_b/N_0 손실

Table 2. E_b/N_0 loss versus ESR (BER of 1×10^{-2} , $\rho = 0.7$).

ESR	Doppler freq.	
	60 Hz	140 Hz
-20 dB	0.6 dB	0.7 dB
-15 dB	1.5 dB	3.7 dB
-10 dB	7.6 dB	Infinite
-5 dB	Infinite	Infinite

Hz, $\rho = 1$). The channel estimation error causes the beam-forming error and consequently decreases the beam-forming gain. We can conclude that the performance degradations caused by the channel estimation errors are greater than that caused by the channel correlation.

The E_b/N_0 loss versus ESR at BER of 1×10^{-2} with $\rho = 0.7$ is given in Table 2. This table shows that the BER of 1×10^{-2} can not be reached with 5 dB and 10 dB of ESR at $f_D = 60$ Hz and 140 Hz, respectively.

VI. CONCLUSIONS

The effect of the channel fading correlation and the channel estimation error is considered in CTD system under Rayleigh flat fading. The E_b/N_0 degradation with the correlation coefficient of 0.5 is negligible irrespective of the fading index. However, when the

correlation coefficient approaches to 1, the degradation increases to 3.3 dB and 9.8 dB at $f_D = 60 \text{ Hz}$ and 140 Hz, respectively at BER of 1×10^{-2} . The E_b/N_0 loss due to the channel estimation error without fading correlation is about 0.8 dB with 15 dB of ESR, while the loss increases to 4.8 dB with 10 dB of ESR at the BER of 1×10^{-2} . From the numerical results, we noticed that the channel estimation error causes the sub-optimal beam-forming and degrades system performance. Compared with the fading correlation, the channel estimation error is more sensitive to the system performance in CTD system.

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