Non-Stationary Response of a Vehicle Obtained From a Series of Stationary Responses

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Ride characteristics of a vehicle moving on a rough ground with changing travel velocity are analyzed in this paper. The solution is difficult due to the non-stationary characteristics of the problem. Hence a new technique has been proposed to overcome this difficulty. This new technique is employed in the analysis of ride characteristics of a vehicle with changing velocity in the time/frequency domain. It is found that the proposed technique gives successful results in modelling non-stationary responses in terms of a series of stationary responses.

Key Words : Ride Characteristics, Random Vibrations, Nonstationary Processes

1. Introduction

Vibration may result in passenger's discomfort as well as untoward effects on vehicle components. Different suspension systems have been developed to reduce vibrations to predetermined levels since the invention of first car. The design and application of suspension systems are still an active area of research as there are some unsolved problems. In recent research activities, some studies on suspension systems focus on active suspension, but passive suspension systems are also studied as they are cheap and reliable.

Responses of a vehicle to changes in suspension parameters ought to be known in order to design an effective passive suspension system. Particular results for each suspension parameter under investigation should be obtained as systems cannot be designed without them.

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There are two important purposes in suspension system design ; one is to ensure passenger comfort and the other is to find the continuous force transmission through the suspension system between the vehicle and road, i.e., handling. These two characteristics are inversely proportional in terms of suspension system characteristics. In spite of the fact that ride and handling problems are closely connected, they should be studied separately due to the complexities of the problem. However, to obtain the optimum values of suspension parameters, ride and handling characteristics have to be studied simultaneously. There are some studies (Mitschke, 1962 ; Sharp and Crolla, 1987 ; Elbeheiry et al., 1995) on the effect of vehicle weight, suspension stiffness and damping ratio on ride and handling characteristics in the literature but more studies are still necessary with real road data, i.e., stochastic analysis, to improve the efficiency of suspension systems.

The studies on vehicle dynamics have started with the invention of the first wheeled motor. Summary of basic advances from these years to the 90s are given by Crolla (1996). A pioneering study on ride characteristics has been done by Mitschke (1962), who mathematically modelled vehicle and investigated effects of some vehicle parameters on ride characteristics.

Vehicle ride can be examined in the time domain, frequency domain or both. Deterministic inputs are commonly used in the analysis ; however stochastical inputs are employed in most of the recent studies, which generally assume a constant vehicle velocity in order to reduce the stochastical analysis to a stationary analysis. As the vehicle travels with changing velocity, the time/frequency analysis becomes indispensable, i.e., time or frequency analysis alone is not enough.

The main vibration excitation source of a vehicle is the road roughness, which is caused by cavities, bumps, construction errors and pavement material inherent properties (Gillespie, 1992; Newland, 1997). Road roughness as "broad band signal" input is defined directly by itself or with its statistical properties (Newland, 1997 ; Sayers and Karamihas, 1996 ; Sayers and Karamihas, 1998 ; Bui et al., 2002). Vehicle velocity is assumed to be constant in order to decrease the complexity of the problem in the stochastic ride characteristic investigation, so that the random vibration problem becomes time invariant, i.e. stationary (Newland, 1997). Nonstationary problem is difficult to solve, so Virchis and Robson (1971) and Sobczyk and Macvean (1976) used the time domain approach and obtained the second order stochastic analysis of vehicle response to a nonstationary base excitation. Macvean (1980) derived velocity response and investigated complex dependence on parameters of the nonstationary response of a vehicle with a similar method.

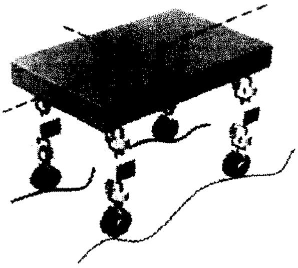
Yadav and Nigam (1978) investigated response of one degree of linear and nonlinear vehicle models to road excitations by expressing the solution with a space dependent frequency response function. Hammond and Harrison (1981) calculated the response variance of one degree of freedom linear vehicle model response to road profile, which is represented as a "spatial" shaping filter. This approach was also used to find multi-wheeled vehicle response by the same authors (Harrison and Hammond, 1986a). Harrison and Hammond (1986b) made use of the time/frequency analysis in the solution of frequency modulated processes with a new method named "covariance equivalence," and give the solution procedure on a vehicle moving on an irregular ground. This method is also employed by other researchers (Hammond and White, 1996; Dalianis and Hammond, 1997) to different problems.

Peter and Bellay (1986) suggested deforming road profile spectra according to velocity for the analysis of ride response of varying velocity vehicle, especially in traffic conditions. They derived a new integral transformation for this purpose. Road profile is also modeled as mathematical functions of time, displacement or frequency using curve fitting operations for further simplification of the ride characteristic problem by Marcondes et al. (1991). However, since road profile as a random process, employment of mathematical functions causes some information losses. If the road profile is defined with all properties and vehicle response to this actual profile is calculated, then the actual ride characteristic may be obtained. However, this procedure is very cumbersome for researchers.

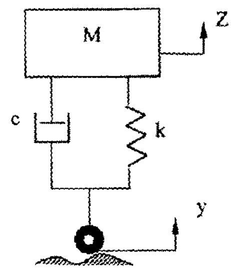
In this paper a different time/frequency analysis technique is suggested in order to determine nonstationary responses of a vehicle, and the research results are compared with previous findings of other researchers.

2. System Stochastic Model

Different vehicle models are used to investigate ride characteristics. These models are full-car, half-car and quarter-car with 1 to 7 degree of



(a) Full-car



(b) Quarter-car

Fig. 1 Vehicle models

freedom (Fig. 1). Use of a simple quarter-car model is common (Fig. 1(b)), and modelling and derivation of its equations of motion are available in the literature (see e.g. Mitschke (1962)). This simple model gives vertical vibration responses, which are important for ride characteristic analysis. Equation of motion for the model is:

$$M\ddot{Z}+c(\dot{Z}-\dot{y})+k(Z-y)=0 \tag{1}$$

Response of the vehicle model is $\ddot{Z}$, and ride comfort is calculated from this acceleration. The transfer function between road roughness y and vehicle response (vertical displacement) Z is:

$$H(j\omega)=\frac{cj\omega+k}{-m\omega^2+cj\omega+k} \tag{2}$$

Eq. 2 is in complex form with the real and imaginary components. A typical frequency response function (response gain) is shown in Fig. 2 for a frequency range of interest.

For a constant vehicle velocity, stochastic road profile processes are stationary Gaussian up to second order (Peter and Bellay, 1986). If the system is assumed to be stationary then Power Spectral Density (PSD) of a linear system with single input-single output can easily be defined as:

$$S_y(\omega)=|H(\omega)|^2S_x(\omega) \tag{3}$$

and for multi input-single output case, PSD is given by:

$$S_y(\omega)=\sum_{r=1}^{N}|H(\omega)|^2S_{x_i}(\omega) \tag{4}$$

Hence, once the road profile is obtained by the use of Eqs. 3 and 4, the response of a quarter-car can easily be obtained.

3. Nonstationary Response

The stationary process does not reflect the real world completely as explained above, in this study a nonstationary process is, therefore, employed using the "rubber band model" (Peter and Bellay, 1986; Marcondes et al., 1991; Farkas et al., 1980). First of all, PSDs for stationary processes, i.e. for constant travel velocity, are calculated for different travel velocities and then these PSDs are put one after another for obtaining the PSD surface of road. In other words, road profile is elongated or shrunk according to vehicle instant velocity, so that nonstationary process on this road is obtained from a series of stationary responses.

Acceleration PSDs of road for different travel velocities are given in Fig. 3 for typical examples, obtained from Transportation Research Institute, University of Michigan, USA (2000). MATLAB®(2000) software is used in the analysis of the data.

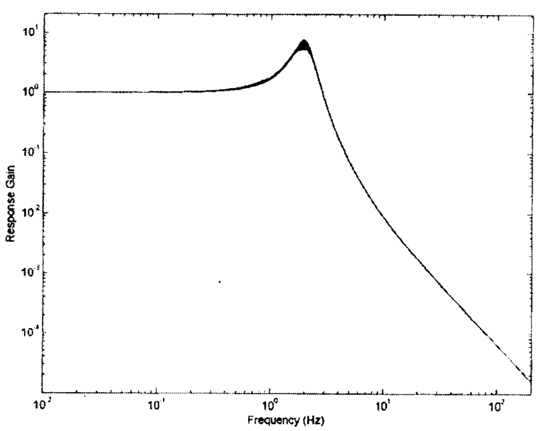
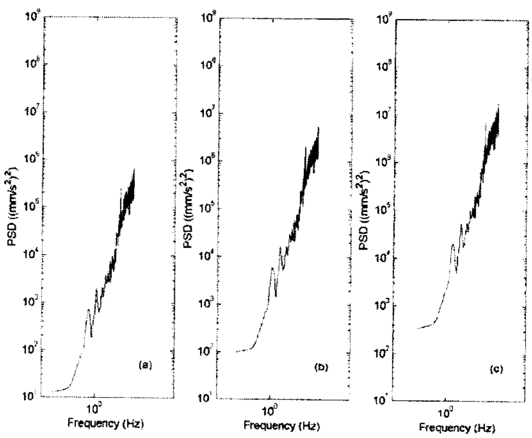


Fig. 2 Typical response gain of a quarter-car model



(a) 10 km/h (b) 20 km/h (c) 30 km/h

Fig. 3 Acceleration PSDs for constant vehicle velocity

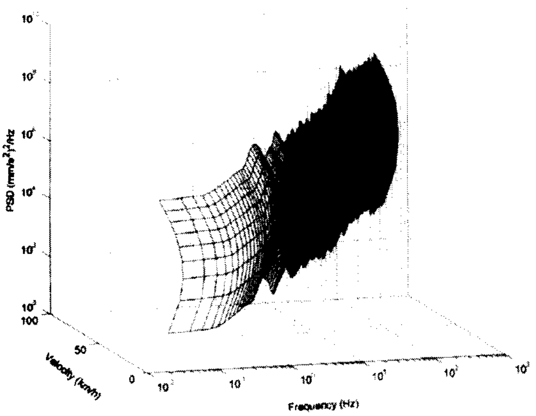


Fig. 4 Road excitation PSD (10-90 km/h)

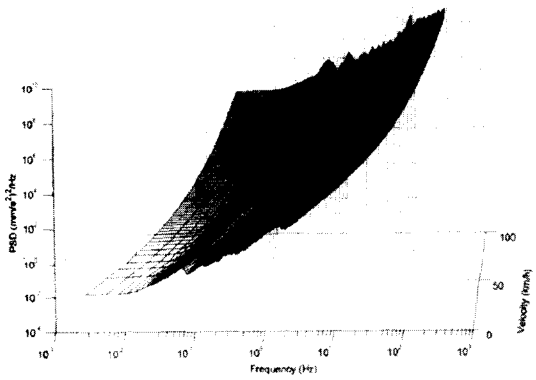
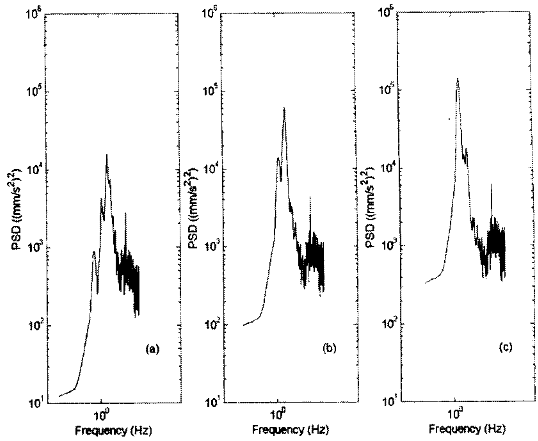


Fig. 5 Road excitation PSD (1-100 km/h)



(a) 10 km/h (b) 20 km/h (c) 30 km/h

Fig. 6 Acceleration response of vehicle for constant velocity runs

Stationary PSDs for constant velocities are arranged according to changing velocity using rubber band model, and nonstationary PSD surface of road between 10 to 90 km/h with 10 km/h increments are shown in Fig. 4.

If velocity difference is reduced to 1 km/h, in order to increase the resolution of PSD, Fig. 5 is obtained. More tight velocity differences could be used for higher resolutions but this increases the computer processing time, hence a 1 km/h difference is chosen to be sufficient to express the surface as seen in Fig. 5.

As the speed increment decreases the gap between stationary processes reduces and analysis gets closer to the nonstationary response. However, as explained above, 1 km/h velocity increment is assumed to be enough for adequate results, otherwise computational processing time will be unnecessarily long and the resulting improvement will be relatively low.

The constant speed at the start for the PSD calculation is 1 km/h, because there is a discontinuity at 0 km/h in the mathematical solution of the problem. Therefore, the solution is not for a vehicle starting from the still condition, instead, acceleration starts vehicle from 1 km/h constant velocity. On the other hand it is clear that when the vehicle is still, PSD values are zero for all frequencies, because there is no excitation from the road. Moving vehicle from this stationary position to 1 km/h is rather difficult in terms of the mathematical modelling.

The road profile acceleration PSD is multiplied by response gains of a one degree of freedom quarter-car model for each velocity to find response PSD according to Eq. 3. Fig. 6 shows three examples of stationary PSD acceleration responses of the vehicle.

4. Results and Discussion

Acceleration response PSD surface for velocities between 1-100 km/h, with 1 km/h increments, is determined using the rubber band model as shown in Fig. 7 for a quarter-car model. Parameters of a quarter-car model are a vehicle mass, m, of 500 kg, a suspension stiffness, k, of 2000 N/m and a damping constant, c, of 400 Ns/m.

These values are selected to satisfy a damping ratio, ζ, of 0.2 and an undamped natural frequency, ω_n, of 2 Hz based on studies by Gillespie (1992) and Gobbi and Mastuni (2001).

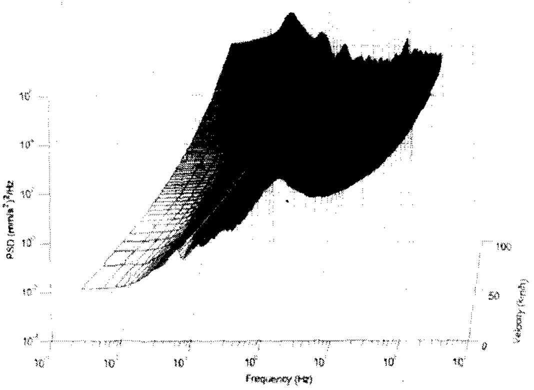


Fig. 7 Acceleration response PSD of quarter-car model

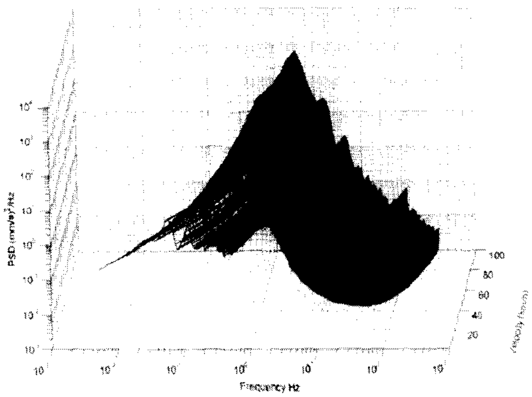


Fig. 8 Velocity response PSD

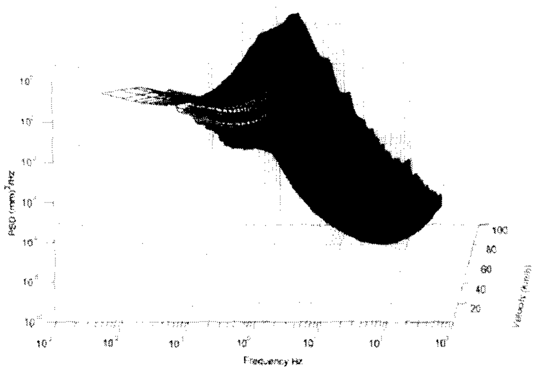


Fig. 9 Displacement response PSD

Velocity and displacement response PSD of vehicle are also shown in Figs. 8 and 9. Road profile velocity PSD does not significantly change for the working frequency range, but vehicle response gain rapidly decreases with increasing velocity. As a result, the velocity response PSD rapidly decreases with increasing frequency as seen in Fig. 8. On the other hand the velocity response PSD increases with increasing travel velocity as also in acceleration response PSD. Vehicle displacement response PSD is concentrated at low frequencies. Another important point is, contrary to acceleration and velocity response PSD, the displacement response PSD rapidly decreases with increasing travel velocity, i.e., vehicle is nearly insensitive to displacement excitation at high velocities. This event can be seen more clearly in Fig. 10 with PSD axis linear instead of logarithmic.

When PSD graphs are examined, it can be observed that frequencies depend on travel velocity and are independent of velocity, resonance frequencies. Velocity dependency of frequencies is related to road properties such as pavement material particle size, slab distance, etc., and independent frequencies are related to the vehicle, such as natural frequencies.

If the vehicle constant acceleration is known, calculated response PSD can be translated to time/frequency domain by dividing velocity axis of PSD by the vehicle acceleration. As an example, acceleration PSD in Fig. 11 is given for

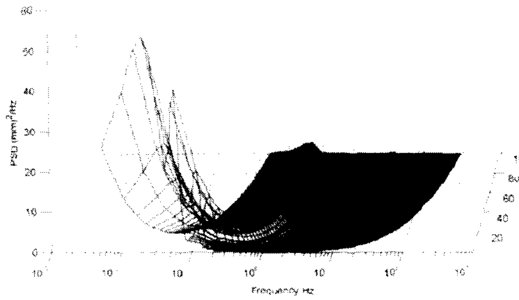


Fig. 10 Displacement response PSD (PSD axis is linear)

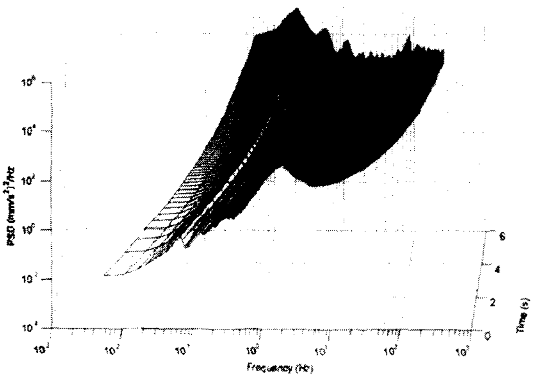


Fig. 11 Acceleration response PSD (for a vehicle acceleration of 5 m/s^2)

5 m/s^2 vehicle acceleration. This analysis could also be repeated for velocity and displacement responses. Although this analysis is done for a constant acceleration, spectral analysis of vehicle with known acceleration profile could be determined by changing algorithm of the computer program.

These results quantitatively agree with the results of Harrison and Hammond (1986b) who showed that as the velocity increases from 1 km/h to 100 km/h there is a spreading out of the excitation energy along the frequency axis, and hence the peak response builds up. Similar behaviour can also be observed in Fig. 9 for logarithmic scale and Fig. 10 for linear scale. In addition, Tamboli and Joshi (1999) point out that when the vehicle velocity increases, the acceleration response value proportionally increases for their half-car model. Same behaviour can be observed in Fig. 7 for the quarter car model of this study.

5. Conclusions

In this investigation a new solution and analysis technique for nonstationary random processes is developed by using the rubber band model, and employed in determining nonstationary ride characteristics of a vehicle. It is observed that this new technique can be used for investigating ride characteristics of a vehicle as well as other nonstationary systems. The comparison of the results obtained by this technique and results of previous researches show a good qualitative agreement.

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