論文
3-3

A comparative study of galloping cable and torsional oscillations in suspension bridge
(갤럽핑 케이블과 현수교의 뒤틀린 진동에 관한 비교 연구)

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ABSTRACT

This paper presents the common and different results between the galloping cable and torsional oscillations in suspension bridge. Numerical results of the galloping cable and torsional oscillations in suspension bridge are presented by using the second-order Runge Kutta method under the initial conditions. This paper shows that large amplitude solution can coexist with the small amplitude one as the frequency and amplitude of the oscillation change. The differences in symmetry and transient effects are presented.

요약

이 연구는 갤럽핑 케이블과 현수교의 뒤틀린 진동에 관한 공통된 결과와 다른 결과를 제시하고 있다. 갤럽핑 케이블과 현수교의 뒤틀린 진동에 관한 수치적인 결과는 2차의 Runge Kutta 방법에 의한 것이다. 진동의 주파수와 진폭이 변함에 따라 다중근의 존재를 확인할 수 있으며, 대칭성이 벗어나고 일시적인 현상에서의 차이를 볼 수 있다.

1) 정회원 :시립 인천전문대학 컴퓨터정보과 교수

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1. Introduction

Tacoma Narrows suspension bridge is a classic movie of the science of mechanics. The collapse of the prior to 10:00 A.M. on November 7th in 1940, there were no recorded instances of the oscillations being otherwise than with the two cables in phase and with no torsional motions. Suddenly, at approximately 10:00 A.M., the center span developed a torsional movement with a node at mid span. The frequency suddenly changed from 37 to 14.

Galloping is the large amplitude up and down motion that can break lines and cause catastrophic failure towers of their supporting according to the Electric Power Research Institute.

We argue that the motions of suspension bridge are governed by nonlinear partial differential equations and that the inherent nonlinearity gives rise to large amplitude oscillations.

2. Description of the model

We describe the equations of torsional oscillations in suspension bridge. If a spring with spring constant K is extended by a distance y, the potential energy is $Ky^2/2$. If a rod of mass m and length $2l$ rotates about its center of gravity with angular velocity $\dot{\theta}$, then, its kinetic energy is given by $(1/6)ml^2(\dot{\theta})^2$. Assume the rod is suspended by springs that resist expansion with a spring constant K at each end. Let y denote the downward distance of the center of gravity of the rod from the unloaded state. Let θ denote the angle of the rod from the horizontal. Let y^+ be the positive part of y, that is, $y^+ = \max\{y, 0\}$.

The potential energy due to gravity is $-mgy$. The extension is $(y - l\sin\theta)^+$ in one spring and $(y + l\sin\theta)^+$ in the other. Then, the total potential energy is

$$V = (K/2)(((y - l\sin\theta)^+)^2 + (y + l\sin\theta)^+)^2 - mgy$$

and the total kinetic energy is

$$T = m\dot{y}^2/2 + (1/6)ml^2\dot{\theta}^2$$

Let $L = T - V$ and put $(d/dt)(\delta L/\delta\dot{\theta}) = (\delta L/\delta\theta)$ and $(d/dt)(\delta L/\delta\dot{y}) = (\delta L/\delta y)$.

We obtain the equations

$$(1/3)ml^2\ddot{\theta} = (Kl)\cos\theta((y - l\sin\theta)^+ - (y + l\sin\theta)^+)$$

$$\ddot{y} = -K((y - l\sin\theta)^+ + (y + l\sin\theta)^+) + mg$$

Simplifying and adding a small viscous damping term $\delta\dot{\theta}$ to the first equation and $\delta\dot{y}$ to the second, and adding an external forcing term $f(t)$ (to be determined later), to the torsional equation, we end up with the system

$$\ddot{\theta} = -\delta\dot{\theta} + (3K/ml)\cos\theta[(y - l\sin\theta)^+ - (y + l\sin\theta)^+] + f(t)$$
$$\ddot{y} = -\delta\dot{y} - (K/m)[(y - l\sin\theta)^+ + (y + l\sin\theta)^+] + g$$

We put $\sin\theta = \theta$ and $\cos\theta = 1$. Finally, we have the two equations

$$\ddot{\theta} = -\delta\dot{\theta} - (6K/m)\theta + f(t)$$
$$\ddot{y} = -\delta\dot{y} - (2K/m)y + g.$$

Now, we describe the equation of the hanging cable. The cable treats as a series of equally distributed point masses connected by nonlinear springs with the same unstretched lengths. To model the cable behavior, the springs resist extensions but not compressions.

We consider a cable which is hung

between fixed points at the same vertical level and distance L apart. Let the line joining the two supports be the x-axis with these two fixed points located at $x=0$ and $x=L$. Let the instantaneous position of the i-th particle be $(x_i(t), y_i(t))$ at time t with the positive directions for x and y.

Then, Newton's second law and Hooke's law immediately give rise to the following equations,

We investigate the torsional oscillations in suspension bridge. At (1), we use $m=2500kg$, $K=1000$, and $\delta=0.01$. We repeat some of these experiments for the forcing term $\lambda\sin\mu t$, then we have the following

$$\ddot{\theta}=-0.01\theta-2.4\cos\theta\sin\theta+\lambda\sin\mu t.$$

Here, we choose initial conditions $\theta(0)=1.2$ and $\dot{\theta}(0)=0$. We vary λ and μ from $\mu=1.2$ to $\mu=4$.

$$\frac{d^2x_i}{dt^2}=-\frac{k}{\rho l}\left(\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}-l\right)^+\times\frac{(x_i-x_{i-1})}{\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}}$$
$$+\frac{k}{\rho l}\left(\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}-l\right)^+\times\frac{(x_{i+1}-x_i)}{\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}}$$
$$-\frac{c}{\rho}\frac{dx_i}{dt}$$

$$\frac{d^2y_i}{dt^2}=-\frac{k}{\rho l}\left(\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}-l\right)^+\times\frac{(y_i-y_{i-1})}{\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}}$$
$$+\frac{k}{\rho l}\left(\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}-l\right)^+\times\frac{(y_{i+1}-y_i)}{\sqrt{(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2}}$$
$$-\frac{c}{\rho}\frac{dy_i}{dt}+g+\frac{1}{\rho}f(\tilde{x}_i,t). \qquad (2)$$

The term ρ denotes the mass per unit length of unstretched cable and the term l the unstretched length of the spring between two point masses. The spring constant is denoted by $k=EA/l$, where E is Young's modulus and A is the cross sectional area. The damping coefficient per unit unstretched length is denoted by c, and the acceleration due to gravity by g.

3. Numerical Experiments

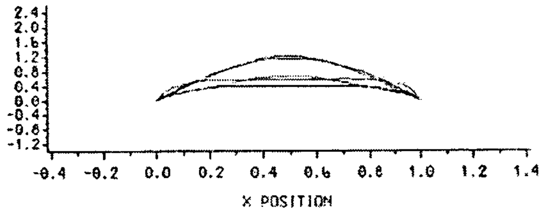
We run the initial value problems for one thousand periods and see what the system has settled down to.

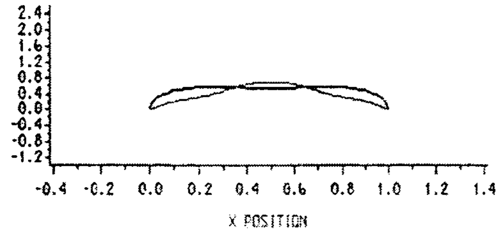
For the galloping cable, we search for periodic solutions by solving the initial value problem for various initial values and allowing the solution to run for large time. At (2), we let the total mass of the cable be 5, the unstretched length of the cable be 1.2, $c=\frac{1}{3}$ and $EA=19.2$, $N=63$.

We use the forcing $f(\tilde{x}_i,t)=\lambda\sin\mu t\sin n\pi\tilde{x}_i$. We investigate the y-magnitude of the mid particle with small and large initial conditions so μ is varied from 3 to 6 in increments of 0.2 and λ from 0 to 2 in increments of 0.2.

3.1 Common Results

In this section, we shall consider common results between the galloping cables and torsional oscillations in suspension bridge. We find that the large amplitude solution can coexist with the small amplitude one.





Let us observe the galloping cable. Figure 1 shows two solutions at $\mu = 5.2$ and $\lambda = 0.4$.

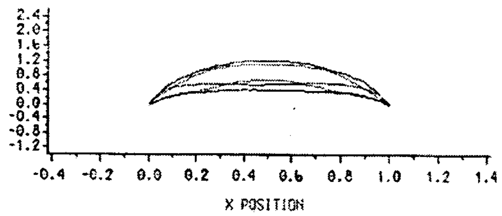
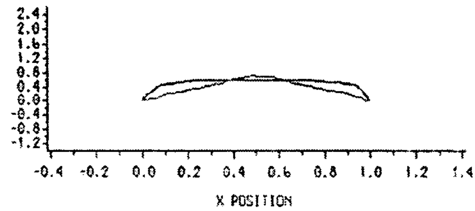
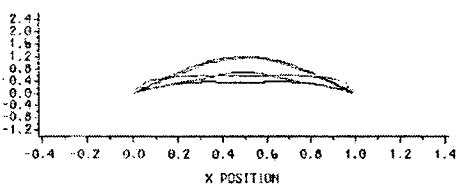


Figure 1. The small amplitude solution and the first half and the second half of the large amplitude solution at $\mu = 5.2$ and $\lambda = 0.4$.
[그림 1] $\mu = 5.2$, $\lambda = 0.4$ 에서 작고 큰 진폭의 주기근.

Figure 2 shows two solutions at $\mu = 5.2$ and $\lambda = 0.6$. We use the forcing term $f(t) = \lambda \sin \mu t$ on $P_0(0,0)$ and $f(t) = -\lambda \sin \mu t$ on $P_{N+1}(1,0)$. This might be the force exerted by vibrating towers with towers out of phase. Figure 3 shows two solutions at $\mu = 4.8$ and $\lambda = 0.1$ when two supports with forces out of phase move. Figure 4 shows the large and small amplitude solutions of the forcing $f(\tilde{x_i}, t) = \lambda \sin \mu t \sin 3\pi \tilde{x_i}$ for $\mu = 4.8$ and $\lambda = 4.6$.

Now, let us observe the torsional oscillations in suspension bridge. We choose $\theta(0) = 1.2$ and $\dot{\theta}(0) = 0$ and $\mu = 1.2$ and $\lambda = 0.06$. Figure 5 shows the large and small amplitude solutions. Figure 6 shows the large and small amplitude solutions with $\mu = 1.3$ and $\lambda = 0.02$. Figure 7 shows the large and small amplitude solutions with $\mu = 2.6$.





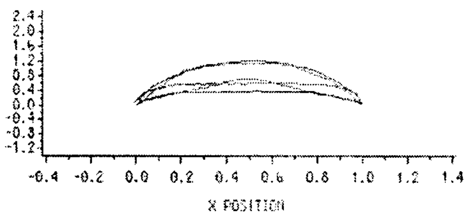


Figure 2. The small amplitude solution and the first half and the second half of the large amplitude solution at $\mu = 5.2$ and $\lambda = 0.6$.

[그림 2] $\mu = 5.2$, $\lambda = 0.6$ 에서 작고 큰 진폭의 주기근.

3.2 Different Results

In this section, we shall consider different results between the galloping cables and torsional oscillations in suspension bridge.

Figure 8 shows that the galloping cable breaks the symmetry at $\mu = 5.2$ and $\lambda = 1.0$.

But the torsional oscillations in suspension bridge do not break the symmetry and just appear transient results. Figure 9 shows that the result of a small forcing term starting at equilibrium and it starts to die down immediately. Figure 10 shows the result of a large push from equilibrium in both the linear and correct models, with no forcing term. There is little real difference, as the damping takes over and both systems settle back to equilibrium.

Now, do the same experiment starting with initial conditions at equilibrium, but with a small forcing term $\lambda = 0.05$. Again, the two systems give close results. This is shown in Figure 9. If we combine the previous two effects, the principle of superposition predicts that the linear system will die down as before. Figure 11 shows this, as well as the huge difference caused by doing the correct trigonometry.

4. Conclusion

This research has discussed the common and different results between galloping cable and torsional oscillations in suspension bridge. Over a wide range of frequencies and amplitudes, the long term behaviors of two systems are highly initial condition dependant. If the initial conditions are close to equilibrium, the solution will be close to a linear small-amplitude solution.

On the other hand, if there is a large initial displacement, the solution after large time may either converge to the small linear solution or may remain in a large amplitude solution.

Common results are that the large amplitude solution can coexist with the small amplitude one. This research also includes the effect of moving the supporting towers out of phase.

Galloping cable has the phenomena that the symmetry breaks but the torsional oscillations in suspension bridge do not. Instead of the symmetry breaking, the torsional oscillations in suspension bridge have the transient results.

If we solve the initial value problem with no forcing but with large initial conditions, the results over the first hundred periods for the two problems are shown in Figure 10. There is little real difference, as the damping takes over and both systems settle back to equilibrium. The torsional oscillations in suspension bridge show the principle of superposition which predicts that the linear system will die down.

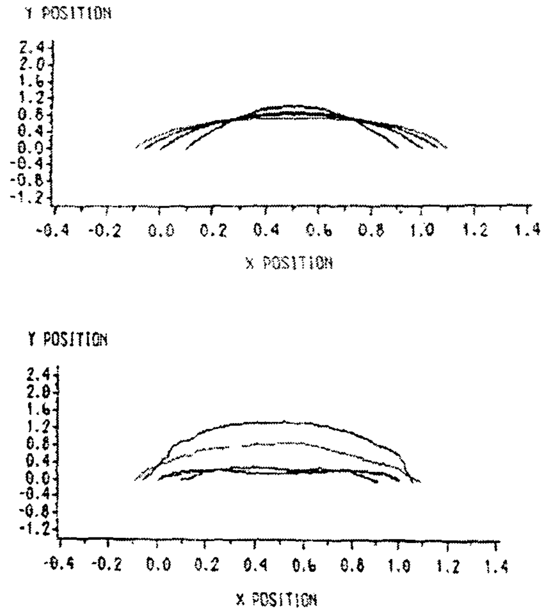


Figure 3. The small and large amplitude solutions at $\mu = 4.8$and $\lambda = 0.1$.

[그림 3] $\mu = 4.8$, $\lambda = 0.1$ 에서 작고 큰 진폭의 주기근.

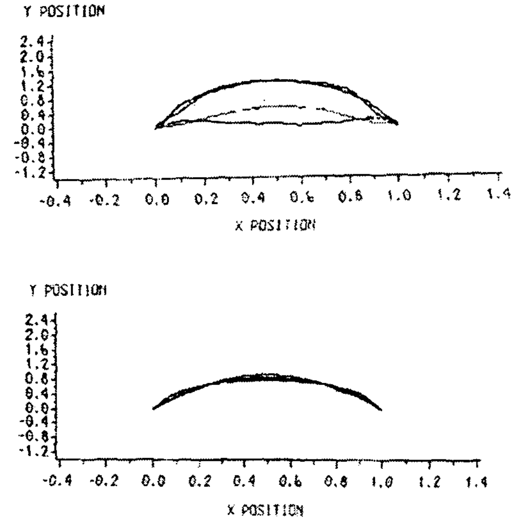


Figure 4. The small and large amplitude solutions at $\mu = 4.8$ and $\lambda = 4.6$.

[그림 4] $\mu = 4.8$ and $\lambda = 4.6$ 에서 작고 큰 진폭의 주기근.

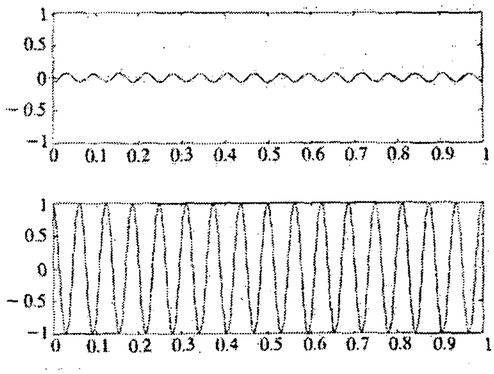


Figure 5. The small and large amplitude solutions at $\mu = 1.2$ and $\lambda = 0.06$.

[그림 5] $\mu = 1.2$, $\lambda = 0.06$ 에서 작고 큰 진폭의 주기근.

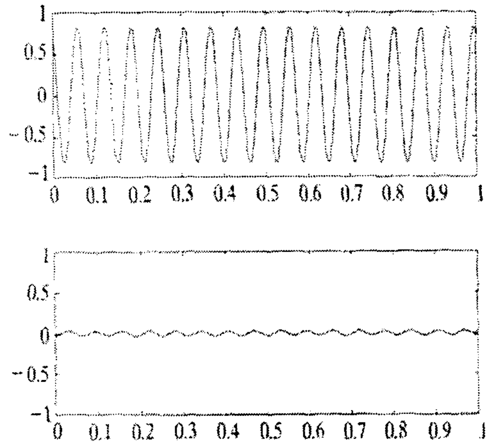


Figure 6. The small and large amplitude solutions at $\mu = 1.3$ and $\lambda = 0.02$.

[그림 6] $\mu = 1.3$, $\lambda = 0.02$ 에서 작고 큰 진폭의 주기근.

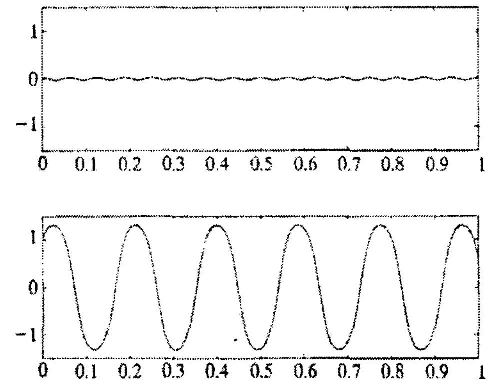


Figure 7. The small and large amplitude solutions at $\mu = 2.6$.

[그림 7] $\mu = 2.6$ 에서 작고 큰 진폭의 주기근.

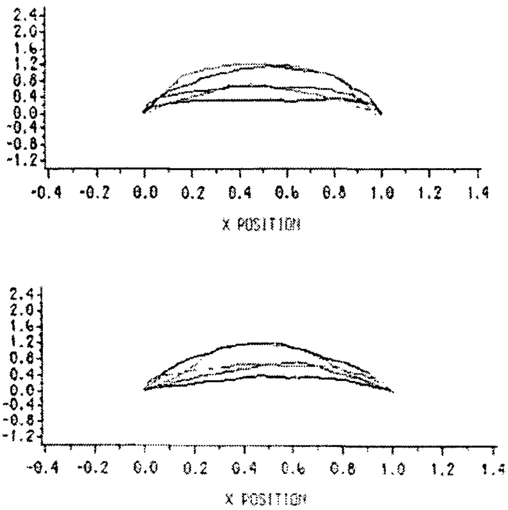


Figure 8. The small amplitude solution and the first half and the second half of the large amplitude solution at $\mu = 5.2$ and $\lambda = 1.0$.

[그림 8] $\mu = 5.2$, $\lambda = 1.0$ 에서 작고 큰 진폭의 주기근.

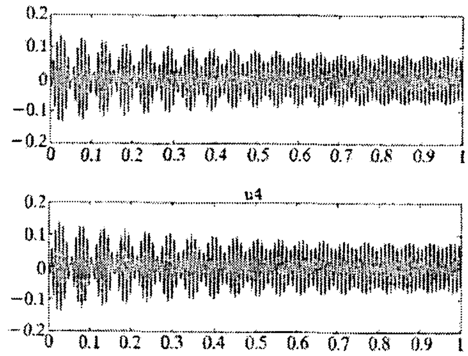


Figure 9. The solutions of a small forcing term starting at equilibrium.

[그림 9] 평형 상태에서 작은 힘을 가했을 때의 근.

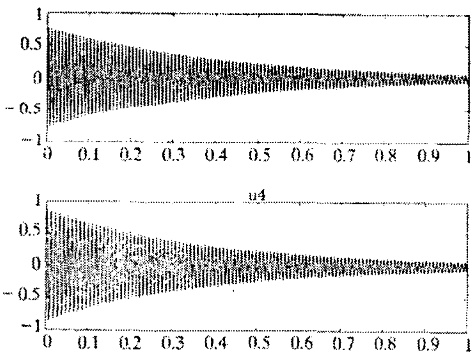


Figure 10. The solutions of a large push from equilibrium with no focing term.

[그림 10] 평형 상태에서 힘항 없이 큰 초기조건을 이용했을 때의 근.

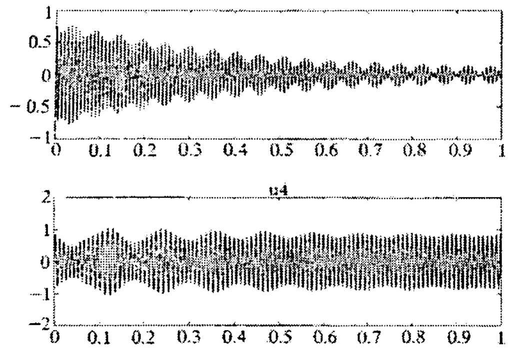


Figure 11. The transient behavior resulting from combining the two influences in Figure 9 and Figure 10.
[그림 11] 그림 9와 10을 합성 했을 때 일시적으로 나타나는 근.

5. References

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