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The "Two Basics" Mathematics Teaching Approach and the Open Ended Problem Solving in China[1]

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There is a tradition of advocating the "two basics" (basic knowledge and basic skills) in Chinese mathematics education. The direct consequence is that Chinese students are able to produce excellent performance in the international mathematics examinations and outstanding results in the international mathematics competitions. In this article, we will present why and how Chinese teachers teach the "two basics," and how combine the pupil's creativity with their "two basics." Open ended problem solving is a way to meet the goal. The following topics will be concerned: Culture background; the speed of computation; "Practice make perfect"; Efficiency in classroom; Balance between "two basics" and personal development. In Particular, Chinese mathematics educators pay more attentions to the link between open ended problem solving and the "two basics" principal.

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0. INTRODUCTION

Due to historical reasons, in many Eastern countries, including Japan, Korea,

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Singapore, and China (Mainland China, Taiwan, and Hong Kong), even Russia, mathematics educators emphasize more on the importance of foundation training. The principal of the "two basics" (basic knowledge and basic skills), however, is the most typically observed in Mainland China.

The direct consequence of the implementation of the "two basics" principal in Mainland China is: acquiring the leading position in numerous international mathematics assessment and contests, for example, topping the accuracy rate in the 1989 International Assessment of Education Progress (IAEP), and achieving outstanding results in the past International Mathematical Olympiads (IMO).

There is a paradox. On one hand, Chinese students are successful in many international mathematics tests. On the other hand, mathematics teaching and learning in China seems to be oriented rote memorization and repeat exercise. In our opinion, the notion of "two basics" teaching approaches may be the key to solve this paradox.

In this paper, we will illuminate the meaning of the "two basics" mathematics teaching approach, and the link between the "two basics" principal and the "open ended problem solving" in Mainland China.

I. THE FOUR DIMENSIONS OF THE "TWO BASICS" PRINCIPAL

In Mainland China, the "two basics" principal in mathematics teaching is a broad and loose concept without a strict definition. Its general meaning is that in the two aspects of "solid foundation" and "individual developing and creativity," although both are important, but more important is the foundation. Most Chinese mathematicians and mathematics educators believe that "although a tower is beautiful, but the groundwork is more important."

In fact, primary and secondary education is foundational education. Establishing a good mathematics foundation is the main task of fundamental education. Especially, this foundation can only be laid properly when one is young, or else it would be too late. We would treasure our childhood mathematics practice, just like language and piano-playing practice. Therefore it is of foremost importance that a good foundation must be laid during school years. Without a solid foundation, it is impossible to realize creativity, and ultimately let alone students' differentiated individual development.

The educational idea of "two basics" mathematics teaching approach could be shown in the following four dimensions: Calculation speed: speed lead to efficiency; Procedure memorization: understanding through memorization; Accuracy expression: based on logical analysis; Exercise doing: repeat with variation.

1. Calculation speed: lead to efficiency

It is an important factor of the "two basics" mathematics teaching approach. As well known, calculation skill is a program and mathematics thinking is a process. In order to enhance the efficiency of mathematics thinking, we must keep the adequate speed of calculation. In fact, speed will save working memory space for thinking at higher level, *i.e.*, speed lead to efficiency. In Chinese primary schools, fast and accurate calculation with the four operations involving integers, decimals and fractions is an essential requirement.

Let us refer to the Fenghua Survey[2].

Test contents: First graders (7 years old): A total of 90 sums to be completed within 15 minutes.

(1) 50 questions on addition and subtraction under 100

3+13=	16+4=	50+5=	12+60=	2+57=
60+33=	53+9=	28+10=	30+48=	7+12=
9+17=	40+45=	23+8=	12+34=	86+8=
78–30=	20–12=	15–5=	20–15=	81–80=
100–40=	95–70=	27–4=	40–20=	18–12=
16–9=	36–16=	26–11=	47–8=	97–14=
45+1=	96–4=	63+7=	16–2=	37+6=
99–2=	38+50=	76–9=	8+25=	63–6=
80–40+30=	17+8+30=	49+20–60=	44–10+13=	35–20+15=
18–0+9=	10+60–8=	32+9–20=	98–30+11=	20–8–11=

(2) Fill in the 40 blanks:

2 + 5 = ( ) – 5 = ( ) + 7 = ( ) + 2 = ( )
11 – 2 = ( ) + 9 = ( ) – 7 = ( ) + 6 = ( )
15 – 8 = ( ) + 2 = ( ) – 3 = ( ) + 18 = ( )
85 – 40 = ( ) – 20 = ( ) – 5 = ( ) – 11 = ( )
12 + 7 = ( ) + 9 = ( ) – 17 = ( ) + 16 = ( )
19 + 7 = ( ) – 8 = ( ) + 13 = ( ) + 7 = ( )
76 – 20 = ( ) + 14 = ( ) – 30 = ( ) + 9 = ( )
95 – 61 = ( ) + 18 = ( ) – 12 = ( ) +31 = ( )
31 + 16 = ( ) – 15 = ( ) + 42 = ( ) – 50 = ( )
41 – 20 = ( ) + 19 = ( ) – 14 = ( ) + 5 = ( )

[2] It was conducted by Zhou Leiming and Hu Yixiang in 2002.

Conclusion: Chinese primary students can complete 10 test questions of addition and subtraction within 100 in one minute. Is it necessary? We are not sure. However, this is an ordinary speed in China. See Table 1.

Table 1. Results and analysis of mathematics ability of first graders

School	Sample size	Quickest	Slowest	Average time used	Average grade	Excellence ratio	Passing rate
Jinpeng Town Centre Sch.	48	6min	15 min	9 min 38sec	90.1	46.65%	100%
Xi'qi Sch.	23	6min 3sec	14 min	9 min 52sec	87.4	36.7%	96%

- We will provide other two results on the "calculation speed."
  For third graders (9 years old), they can complete 2 test questions per minute in average. A part of the questions are:

125×72=	8500÷50=	4907÷7=	490×20=
8640÷60=	8585÷17=	2821×3=	490÷35=

- In a survey with 4000 sample size, we hope to measure the speed of algebraic manipulation on polynomials. Students are asked to complete 38 test questions in 10 minutes (see Appendix).

Test contents: A part of the test questions are:

Solve questions 1 to 5:

1) $-(2/3)\,ab + (3/4)\,ab + ab =$
2) $-y^2 - 2x^2 - (-3y^2) =$
3) $3\,x^2\,y\cdot(1/2)\,x\cdot(-2xy^2) =$
4) $3x^2\,y + (1/3)\,xy \quad \div(-xy) =$
5) $[(-2n)^{2}]^3 =$

Factorization (1–3):

1) $(a+b)^2 - (x-y)^2$
2) $(x+y)^2 + 5\,(x+y) + 6$
3) $(m-n)^2 + 4(m-n) + 4$

Completing the square:

$(1/2)\,x + x + (2/3)$

Conclusion: We think it maybe too demanding for school students. However, speed requirement still is a goal of mathematics teaching and learning in China. "Quick calculation competition" is held in most school year by year. In fact, calculation speed is the foundation of a higher achievement in any time-limited test. See Table A1 of Appendix.

2. Procedure memorization: memorization through understanding

Nobody deny the importance of memorization in a cognition process. However, there are different views between memorization and understanding. For example, many educators strongly demand that "Don't learn anything by rote!" and others say that "understanding is through with memorization!"

Most Chinese teachers believe that "memorize it at first, and then understand it step by step." For example, although children don't understand why the piano fingers exercise should be, but they must memorize it, and then understand it later. Similarly, we speak mother language just rely on the memorization and imitation, even we don't understand what is the grammar. In China, we usually say: "if you understand it, you should practice; even if you don't understand it well, you have to practice too, in the process of doing, and then you will understand it better and better."

Here are some special cases.

(1) All pupils must memorize and recite the 9×9 multiply table when they are 8–9 years old (in the second-third grade).
(2) The rule "negative time negative equal to positive" ("–"×"–"="+"). To understand this algorithm is more difficult. Up to today, we still are unable to explain why it is correct with convincing. In China, most pupils memorize it at first, and then understand it gradually.
(3) For the trigonometric formulas, in actual teaching, students are required to recite angle-sum formulas, double-angle formulas and half-angle formulas and the formulas for changing sum to product and vice versa, concerning sine, cosine and tangent (Formulas on triple-angles has not been required for students to memorize in recent years). It is the basic knowledge for student's life-long study, especially in the study of Calculus.

These views also shared from Western scholars. For instance, "memorizing what is understanding, understanding through memorization" (Marton 1991). Memorization is a means of deepening understanding or is a precondition for understanding? An interpretation derived from studies of Japanese students is repetition may be as a route to understanding (Hess & Azumas 1991).

3. Accuracy expression: based on logical analysis

In China, mathematics teaching is required to explain mathematics concepts and mathematics ideas by informal approaches. However, to an extent, it is necessary to keep up the mathematics with accuracy, logic and formality. In particular, the mathematical language should be formal. Students have to do more exercise to learn how to make a logical expression.

An example is that mathematics teaching in China stresses rigorous deduction and proof. Between "verification" and "proof" the latter is more valued. Taking a basic piece of geometric knowledge – "Gou Gu Theorem" (Pythagoras' Theorem) as an example, in teaching it must be proved in an abstract manner using algebraic or geometric methods, while cut-and-paste method is not counted as an acceptable way for eventual proof.

Here is an example appeared in the 1991 National College Entrance Examination:

Question 15. Which of the following statements is false?

(A) There exist the values of a and β so that $\cos(\alpha+\beta) = \cos a \cos\beta + \sin a \sin\beta$,
(B) There exists no infinitely many values of a and β, so that $\cos(\alpha+\beta) = \cos a \cos\beta + \sin a \sin\beta$,
(C) For any a and β, so that $\cos(\alpha+\beta) = \cos a \cos\beta + \sin a \sin\beta$.
(D) There exists no values of a and β so that $\cos(\alpha+\beta) = \cos a \cos\beta + \sin a \sin\beta$,

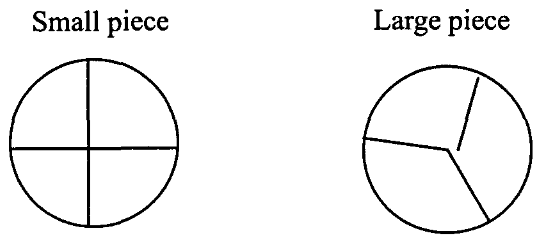
The correct choice is (A). The solution process should confirm to rigorous logical reasoning, with sufficient reason and being expressed in a clear and formal way.

The "two basics" mathematics teaching approach insists to keep mathematics thinking in abstract style. According to Cai (2001), in contrast with pupils from USA, Chinese pupils take a different way to solve the fraction problem. The test question is:

Pizza Ratio Problem: 8 girls share two pizzas and 3 boys share one pizza, each pizza has the same size. Please justify if each girl or each boy get more pizza.

There are three ways:

- Numerical symbols: 2 ÷ 8=1/4. 1 ÷ 3=1/3. 1/3 > 1/4.
- Words: 2/8 equal to 1/4, and 1/3 is greater than 1/4, boy get more pizza.
- Drawing



The report from Cai (2001) show that American pupils like to use drawing way to solve the problem, and very few Chinese pupils handle in this way. See Table 2.

As indicated above, the "two basics" mathematics teaching seems to pay more attentions to logical analysis, abstract thinking and formal expression.

Table 2. Numbers of Students Using Three Ways

	China			USA		
Grade	4	5	6	4	5	6
Correct rate	21	57	93	42	53	59
Visual drawing	4	3	4	65	53	59
Numerical symbol	35	51	58	27	36	47
Written words	61	42	28	8	11	4

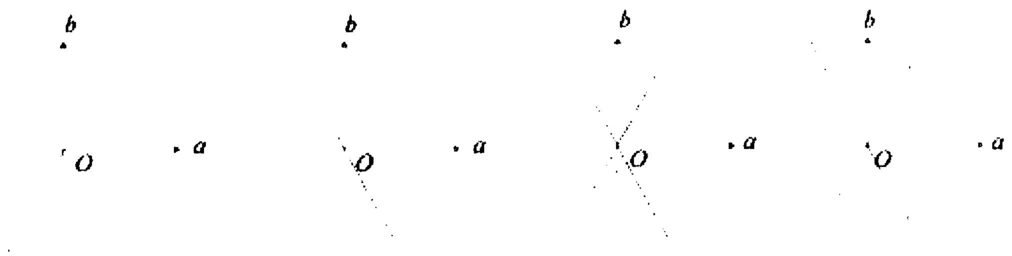
4. Exercise doing: repeat with variation

To attain a mathematical skill must be through frequent practice. Of course, mechanical repeat exercise is unable to achieve individual developing. Recently, a lot of study report that repeat with variation is a Chinese way of promoting effective mathematics learning (Gu, Huang, Marton)

Let us consider a typical example of variation (only one letter change: $c \to a$)

Example. (A test question of college entrance examination in 2003)

If the graphic of function $y=ax^2+bx+a$ has two intersection points with x-axis, then the point (a, b) located in the area of ( ).



(A) (B) (C) (D)

Correct answer is (C), but $a \neq 0$.

Although we only change a letter in the normal expression of $y=ax^2+bx+c$, the function $y=ax + bx+a$ will concern a lot of basic mathematics knowledge:

- Real roots of quadratic equation;
- Discriminate: $b^2 - 4aa = b^2 - 4a^2 = (b + 2a)(b - 2a) \geq 0$;
- Equation of straight line
- Area bounded by lines

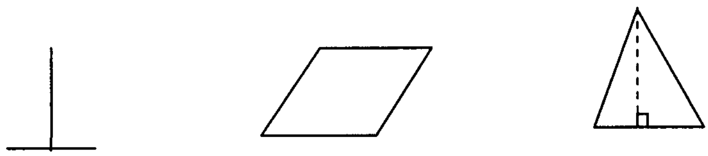
- The except straight line $a \neq 0$.

The major form is "variation method," including logic variation of concepts and the variation in the process of solution. There are explicit variations as well as implicit variations. This sophisticated arrangement of exercise is much more effective than the simple repetition of tasks (Huang 2002). Many studies show that continuous practice with increasing variation which will lead to understanding (Marton 1997).

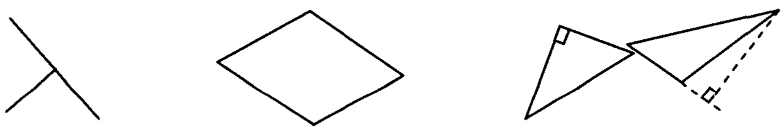
There are two types of variations:

(1) Conceptual Variations

Conceptual Variation (I)

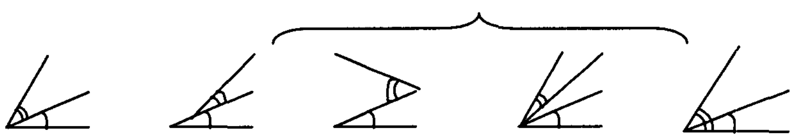


Perpendicular

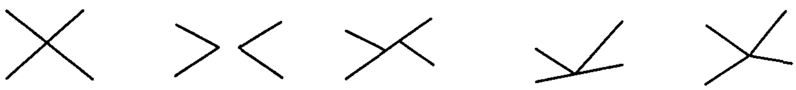


Conceptual variation (II)

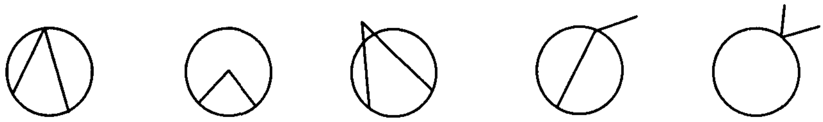
Adjacent angle



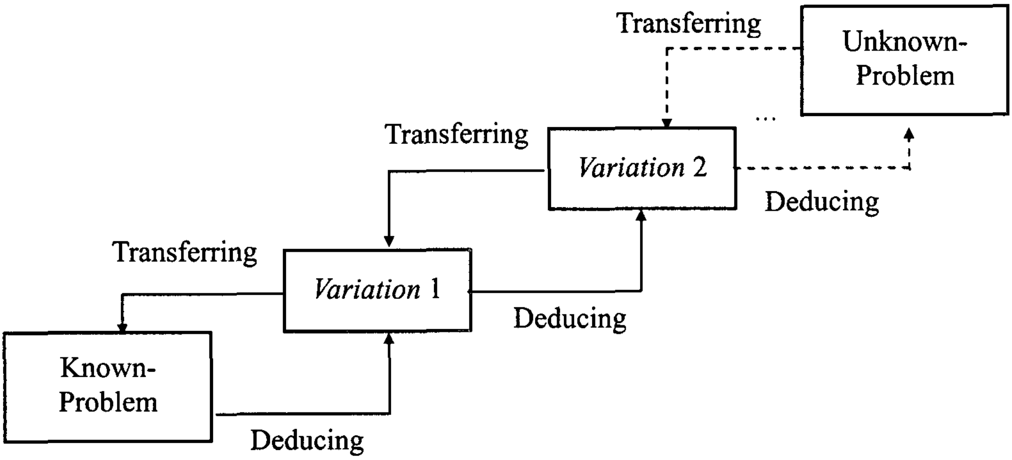
Opposite angle



Angle at the circumference



(2) Process variation



II. CHARACTERISTICS OF MATHEMATICS TEACHING UNDER THE "TWO BASICS" PRINCIPAL

China is a developing country with a population of 1.3 billion. There is a nine years compulsory education system. As the class size can reach as many as 40–50 students, it means in practice that it is impossible for teachers to practice individualized teaching. Under the affect of the "two basics" principal, we can see some characteristics of mathematics teaching in China.

(1) The rhythm of classroom teaching is led by the plan of the teachers. It emphasizes inspiring teaching and opposes spoon-feed teaching. Students are required to cater to the pace of progress set for most students by the teachers. It requires teachers to present the main mathematics contents as quickly as possible so as to avoid students spending too much time for getting into a winding path. However, in the Chinese classroom, teachers do not always give lecture demonstration. Oral questioning is quite frequent. Usually, the teacher presents a series of relative easy questions and asks students to answer in group or individual, and leading them to reach the set target through a number of small steps instead of letting them discover on their own. This is what we called "small step" teaching.

(2) The use of "essential lectures, much practice" principal. On the relationship between comprehension and manipulation, it does not support the proposal of "understanding first", but maintains that both are equally important. The lecture

demonstration for understanding must be short and essential in order to leave more time for solving mathematics problems. For time allocation, there is no need to spend too long time on the comprehension stage, as it is not likely to be accomplished at the first explanation. There must be practice in mathematics. Therefore, doing exercises in mathematics is of course necessary following comprehension, but without thorough understanding, students can still do exercises, and they can develop their understanding through working on the problems.

(3) Emphasizing the logical expressions. The "two basics" principal emphasizes the importance of fostering mathematics thinking, and the effect of repeatedly explaining and training in learning mathematical contents and ideas, such as complete categorization, conversions among the four prepositions, and comprehension of necessary and sufficient conditions; analysis, induction, synthesis, association, and RMI methods etc. In particular, during pre-examination revisions, the emphasis is on learning new things through reviewing the old. Some outstanding teachers use teaching methods that are "intensive, fast-paced, and full of contents", which can help them go over some twenty to thirty problems in a single mathematics revision lesson, and then generalize the mathematical methods employed. This kind of skill training is a demonstration of applying the "two basics" principal in its highest level.

(4) Pay more attentions to the applying of "mathematics method." Through research on "thinking method on secondary school mathematics"(*e.g.*, synthesis and analysis, inductive and deductive, transform between number and shape, modeling, analogy, connection etc), mechanical logical reasoning evolves to logical thinking ability, so one can grasp the overall structure of mathematics thinking at the secondary level, and develop systematic knowledge on it. It has been revealed that well versed and flexible mathematics manipulation can facilitate the formation of mathematics concepts. The ability to apply formulas and patterns in the calculation can convert mechanical manipulation to mathematical calculation ability. Skillful calculation and memorization of formulas can make mathematical thinking more condensed and faster, leading to a progression to a higher level of mathematics thinking (Hess & Azuma 1991; Thomas & Bain 1984; Li 1996).

III. HISTORICAL ROOTS AND SOCIAL ENVIRONMENT FOR THE "TWO BASICS" PRINCIPAL IN MATHEMATICS TEACHING

A good foundation is essential for the construction of buildings. Hence, no one would deny the importance of a good foundation, but the question is to what level or degree we should emphasize it, to which people have different views and practice. Most Chinese educators' these beliefs were gradually formed under the influences of culture accumulated through thousands years.

Let us trace back to the factors of forming these beliefs about the foundation in China's traditional education.

First, thousands years of agricultural culture, especially culture developed from plantation of paddy-field crops, required detailed and crafty artifice. Given a small land area, farmers have to rely on well-practiced and efficient techniques to obtain maximum outputs. It is very different from a nomadic society's culture where people can make a living through the extension of rearing areas. Thus, in the Chinese society, to be quipped with effective and efficient "skills" are of vital importance for survival.

Second, the strict examination system and unified exam questions have driven students to only learn the contents that will be tested in the exams. The system of civil examinations in China can be traced back to as early as the year of 597. Through this system, peasants can become government officials if they can pass the national examination. This is a highly fair and civilized policy. It is hence rooted in the minds of Chinese that examination can determine one's life. After the Ming Dynasty, the test items used in the unified examinations by the government become "Bagu-oriented" (which refers to a set of extremely condensed and procedure-fixed basic knowledge, and very stereotyped and sophisticated of writing skills). Referring to the modern mathematics examinations, most of the contents tested are also mathematics procedures and well-practiced skills. As examinations focused on the "basics", for the purpose of scoring well on the examinations students would also learn the basics only (Bishop 1998; Zhang & Lee 1990).

Third, as an educational wisdom in the Chinese society says, practice makes perfect. It was believed that the primary aim of education is to achieve familiarity. Through familiarity, who can naturally become "skillful" (perfect).

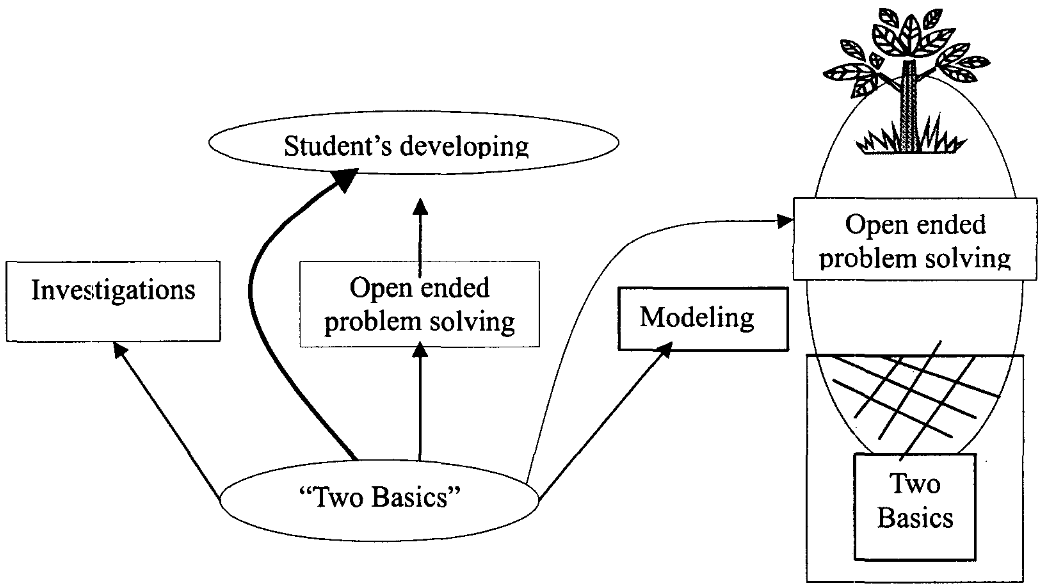
Finally, in the 1950s, the mathematics education in Mainland China was heavily influenced by then Soviet Union's mathematics education. As well known, the mathematics education in the Soviet Union in 1950s had emphasized more on the memorization of rules and regulations of basic knowledge, the rigor of a proof, including the basic training of logic reasoning.

Under the above factors, in 1963, the Ministry of Education of China stated in the mathematics syllabus the following: "mathematics education should strengthen students' learning of basic knowledge and basic skills," and moreover mathematics instruction should foster students' "basic computation ability, spatial imagination ability and logical thinking ability". This idea of emphasizing on the basics is still practiced nowadays.

IV. AN ONGOING DEVELOPMENT: OPEN ENDED PROBLEM SOLVING

It is not complete if we only think that teaching under the "two basics" (not the principal itself) in China means emphasizing memorization, imitation, and manipulation (Watkin & Biggs 1996). Since the 1990s, with the effort of numerous mathematics teachers, "two basics" teaching has been raised to the level of mathematical thinking, giving rise to a series of meaningful, scientific teaching methods that also match students' thinking process.

Quality mathematics education should consist of "solid mathematics foundation" and "progressive mathematics innovation". With modern mathematics education theories, mathematics teaching under the "two basics" principle has gradually entered a new phase. A Special trend is to incorporate the "two basics" of mathematics into the teaching using open-ended problems. Open-ended problem solving has become a fashion in China.



Open ended problem solving is initiated in Japan in 1970s. Then it is introduced into China. In the early of 1990s, mathematics teachers used the "open-ended problems" in their classroom teaching. A key step is that open ended problems appeared in the test paper of many very keen examinations. As a teaching approach, open ended problem solving has been recommended by national curriculum standards in 2002.

How to merge the open ended problem solving into the "two basics" mathematics teaching approaches?

A classical open ended problem is the "marble problem" from Japan: "Throw five marbles into a plane, how to define the diversity of these marbles?"

It is a quite excellent problem. However, it seems too far from the "basic knowledge."

In the past ten years, Chinese educators designed a lot of problems with new style. Here are some examples

Example 1. What is the common figure in the following two expressions: $8a^2b^2c^3$ and $12x^2y^3a^2$?

The possible answers may be: they are both algebraic expressions, they are both single algebraic terms, the coefficient for both the terms is 2, both have a variable with degree 2, etc. In fact, it has many answers, but is closely related to the "two basics"–basic conceptions (Dai 2002).

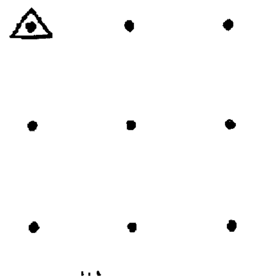
A simple open ended problem is designed for number sense.

Example 2. Please insert three irrational numbers between 50π and 170.

There are infinite answers. Students can use different ways to construct irrational numbers and by which they also acquire the computation skill. Another problem is concerning geometric conception of symmetry and reflection.

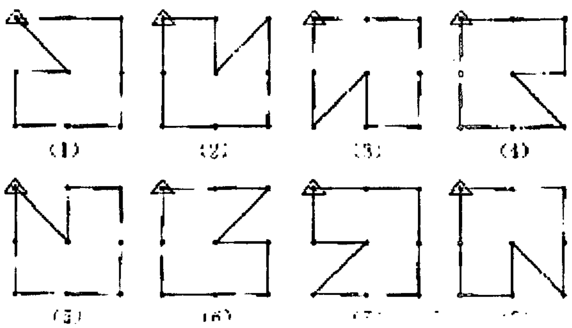
Example 3. Simple post route problem (Jinhui School, Shanghai, 1998)

There are 9 villages located in a square area (figure). The left above corner is the post office. The postman starts from the office, covers all the 9 villages and finally returns to the office.



What is the shortest route, how many routes can you find out?

This is a problem reflecting the combination of the basic geometric knowledge and creativity.



1→2→4→3 by a 90^0 rotation

1→5 by a reflection

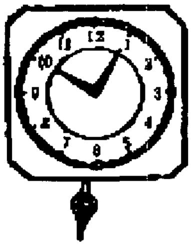
5→6→7→8 by a 90^0 rotation

An interesting example is the clock-face question that occasionally appeared in the junior high school textbooks in 1993. It shows that computation skill can be combined with open ended problem solving.

Example 4. (Clock-face number question)

There are 12 numbers on the clock-face, please add a positive or negative sign before the numbers so the algebraic sum of them becomes zero.

如图，钟面上有12个数字，试在某些数字前添上负号，使钟面上所有数字之和等于零。



Students can give some answers by their computation skill. However, beyond the imagination of students and the authors, there are 124 answers! Indeed, it is quite a good open-ended problem. Obviously, it is impossible for students to quote them all. However, they can come to the pattern that the sum of the positive numbers should be equal to that of the negative numbers. For instance, if we know the sum of numbers with plus (mines) symbol is 78 (–78), more answers can be found out. This is closely related to the basic training they had for addition and subtraction of rational numbers. Many open ended problems just is a mathematical circumstance. Given some conditions, but there are no conclusions.

Example 5. If $\triangle ABC$ is a triangle with right angle C, CD is perpendicular to AB, please find out the relations between the figures, lines, angles of the Figure A as many as possible.

This kind of problem are popularly used in classroom teaching in China, especially in reviewing lessen.

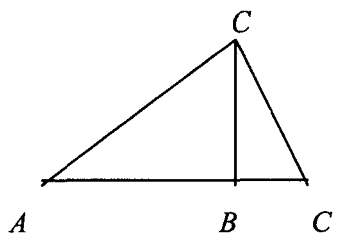
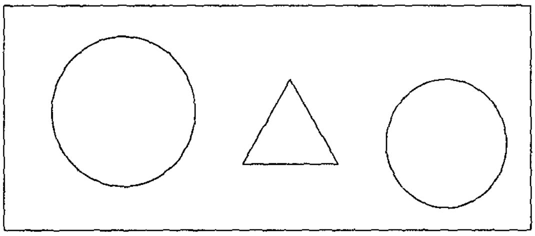


Figure A

Some open ended problems from abroad is also in use of mathematics teaching, if it is more closed to the "two basics." Here is an example from Japan.

Example 6. Flower bed problem (PME-13, Japan, 1993)

There is a rectangle field, in which we will design a flower bed with half area of the field. Please give your various designs.



This mathematics problem is combined well with arts. Especially, it related to the basic geometric knowledge and skill: figures, area, operation of real numbers, equation, square root, etc.

As well known, examination is the baton of mathematics education. Once the open-ended problems have been introduced in the very keen entrance examination papers. Then the use of open ended problem in classroom teaching formed a fashion gradually. Nowadays, every mathematics teacher in China is trying to use open ended problem in class room teaching, and focus on the assessment of test question with this style.

Example 7. (National University and College Entrance examination, 1998) In the figure B, $A_1B_1C_1D_1$-ABCD is a right prism, put a sufficient condition on quadrilateral ABCD so that $A_1C \perp B_1D_1$.

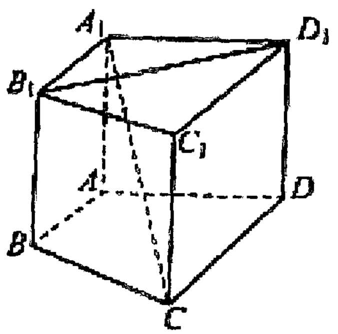
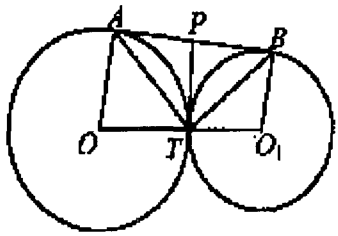


Figure B

It is the first open ended problem which appeared in a national university and college entrance examination. Although this problem is only valued 5 marks in the totals 150 marks of the test paper. However, this problem had caused the teacher's attentions to the applications of open ended problem solving.

Example 8. (High School Eentrance Exam, Hangzhou, 2001) Circle O and O_1 tangent to each other at *T*, *PT* and *AB* is the inner (exterior) common tangent respectively. Please give one of conclusions and its proof. (This problem is valued 12 marks. You will be evaluated according to the difficulty degree of the conclusion you gave.)



Finally, the authority of examination center made the evaluation standard as the following:

■ (6 marks) if student proved:

1. $PA = PT\ (PB = PT)$.
2. $\angle PAT = \angle PTA\ (\angle PBT = \angle PTB)$
3. $\angle OAP = \angle OTP = 90°$

■ (8 marks)

1. $PA = PB = PT$.
2. $\angle ATB = 90°$
3. $\angle AOT + \angle APT = 180°$
4. $OA \,//\, O_1B$

■ (10 marks)

$\triangle OAT \sim \triangle PTB \quad (\triangle PTA \sim \triangle O_1BT)$

■ (12 marks)

$PA \cdot PB = OT \cdot O_1T$

$(PA \cdot PB = OA \cdot O_1B)$

A very interesting open problem is appeared in the "National Uuniversities and Colleges Examination" in 2002.

Example 9. Given two pieces of equilateral triangular papers with the same area, please try to:

1. Cut one paper (Figure 1) into several pieces to form the faces of a regular tetrahedron;
2. Cut another paper (Figure 2) into several pieces to form the faces of a triangular prism.

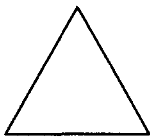


Figure 1

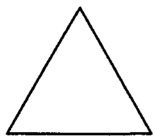
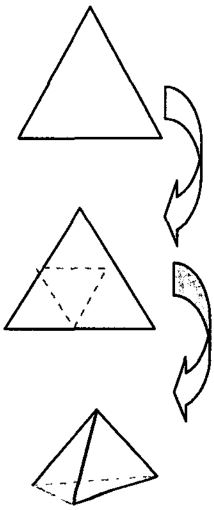
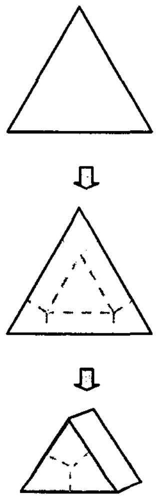
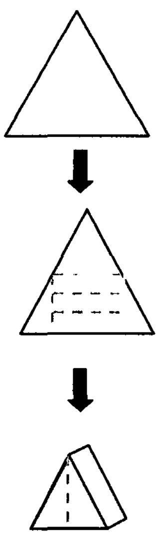


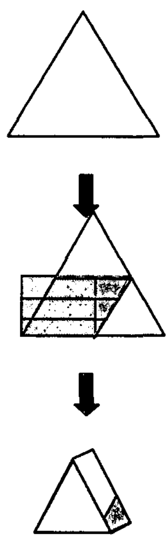
Figure 2





It seems the easiest solution of the problem. However, there are many answers as the following:





In fact, the answers are infinite. We can recall the "Bolyai-Gerwien" Theorem (1832):

Bolyai-Gerwien Theorem. If two simple polygons of equal area are given, one can cut the first into finitely many polygonal pieces and rearrange the pieces to obtain the second polygon. "Rearrangement" means that one may apply a translation and a rotation to every polygonal piece.

Janos Bolyai
1802 - 1860

CONCLUSION

The eastern and western mathematics education is looking for a balance between fundamentals and development (Leung 1998; Lim-Teo 1998). The teaching under the "two basics" in China is leaning out for new development on top of the characteristics it already possessed, striking a balance between foundation and development in classroom teaching. In any case, "basics knowledge and basic skills will be important forever in one's life." Foundations need to develop with time.

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APPENDIX

Research of Rural and Urban Secondary Students' Identity Calculation at Jiangsu Province3

(1) Test subject and method

In 1998, a test was done on 2049 students from 8 schools, a total of 43 classes in Jiangsu Province, China. The content is mathematics equations (included grouping, completing the square and factorization).

38 questions of various levels of difficulties are arranged, and the time limit is 10 minutes. It was a test of students' basic mathematics skills, especially the speed of completion.

(2) Test statistics

Results are shown in the following table, X is the average number of correct answers, σn is the standard differentiation of the number of correct numbers, $Y(\%)$ is the a ccuracy ratio.

The definition of accuracy is correct answers of the tested form divided by the number of group members multiply by the number of questions time 100%.

Table A1. Average Number of Correct Questions, Standard Deviation and Accuracy

Grades / Variables	8th	9th	10th	11th	12th	Total
X	17.82	20.73	28.11	29.43	31.00	25.78
σ_n	6.08	7.05	4.22	4.34	4.20	7.29
Y(%)	46.89	54.55	73.97	77.45	81.58	67.84

The testing criteria of secondary students' equation solving ability

In mathematics teaching, it is a major teaching goal to build students' basic knowledge and accompany it with solid training so as to form high standards of mathematics ability.

Based on the statistical analysis of the above samples, and combine it with the experiences of Mathematics teachers, 3 indexes can be concluded, they are: the average, the pass and the excellent grading. According to statistical results, the following reference standard is compiled:

3 Conducted by Tian Zhong, Changshu College, 2003.

Table A2. Standard Reference Chart of the Equation Solving Test Results

Grades \ Standard	Average*	Pass*	Excellent*
	Correct answers	Correct answers	Correct answers
8th	17.90	≥13	≥27
9th	21.84	≥16	≥32
10th	27.67	≥24	≥34
11th	30.06	≥26	≥36
12th	31.29	≥28	≥36

* Number of correct answers: The numbers of problems is 38.

Calculation test paper

(I) Solve questions 1 to 5:

1. $1 -3x^2y + 5x^2 y =$
2. $(1/4)ab^2 -2ab^2 =$
3. $(1/3)xy^2 \cdot (-6x^2y) =$
4. $6ab^2c \div (-9ac) =$
5. $(-3xy^2)^3 =$

Factorization (6–8):

6. $(x2^m-9) =$
7. $x^2-3x +2 =$
8. $y^2 + y + (1/4) =$

Completing the Square (9):

9. $x^2- 3x +1 =$

(II) Solve questions 10 to 14:

10. $-(2/3)\, ab + (3/4)\, ab + ab$
11. $-y^2 - 2x^2 - (-3y^2)$
12. $3x^2 y \cdot (1/2)\, x \cdot (-2xy^2)$
13. $3x^2 y + (1/3)\, xy \div (-xy)$
14. $[(-2n)^2]^3$

Factorization (15–17):

15. $(a + b)^2- (x-y)^2$
16. $(x + y)^2+ 5\,(x + y) + 6$
17. $(m - n)^2+ 4(m - n) + 4$

Completing the square (18):

18. $(1/2)\,x + x + (2/3)$

(III) Solve question 19 to 20:

19. $4x^3 - (-6x^3) + 9x^3$

20. $2a^2\,by\,[(-1/3)by] \div a^2\,by$

Factorization (21–23)

21. $a^2 - ab + ac - bc$

22. $m^2 - n^2 + am - an$

23. $x^2 + 2xy + y^2 - z^2$

Completing the square (24)

24. $x + px + q$

(IV) Solve question 25 to 30:

25. $(-1/2)\,ab^2\,(b^2 + 3a^2b)$

26. $(x - 2y^2)\,(-2x^2y)$

27. $(m^3\,n + mn^2) \div (1/3)mn$

28. $(a^3\,b^4\,c - 2ab^3c) \div (-2ab)$

29. $(-2ab^2 + a^2b + 3ab^2)^2$

30. $(-a^2b)^5 \div a^6\,b^2$

(V) Solve question 31 to 32:

31. $(4x^2y - 5xy^2) - (3x^2y - 4xy^2)$

32. $6ab^2\,[(-1/3)ab^4] \div 2a\cdot(-ab^2)$

(VI) Solve question 33 to 38:

33. $2s^2t + (1/2)\,s^3t^2 \div (3/2)st + (2/3)\,s^2t$

34. $ab^2c^2 - ab\,(1/2)\,bc^2 - ab^2c^2$

35. $xy^2z - xy\cdot x^2y\cdot(-xz)$

36. $-m^5n^3\,(1/3)\,m^2n^2mn + 4m^2$

37. $(21a^2b^2 - 35\,a^3b^3) \div 7a^2b^{\,2}\cdot 2ab$

38. $2x^5y^4 \div (1/2)\,xy^2 + (-3x^2y^2)\,2x^2$