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Early Mathematical Giftedness and its Social Context: The Cases of Imperial China and Soviet Russia[1]

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The paper discusses the phenomenon of mathematical giftedness, especially manifested at early stages of life of future outstanding mathematicians, taken in its socio-cultural context. The authors suggest that the images of mathematical giftedness are formed differently in various cultural contexts and thus can imply different settings of the educational institutions that can accordingly ignore, encourage, or restrain the students considered gifted. The paper focuses on the cases of traditional mathematics in several Asian countries (China, Vietnam, and Japan) and of modern mathematics in Soviet Union/Russia in order to provide examples of different patterns of forming the image of mathematical giftedness and of the corresponding educational approaches.

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INTRODUCTION

The reader is certainly familiar with the famous anecdote of Gauss who at age of 9 was asked by his teacher to add the natural numbers from 1 to 100 and solved the problem almost immediately. This story is often told in standard textbooks on the history

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of mathematics and is generally considered relevant to Gauss' prominent position in the 19th century mathematics, even though its relevance has never been discussed in detail[2]. Several other famous mathematicians are also believed to be mathematical prodigies, or at least to have developed strong interest in mathematics at early age; one can immediately recall the cases of Euler[3], Galois[4], Abel[5], Ramanujan[6], Erdös[7], Ulam[8], Wiener[9], and Wiles[10], among others. However, historical data suggest that a number of outstanding mathematicians did not develop particular mathematical skills at very early age (Newton)[11], or were below the average level (Courant[12] and Halmos[13]), while the circumstances of the personal life of many other mathematicians at early age simply remain unknown[14]. However, a considerable number of modern mathematicians share the opinion that the major mathematical discoveries are usually made at relatively early age[15], and therefore the mathematical giftedness manifested by children in school appears to warrant a success in mathematical career.

In the present paper, we are not going to discuss the validity of this hypothesis; instead, we are going to investigate how the vision of early giftedness commonly shared within a given culture may affect the educative strategies and influence, if not shape, the system of mathematics education. To make more visible the differences in the concepts of

[2] See, for example, Katz (1988), especially see p. 654; Katz refers to Bühler (1981).

[3] As a child, Euler had an outstanding ability for mental calculations, see Dunham (1999).

[4] See Toti Rigatelli (1996).

[5] See Ore (1957).

[6] See Ranganathan (1967), Kanigel (1991); for a short biography see Berndt (1989).

[7] By the age of three Erdös could mentally multiply five digit numbers, by four, he independently discovered and mastered negative numbers, and by seven he found more than thirty proofs to the Pythagoras' Theorem; see Schechter (1998), Hoffman (1998).

[8] See Cooper (1987).

[9] Wiener was a fluent reader by the age of three, completed high school in two years by age eleven, and obtained his doctorate in mathematical logic by nineteen; see Wiener (1956).

[10] A. Wiles was ten years old when he developed an interest in Fermat's Last Theorem; see Singh (1997).

[11] I. Newton (1642–1727) started studying Descartes, Vieta, Wallis and Gallileo only after his enrollment at Cambridge University (1661); however, at this stage Newton showed no great talent and obtained his first major mathematical results only in 1665–1667; see Whiteside (1964, 1966), Westfall (1980, 1993).

[12] At school, R. Courant was ranked well below average, especially in mathematics; see Reid (1976).

[13] When P. Halmos completed his undergraduate degree, he felt that "in mathematics [he] was not only uninspired, but shockingly ignorant", see Halmos (1985, p. 35).

[14] For example, it is unknown at what age Pierre Fermat (1601–1665), arguably one of the greatest mathematicians, began studying mathematics and how good he was at it; he joined the scientific community for the first time as late as 1636 when replying to Mersenne; see Mahoney (1994).

[15] This conviction is apparently reflected in the age restriction for the Fields medal recipients; for the history of the establishment of the prize, see Monastyrsky (1997).

giftedness in various cultural settings, we are going to describe two clearly distinct mathematical and didactical traditions: (1) the traditional Chinese mathematics and its local modifications in Vietnam and Japan, and (2) the modern school mathematics as taught in the USSR (currently Russia) prior to 1991. In both cases, we will pay attention to the perception of the "gifted" children and the ways the established systems of mathematics education dealt with them.

1. TRADITIONAL CHINESE MATHEMATICS AND MATHEMATICS EDUCATION

By traditional Chinese mathematics we mean the tradition that originated in China by the second half of first millennium BC. There are very few literary sources that survived from the earliest period of the development of the tradition, such as the unearthed in the 1980s mathematical treatise *Suanshu shu*[16] (算數書: Writing on Calculations with counting rods) (see Peng 2001) written no later than the early second century BC and the mathematical treatise *Jiuzhang suanshu*[17] (九章算術: Computational procedures of nine categories) compiled most likely in the first century AD on the basis of older literary sources. The latter treatise, together with 11 other mathematical books (*e.g., Suanxue*: 算學), was used from the 7th century AD onwards as one of the principal textbooks in the Mathematical College of the State University located in the capital of China. The state system of mathematics education existed in China from the late 6th or early 7th century to 13th century AD, and its decline occurred simultaneously with the general decline of Chinese mathematical tradition[18].

The instruction in the Mathematical College lasted 7 years, and the age of enrolment varied from 14 to 19 years. The teaching and testing were described only briefly in the extant sources, and yet these descriptions allow a reasonable reconstruction that shows that the instruction was not focused on mere memorisation, as some authors suggested Martzloff (1997), but was rather a set of activities combining memorisation of the textbooks with interpretation and understanding of their contents. To test the knowledge

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[16] In this paper we adopted the *pinyin* transcription system of Chinese terms.

[17] The title of this treatise has been rendered in various ways, such as Nine Chapters of/on Mathematical Art, Nine Chapters of Computational Procedures, etc. For an English translation of the treatise see Shen, Crossley & Lun (1999).

[18] In the early 17th century the first six books of (an adapted version of) the *Elements* of Euclid were rendered into Chinese by Matteo Ricci and Xu Guangqi, and this event marked the beginning of the introduction of the Western mathematical tradition into China, see Engelfriet (1998); however, to the best of our knowledge, no research has been done concerning the continuation of Chinese *didactical* tradition in the context of the newly adopted European mathematics.

of the students, oral and written tests were given (routine quizzes were given on every 10th day), and their extant brief descriptions again suggest that the testing procedures may have focused on the understanding of the material taught rather than memorisation, even though a memorisation test was a part of the examinations[19].

In the present paper, we will also consider one representative of Vietnamese mathematical tradition which was a continuation of the Chinese one, as far as the institution of state examinations is concerned. Vietnamese (state) mathematics was practiced until the late 19th century, or even later[20], and we assume that social and cultural context of Vietnamese mathematics was similar to that of its Chinese predecessor.

Our preliminary investigation of the received (auto-) biographical data of the most outstanding Chinese mathematicians is insufficient to claim that any of them were mathematical prodigies at early age. In one case, a mathematician (monk Yixing, see below) demonstrated an extraordinary memory, yet his age was not specified and the ability was not, technically speaking, the mathematical one. In some cases (*e.g.*, Liu Hui: 劉徽) mathematicians manifested interest to mathematical books "at early age", but neither the age nor the type of mathematical work undertaken by the mathematician remained was specified. In what follows we are going to provide a cursory presentation of several cases that appear relevant to the discussion.

Liu Hui, Rong Fang, Chenzi and the ways of learning

Liu Hui (p. 263) is the commentator of the *Jiuzhang suanshu* and arguably one of the greatest mathematician in the world history of mathematics. Nothing is known about his personal background, yet his *Preface* to the *Jiuzhang suanshu* contains the following line:

徽幼習九章。長再詳覽[21].

Given that the *Jiuzhang suanshu* with commentaries of Liu Hui was one of the central mathematical treatises in the Mathematical College, and its study (together with another treatise, the *Haidao suanjing* 海島算經 by Liu Hui) lasted for three years, that is, longer

[19] Siu and Volkov (1999), see also Siu (1999), Volkov (2004).

[20] Historical documents contain mentions of the state mathematics examinations that took place in in 1077, 1179, 1261, 1363, 1404, 1437, 1472, and 1505; information is currently lacking for mathematics examinations which may have been held after the beginning of the 16th century, yet it is known that the examinations in mathematics took place in 1698, 1711, 1725, 1732, 1747, 1762, 1767, and 1777. Moreover, a Vietnamese mathematical treatise of 1820 devotes a special section to the way in which an examination paper is to be written, and this also suggests that the same tradition still existed in the early 19th century; see Volkov (2004).

[21] "[When I, Liu] Hui, was a child, I studied the *Jiuzhang* [*suanshu*], [when] I grew up, I read it again in great detail" (see Guo & Liu 2001, p. 83).

than the other treatises of the syllabus[22], one can suggest that the Preface by Liu Hui provided a paradigmatic example rather than information concerning personal history of his studies, thus suggesting a particular model of learning: the "study" mentioned here is expressed with the word *xi* (習) having the meaning "repeatedly practice" and employed in this way, for example, in the main Confucian classic, the *Analects* (*Lun yu*: 論語)[23]. The Preface thus suggests that the first step consisted of "practice" in the given text, while the second step was the "detailed reading" that would lead to the "understanding" mentioned later in the Preface. The "true understanding" thus was supposed to happen only at the stage of "maturity" of the mathematician, and not earlier.

This interpretation of the procedure of learning is supported by the dialogue between two legendary mathematicians of antiquity; their existence is highly problematic, yet their conversation, taken in context, sets another paradigmatic example of interaction between student and teacher. In this dialogue, the disciple, Rong Fang (榮方), asked his mentor, Chenzi (陳子), a question related to astronomy. Instead of explaining the details asked about by Rong Fang, Chenzi suggested his disciple to "think more" about how to "apply mathematical procedures" to answer his own question. The student returned back home and passed several days in search of the answer, yet gave up, and asked the teacher for further explanations again. The Master replied, yet instead of a detail exposition he lectured his disciple on the general methodology of learning. The latter returned back home and stayed there for several days trying to apply the depicted methodology. As the reader can guess, he failed again and had to return to his Master for explanations, which were finally granted[24].

If we try to see the general pattern behind the stories, we can suggest that the "learning" in both cases includes an interaction between the "source" of knowledge, be it a book or a master, and a multiple "questioning" the source, coming together with personal efforts, until the required knowledge is obtained. The early giftedness, certainly, can fit into the described scheme, yet it will play a particular role, namely, the "gifted" students will understand the hints of the book or of the master faster than a "regular" one. Below we will see that most likely the interaction with the master/book plays an important, if not the central, role in the scheme of traditional education, and the mathematical giftedness of the student plays the complementary, yet also important role.

The actual careers of mathematicians in tradictional China

It is not known whether Zu Chongzhi (祖沖之: 429–500) and Li Chunfeng (李淳風:

[22] There were two syllabi, one regular and one advanced; see Siu and Volkov (1999). The *Jiuzhang suanshu* was the most difficult book in the regular syllabus.

[23] Ware (1980), Chapter 1.

[24] For the original text, see Guo and Liu (2001, pp. 37–38); for a translation, see Cullen (1996).

602–670), two outstanding Chinese mathematicians, were mathematical prodigies, yet they both manifested their mathematical talents (especially in the field of calendrical astronomy) relatively early, in their mid-twenties or early thirties. In ca. 462 Zu Chongzhi submitted to the throne a memorandum criticizing the then running calendar and suggesting a new calendar. Similarly, in 627 Li Chunfeng criticized the calendar introduced in 619, and his criticism won the support of the court scholars; consequently Li was honored by the Emperor with a prestige title and given an office in the Astrological Bureau.

If one recalls the age of admission to the Mathematical College was set between 14 and 19, and that the studies normally lasted for 7 years, one can conclude that the average official career was designed in such a way that a successful doctoral candidate in mathematics examinations could have obtained the official position in his early or mid-twenties. Given that all the textbooks of the syllabus implied perfect command of arithmetical operations, one can suggest that the depicted education system was dealing with students who began studying mathematics at rather early age. At the same time, the system was designed in the way precluding from studying mathematics in an "unorthodox" way: the set of the textbooks was especially edited for instruction and was given the Emperor's formal approval. At the same time, the private teaching of astronomy (most likely including various mathematical subjects) was officially proscribed see Volkov (2002a). However, multiple pieces of evidence suggest that the official mathematics education at the Mathematical College was not the only channel of transmission of mathematical expertise: there existed networks of private teachers and independent experts, often belonging to religious (Daoist and Buddhist) sects see Volkov (1997).

Lương Thế Vinh and other cases of "giftedness"

Lương Thế Vinh (梁世榮: 1441–1496?), a high-rank official of the Vietnamese Lê (黎) dynasty (1428–1789), is conventionally considered the author of the oldest extant mathematical treatise *Toán pháp đại thành*[25] (算法大成). Even though Lương Thế Vinh's authorship of the treatise is problematic[26], his image as a patron saint of court mathematicians in the 18th and 19th century appears relevant to the present discussion.

There exists a legend about the childhood of Lương Thế Vinh portraying him as a particularly gifted child. According to the legend, all the children of his village were given a problem by a stranger: how can one remove an apple from a narrow and deep cavity without using a stick? Only Lương Thế Vinh, the future winner of the Imperial

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[25] On the contents of the treatise, see A. Volkov (2002b).

[26] See A. Volkov (2004 forthcoming).

examinations, was able to solve the problem[27].

This episode does not appear to have any particular mathematical relevance; it rather speaks about the general lucidity of the mind of the future great scholar. However, interestingly enough, similar stories exist in other East Asian traditions. For example, the most outstanding Japanese mathematician Seki Takakazu (關孝和: ?–1708), according to a legend, was able to solve a problem that was not, strictly speaking, mathematical: he indicated how to cut a tree in such a way that each piece would have a given weight[28]. The age of Seki at the moment when he offered the solution is not specified, yet it appears that he was not portrayed as a mathematical prodigy by his biographers[29]. However, the plot of the story is rather similar: the giftedness of the mathematicians is manifested in their solutions for "non-standard" problems.

One more similar story concerns the famous Chinese mathematician Yixing (一行: lay name Zhang Sui 張遂, 683–727); according to a legend, he memorized a difficult text after having it read only one time, and this event made the author of the text suggest that Yixing must leave the monastery where he studied in search for a suitable teacher[30]. The latter case brings us back to the familiar framework of the situation of transmission of knowledge from a master to a disciple in which the giftedness of the student is perceived as a prerequisite for the successful transmission of knowledge. This pattern is constantly repeated in the stories about the most outstanding mathematicians. One can suggest that this perception of education and giftedness was interrelated with the differentiation of the students on the basis of their capacities.

Those who were less gifted could follow a standard mathematical career imbedded into the framework of state-run education and aiming at official career, while the talented could choose between the official education and a private apprenticeship within a small group of disciples of an outstanding master[31].

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27 See the unpublished MSS Trần Tiến 陳璡, *Đăng khoa lục sưu giảng* 登科錄搜講 (In search for explanations for the Records of successful examinees), pp. 5a–10a; Anonym, *Đại Việt đỉnh nguyễn Phật lục* (大越鼎元佛錄: Records of the Majestic First [Candidates] of the Great Viet [Who Reached] Buddhahood), pp. 14b–17b; Vũ Văn Lập, *Nam sử tập biên* (南史輯編: Histories of the South [= Vietnam], collected and edited), pp. 42a–44b.

28 The story is found in the unpublished treatise Burin inkenroku 武林隱見錄 (Records of warriors, hidden and manifested) by Saitô Yajin, quoted in Horiuchi (1994, p. 130).

29 Unlike his closest disciple Takebe Katahiro (建部賢弘: 1664–1739) who discovered mathematics at the age of 13 and studied it "from dawn to dusk" and was no less talented than his Master; see Horiuchi (1994, p. 148).

30 See Zheng (1986, pp. 521–522); for an English translation see Needham (1959, p. 38).

31 Guo Shirong lists 50 most outstanding Chinese mathematicians and astronomers who were active during the time-span from the first millennium BC to the end of the 19th century, and remarks that only two of them graduated from the State educational institutions, and only four graduates of these institutions are to be found among the 150 scholars listed in the Chouren zhuan (疇人傳: Biographies

It is certain that in each individual case the biography of a mathematician depended largely on concrete circumstances and could not be reduced to only two patterns, yet the vision of giftedness as of one of two components necessary for becoming an outstanding mathematician appears to be clearly articulated in the aforementioned narratives.

Figure 1. A teacher of mathematics with two disciples.

It appears also that the second indispensable component was the meeting with the "right" master. The extant mathematical books and (often legendary) biographies repeatedly depict the image of a gifted (and, supposedly, young) mathematician searching for "his" master. Again, the example is provided with the case of Yixing. When Yixing successfully calculated the leaves of the tree, his superior in the monastery where he stayed at that time ordered him to leave the monastery and to travel in order to find an appropriate teacher. Finally, Yixing discovered a "predestined" master waiting for him in another monastery[32].

The pattern one can see here is as following: the solution of non-standard

of astronomers [and mathematicians]) by Ruan Yuan 阮元 (1799); see Guo Shirong (1991).

[32] For a biography of Yixing see Chen (1992).

(mathematical as well as non-mathematical) problems at an early age (*i.e.*, giftedness) symbolizes the particular potential of the individual, which can be fully developed only after his meeting a "special" teacher. One can find the same pattern in many other disciplines of the traditional Chinese science, *e.g.*, in the transmission of alchemical knowledge. This importance of the figure of mathematical teacher is reflected in the piece of late Ming dynasty (1368–1644) iconography presented in Figure. 1 Cheng (1990, p. 804): here the Master is portrayed with certain attributes of a saint and/or a Confucian sage (such as a deer, a miraculous mushroom, a pine-tree, etc.).

2. SOVIET MATHEMATICS EDUCATION: WORK WITH GIFTED CHILDREN IN THE CONTEXT OF AN "EGALITARIAN SCHOOL"

As we move to the modern time, Soviet mathematics education provides another example of a coexistence of two different approaches to mathematics education, one imbedded into the general lay public educational system implementing the blueprint based on the European concepts of the late 19th century, and the other one focusing mainly on gifted children and having flourished starting from 1950s onwards. The latter took the form of a complex network of activities including "mathematics clubs for advanced children" (Russian "кружки" (*kruzhki*), lit. "circles" or "rings", usually affiliated with schools but also home-based), Olympiads, team mathematics competitions, (*mat-boi*, literally "mathematical fight"), extracurricular winter or summer schools for gifted children, publication of magazines on physics and mathematics for children (the most famous being the *Kvant*, lit "Quantum"), among others.

All these activities were free for children; the majority of them were not paid and were based solely on the enthusiasm of mathematics teachers or university professors. This process led to the creation of a system of formation of a "mathematical elite"[33] in the former USSR focused first and foremost on "gifted children", which was in a sharp contrast with the "egalitarian" regular state-run schools targeting "average student" and thus neglecting the needs of all those above average level. To deal with this phenomenon we are going to present the historical background in Russian and later Soviet mathematics education. The Soviet regime inherited the scientific legacy from the 'old' Imperial Russia of which mathematics was one of the most developed disciplines[34]. Under the influence of such famous mathematicians like D. Euler and L. Bernoulli who worked in

[33] A similar process took place in other disciplines, in particular, physics, but in the present paper we will focus on mathematics education.

[34] See, for example, Graham (1987, p. 29): "The two greatest traditions of Russian science established in the eighteenth century were in mathematics and in the study of natural resources".

Russia, 'native Russians were beginning to show genuine talent' by the end of the 18th century. Moreover, in the 18th century several higher education institutions with strong mathematics curriculum including arithmetic, geometry, and mathematical geography appeared in Russia[35]. Along with the state elite schools, the regular school system was created, yet it was rather limited, formal and slow[36]. And yet, according to Vogeli[37], development of mental capacities of the pupils was the dominant didactical principle in mathematics education. The painting of N. Bogdanov-Belskii (1868–1945) "Ustnyi schet" (Mental calculation) is a beautiful illustration of this method; see Figure 2.

Figure 2. N. Bogdanov-Belskii 'Ustnyi schet'[38]

[35] B. Vogely gives three examples of such schools: The Moscow School of Mathematical and Navigational Sciences, founded by Peter the Great in 1701, St.-Petersburg gymnasium (1726) and the Smol'nyi Institute for "young ladies of noble birth", see Vogeli (1968, p. 697).

[36] Vogeli stresses that mathematics instruction was formal to the extreme and differed little from instruction in the linguistic forms and syntax of classic Greek and Latin (Vogeli 1968).

[37] Vogeli (196) with reference to Semushin (1963).

[38] The painting portrays a countryside school and several peasants' sons solving a problem set by an old mathematics teacher. The problem is written on the blackboard, it reads $(10^2 + 11^2 + 12^2 + 13^2 + 14^2)/365$ and the pupils are supposed to solve it mentally. The age of the pupils most likely varies

As regular school system usually did not pay much attention to gifted children, only a few gifted children such as young A. Kolmogorov were able to benefit from a unique extra-curriculum pedagogical environment allowing them to enjoy the beauty of mathematical discovery. He attended a small private school organized by his grand-mother at home for a small group of students of various ages in which the teachers used the most recent pedagogical innovations. The report of the Kolmogorv's 'mathematical discovery' was published in the school magazine[39].

An interesting episode in Kolmogorov's story is related to his further schooling at a private gymnasium organized by "radically oriented intellectuals."[40] A mixed school (for girls and boys) with the curriculum of "boys' gymnasium" (*i.e.*, a college) gave the students an opportunity to study on according to their own interests and levels (Kolmogorov could, for example, take a math course from one grade higher). At the same time, students felt responsible to study hard and to get best results for the tests and state examinations. It is not surprising that a school of this kind was in contradiction with "regular" state schools and thus was under constant threat of closing by the officials.

After the revolution of 1917, the new Soviet government closed all private schools and established a completely new school system with new curriculum. The system was aimed at offering a basic education to the entire population and, at the same time, making the education more practice-oriented. As result of implementation of these ideas, "the mathematics program lost much of its theoretical content. Pupils studied mathematical 'recipes' applicable in specific practical situations, often without consideration of their theoretical bases"[41]. It remains unclear what was the situation of gifted students in those years, yet the sources point at the lack of knowledge displayed by those who graduated from the schools based on such "innovative" approaches as the so-called "brigade-project" organization[42].

As reaction to this gloomy situation, the government declared these methods as 'errors' and ordered to make necessary corrections in the school curriculum in the early 30s. Thus, "pre-Revolutionary mathematics texts were resurrected, revised, and made

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from 7–8 to 12–13.

39 A. Kolmogorov writes that at the age of 5 or 6 he was pleased with his discovery of the regularity of the sum of odd numbers: $1=1^2$, $1+3=2^2$, $1+3+5=3^2$, etc.; see Kolmogorov (1988).

40 "Радикльно настроенная интеллигенция", that is, a group of highly educated persons with radical progressive views and highest demands concerning the quality of the education given to their children.

41 Vogeli mentions that "the 1921 syllabus emphasized the value of creative activity in teaching of mathematics, the need to broaden pupils' mathematical background, and the desirability of relating mathematics to life", Vogeli (1968, p. 4).

42 Vogeli (1968) with reference to Bradis (1954, p. 38).

official standard"[43].

Resuming our analysis, we can state that keeping explicitly egalitarian approach to education, the Soviet education system at one moment (namely, in earlier 1930s) began spending much more money and effort to identify promising individuals and to provide them with opportunities to develop their talents (Blazer 1989)..

As the officials struggled to meet the needs of growing economy and to maintain the access to schools for everybody, numerous initiatives came from prominent mathematicians and scientists. One of the striking examples was the first mathematical Olympiad for schoolchildren organized in biggest Soviet cities: Leningrad, Tbilisi, and Moscow in 1934–35[44]. The Olympiads helped to build traditions that went far beyond the officially stated goals (such as a high quality education). Instead, as their participants recall, they became actual festivals of mathematicians of all generations, schoolchildren, university students, school teachers, young high-school professors, and prominent scientists.

The Olympiad problems were not oriented on a mere application of school knowledge but required a capacity to find original way of thinking, ability to reason logically in a non-standard situation. The Olympiad was usually followed by a lecture with analysis of typical mistakes and by individual meetings of the participants with the members of the jury. The Olympiad was not the only a way to work informally with young talents but also a way to motivate school children to learn mathematics in a more systematic way by participating in "mathematics circles", attending public lectures given by outstanding mathematicians and by self-study of mathematical books.

Looking at the social context of this phenomenon, we can consider it as a personal mission of mathematicians contributing to the society in order to promote and popularize mathematics and emphasize the value of mathematical work, to search and to support young talents and give them the best of their knowledge. A mathematical community was created beyond regular educational system, and the explicitly stated goal of this community was to maintain the highest level of mathematics and to promote

[43] Ibid., see also И. Логвинов, О реформе школ и педагогической науке: забота о просвещении народа, или… (I. Logvinov, On the school reform and the pedagogical science: the care of the education of population, or…, in Russian), Moscow, 1990. Logvinov argues that when one compares this system to the old Russian one (*i.e.*, prior to 1917 revolution), some striking similarities would appear, and the new system is no less than a clear-cut return to the tradition of the classical "gymnasia" (Colleges) and the "real schools" ("реальные училища"). Comparatively to the old system, the Soviet school lost such subjects as religion, philosophy and several foreign languages, but kept the old tradition of the high quality of teaching mathematics and natural science. This high quality was possible due to the teachers of the 'old' school many of whom were still active.

[44] The *Encyclopedia of Young Mathematician* (*Энциклопедический словарь юного математика*, Moscow: Pedagogika, 1985, in Russian, p. 187) mentions the famous mathematicians B. N. Delone and P. S. Alexandrov as the intiators and organizers of these competitions.

attractiveness of mathematical activity among the population and to support and encourage everyone with talent and interest in mathematics.

We are going to briefly depict the work of three personalities actively engaged in this social mission[45]. All of them had experience as school teachers, and, to a certain extent, belonged to the official pedagogy. Kolmogorov and Schwarzburd were the authors of textbooks for the secondary schools. Kolmogorov was also the "father" of the curriculum reform in 1970s; he wrote textbooks on geometry for grades 6 to 10 and on algebra and beginning of calculus for grades 9 and 10, while Schwarzburd wrote textbooks on mathematics for grades 5 and 6. At the same time, they devoted themselves in various ways to the work for and with gifted students. Schwarzburd participated actively in the organization and teaching of the first experimental secondary school class with specialization in mathematics and computer programming in 1959[46]. In turn, Kolmogorov, who was at that time a famous mathematician and a full member of the Academy of Sciences, was the leader of the Moscow School of Mathematics and Physics for gifted children created in 1963 and managed to teach at school three days each week[47].

Kolmogorov and Schwarzburd both had a clear concept of a "mathematically gifted child". Kolmogorov describes mathematical ability as a combination of an "algorithmic ability" (the capacity to find suitable methods of calculation in non-standard situations), "geometric intuition" (imagination) and "maturity of logical reasoning" (profound understanding of the principle of mathematical induction)[48].

Schwarzburd listed several characteristics of mathematically gifted children, some of them related to the abilities in a general sense of this word, others to skills or to personal qualities. Some of them are mathematically specific, others are more general: spatial vision, abstract thinking, modelling, deductive thinking, analytic thinking, the capacity of applying abstract knowledge in concrete situations, critical thinking, questioning, communication, and perseverance in problem solving[49].

Konstantinov, as energetic organizer of various mathematical competitions (Olympiads and tournaments)[50], brought the element of competition to the concept of education of mathematically gifted children. While he acknowledged that some people are not made to compete, he insisted however on the importance of mathematics competition for

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45 A. N. Kolmogorov, S. I. Schwarzburd and N. N. Konstantinov.

46 Vogeli (1968, p. 11) mentions the role of Schwarzburd in developing of the syllabi and teaching materials (with V.G. Ashkinaze) for the 9th Grade class in school 425 of Moscow.

47 Vogeli (1968, p. 49 refers here to the "Statement to Moscow Mathematical Society" by A. N. Kolmogorov (March, 1964).

48 Krutetskii (1968, pp. 70–71), with a reference to Kolmogorov (1959).

49 See Krutetskii (1968); the latter refers to Shvartsburd (1964).

50 See, for example, the information about the "Tournaments of Towns" posted at http://www.amt.canberra.edu.au/imtot.html

the education of mathematical talents. In his opinion, the Olympiads can help building energetic personality ready to overcome obstacles (Konstantinov 2004).

All three mathematicians represented different epochs, groups of educators, and pedagogical views on giftedness, yet all of them share one common feature: they sacrificed their entire life to the children with special talent in mathematics. It is highly pertaining to the present study to notice that they worked with children not because it was their duty or they were paid for it. As a hypothesis, we would like to suggest that the determination in their search for talents has been linked not only to their personalities but also to the shared representations concerning the gifted children and mathematics education.

CONCLUSIONS

We would like to reiterate that in this paper we do not study the mathematical giftedness as a phenomenon independent of the cultural setting; instead, we study the images and representations related to this phenomenon. These images appear important since they provided, in each given context, a conceptual language for dealing with the phenomenon of early (mathematical) giftedness and suggested the ways it can be dealt with. In this short paper we tired to address the following questions:

(1) how were constructed the socially adopted images of giftedness?

(2) how did these images influence the ways in which the (official and non-official) educational institutions dealt with the gifted children? Our goal was not to answer these questions (we hope to do so in future publications) but rather to develop an approach permitting dealing with the social and cultural constructions related to the *gifted children*.

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