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Activity of a Gifted Student Who Found Linear Algebraic Solution of Blackout Puzzle[1]

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The purpose of this paper is to introduce an activity of student who found purely linear algebraic solution of the Blackout puzzle. It shows how we can help and work with gifted students. It deals with algorithm, mathematical modeling, optimal solution and software.

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ZDM Classification: D40, H60, U70
MSC2000 Classification: 15A03, 97A90, 97U70

INTRODUCTION

In our real life problems, gifted students see some aspects that others don't see. Blackout game, which was introduced in the official homepage of popular movie 'A Beautiful Mind', is a one-person strategy game that has recently gained popularity as a

[1] This paper will be presented at the Ninth International Seminar of Mathematics Education on Creativity Development, held at Daejeon, Korea, October 9, 2004. A part (the mathematical proof) of this paper was presented as Lee, Park, Yang & Kim (2004).

diversion on handheld computing devices. An animated Macromedia Flash version of the puzzle can be found from the official website for the 2001 movie 'A Beautiful Mind'. We will show what was the question and answer that was made by student. We will introduce student's answer which is very elementary and intuitive. We also made a software based on his algorithm. This process only used the basic knowledge of linear algebra and can be extended to the full size *go* board problem and teach how we work with gifted students.

2. BACKGROUND OF BLACKOUT PUZZLE

In my recent linear algebra class we were talking about the movie 'A Beautiful Mind', starring Russell Crowe as Nobel Laureate John F. Nash, Jr. (2001), where Nash was playing Go game with his friend. Some of my students told me that they have played 'the Blackout puzzle' from the Korean official website[2] of the movie.

Figure 1. Blackout puzzle

One of my students asked me "Can we find an optimal solution for the game?" and further "Is there any possibility that we can not win the game if the given setting is fixed?" After a couple of days, one of my young student came to me with his idea. He and I met a couple of times personally and made a Mathematical model of the

[2] JAVA program by CJ Entertainment Inc., Movie "The Beautiful Mind, Blackout" puzzle, `http://www.cjent.co.kr/beautifulmind`

game, and he brought me the right answer. What he found was that we can always win the game. The model is fully based on the basic knowledge of linear algebra. We made a search of the game at that time but we did not find any good reference, so we did it in our own way[3] . At ICME-10, Professor Ole Björkqvist, who is an IPC member of ICMI, told us there was a workshop on the "Blackout Puzzle" in aspect of game and our algebraic method was not known before.

2.1. Introduction of the Blackout Puzzle. Blackout is a one-person strategy game that has recently gained popularity as a diversion on handheld computing devices. An animated *Macromedia Flash* version of the puzzle can be found from the official website for the 2001 movie 'A Beautiful Mind.'

2.2. How To Play. The Blackout board is a grid of any size. Each square takes on one of two colors. (The diagram above used blue and red.) The player takes a turn by choosing any square. The selected square and all squares that share an edge with it change their colors.

The object of the game is to get all squares on the grid (tile) to be the same color (Black or White).

When you click on a tile the highlighted tile icons will change or "flip" from their current state to the opposite state. Remember, the goal is to change all of the tile icons to black (or white).

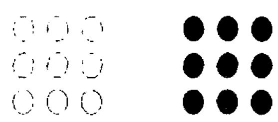


Figure 2. The End of the Game

2.3. How To Solve Any 3×3 Game. The diagram below illustrates the shortest sequence of moves for resolving possible scenarios on a 3×3 board.

[3] Later we found the following web site
`http://home.sc.rr.com/jacobsfam/jared/blackout.html`

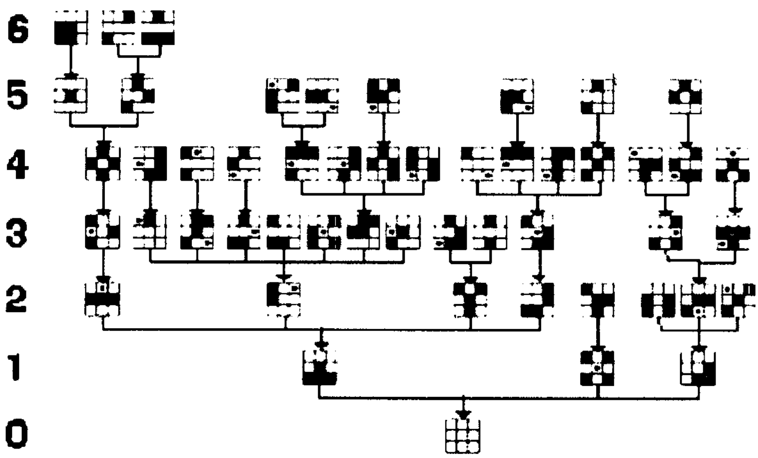


Figure 3. Understanding the Diagram

3. MAIN QUESTIONS

Q 1. "Is there any possibility that we can not win the game if the given setting is fixed?"

Q 2. "Can we always find an optimal solution for the game?"

Q 3. "Can we make a program to give us an optimal solution?"

4. OUR SOLUTION OF THE BLACKOUT PUZZLE

There are $2 \times 2 \times \cdots \times 2 = 2^9 = 512$ patterns of 3×3 blackout grid. Among these 512 patterns, there are $9 \times 2 = 18$ patterns such that we can win the game with only one more click as following. (Twice of the following basic 9 patterns as we can change all initial colors.)

We checked several examples and had enough trial and error to convince us to answer the first question with any given initial condition.

EXAMPLE 1. Assume the following initial condition
The following 3 clicks make it all white.

Our approach to find a winning strategy was to recognize these 18 patterns in Figure 4.

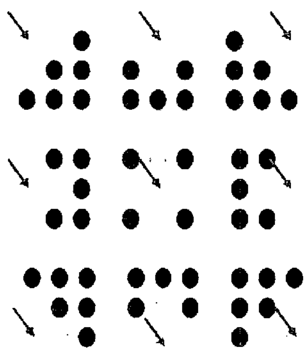


Figure 4. Basic 9 Patterns (Times 2)

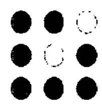


Figure 5. the following initial condition

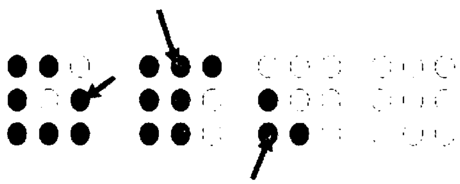
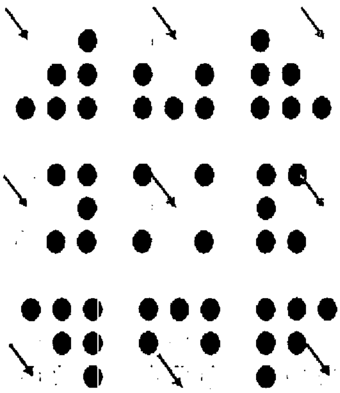


Figure 6. Three clicks

Now we try to make a **Mathematical Model of this game**. Only actions that we can perform are 9 clicks because we only have 9 stones on the board. We assume "the white stone $\equiv 1$ and black stone $\equiv 0$". Then we can classify effects of each action as an addition of one vector (or 3×3 matrix). Any series of our actions results a linear combination of these vectors. Then we use modular 2 arithmetic to make the zero vector or all 1's vector (or matrix, resp.) to finish the game.

So we now have the above 9 vectors (in fact, twice of them) to consider which will end the game with just one more click.

EXAMPLE 2. Suppose we have 5 black (or blue) stones and 4 white(or red) stones in the board as below.



$$\begin{bmatrix}1&1&0\\1&0&0\\0&0&0\end{bmatrix},\quad \begin{bmatrix}1&1&1\\0&1&0\\0&0&0\end{bmatrix},\quad \begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix},$$

$$\begin{bmatrix}1&0&0\\1&1&0\\1&0&0\end{bmatrix},\quad \begin{bmatrix}0&1&0\\1&1&1\\0&1&0\end{bmatrix},\quad \begin{bmatrix}0&0&1\\0&1&1\\0&0&1\end{bmatrix},$$

$$\begin{bmatrix}0&0&0\\1&0&0\\1&1&0\end{bmatrix},\quad \begin{bmatrix}0&0&0\\0&1&0\\1&1&1\end{bmatrix},\quad \begin{bmatrix}0&0&0\\0&0&1\\0&1&1\end{bmatrix}.$$

Figure 7. Nine clicks

Then the given condition can be written as the following matrix

$$B = \begin{bmatrix}0&0&1\\0&0&1\\0&1&1\end{bmatrix}.$$

And we now choose some of 9 positions to take action on it. This can be represented by

$$a\begin{bmatrix}1&1&0\\1&0&0\\0&0&0\end{bmatrix} + b\begin{bmatrix}1&1&1\\0&1&0\\0&0&0\end{bmatrix} + c\begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix} + d\begin{bmatrix}1&0&0\\1&1&0\\1&0&0\end{bmatrix} + e\begin{bmatrix}0&1&0\\1&1&1\\0&1&0\end{bmatrix}$$
$$+ f\begin{bmatrix}0&0&1\\0&1&1\\0&0&1\end{bmatrix} + g\begin{bmatrix}0&0&0\\1&0&0\\1&1&0\end{bmatrix} + h\begin{bmatrix}0&0&0\\0&1&0\\1&1&1\end{bmatrix} + i\begin{bmatrix}0&0&0\\0&0&1\\0&1&1\end{bmatrix} = C$$

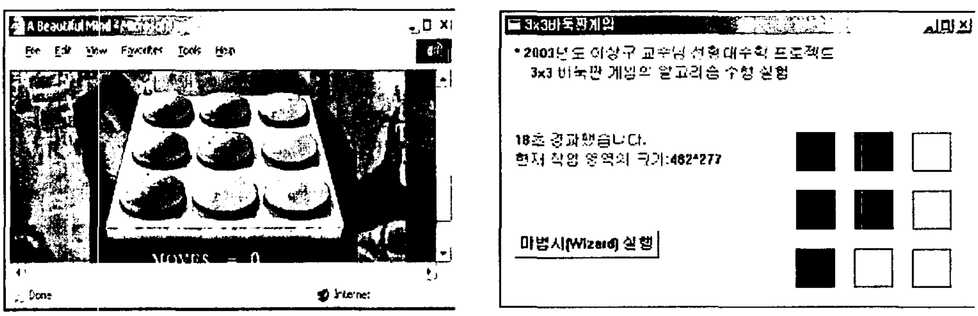


Figure 8. Five blacks and four whites

So, our problem is to find some a, b, c, d, e, f, g, h and i such that

$$B + C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

If

$$a \underset{(1,1)}{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} + b \underset{(1,2)}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}} + c \underset{(1,3)}{\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}} + d \underset{(2,1)}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}} + e \underset{(2,2)}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$
$$+ f \underset{(2,3)}{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}} + g \underset{(3,1)}{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}} + h \underset{(3,2)}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}} + i \underset{(3,3)}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}} + \underset{\text{Initial}}{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$$
$$= \underset{\text{Goal}}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}.$$

then

$$a \underset{(1,1)}{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} + b \underset{(1,2)}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}} + c \underset{(1,3)}{\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}} + d \underset{(2,1)}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}} + e \underset{(2,2)}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$
$$+ f \underset{(2,3)}{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}} + g \underset{(3,1)}{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}} + h \underset{(3,2)}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}} + i \underset{(3,3)}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$$
$$= \underset{\text{Goal}}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} - \underset{\text{Initial}}{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$$
$$= -B \ (\text{or} \quad J - B)$$

We can use any computational tool to obtain. Now we do mod 2 arithmetic to get an answer.

$$I \text{ and } \mathbf{x}' \equiv \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T \quad \text{or} \quad \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (\text{mod } 2).$$

We only need 0 and 1 because clicking $2n+1$ times of one stone is same as clicking once, and $2n$ clicks of one stone is same as doing nothing. So, the answer is

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

This shows that if we click on positions $(1,1),(1,3),(2,1),(3,2)$, we will get all white stones on the board with only 4 clicks. In the following Figure 9, the command "(Wizard)" tells us "1 3 4 8" that indicates which 4 stones we have to click to win. The number "4" shows we won with 4 clicks(MOVE).

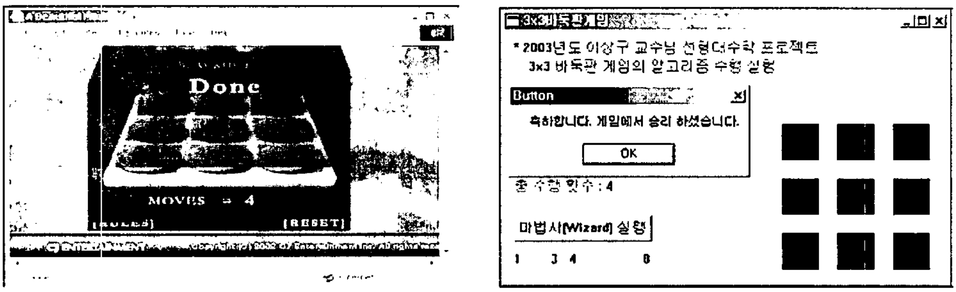


Figure 9. Our Program Down

We can run the program from

`http://matrix.skku.ac.kr/sglee/blackout_win.exe`

This works always. Why does this happen?

So our next question is

Q 2. "Can we always find an optimal solution for the game?"

PROOF. From the 9 matrices

$$\begin{bmatrix}1&1&0\\1&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}1&1&1\\0&1&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix},$$
$$\begin{bmatrix}1&0&0\\1&1&0\\1&0&0\end{bmatrix}, \begin{bmatrix}0&1&0\\1&1&1\\0&1&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&1&1\\0&0&1\end{bmatrix},$$
$$\begin{bmatrix}0&0&0\\1&0&0\\1&1&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&0\\1&1&1\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&1\\0&1&1\end{bmatrix},$$

we can make nine 9×1 column vectors and make a symmetric matrix (because of the symmetry in the board) whose columns are these vectors as follows

$$\begin{bmatrix}1&1&0&1&0&0&0&0&0\\1&1&1&0&1&0&0&0&0\\0&1&1&0&0&1&0&0&0\\1&0&0&1&1&0&1&0&0\\0&1&0&1&1&1&0&1&0\\0&0&1&0&1&1&0&0&1\\0&0&0&1&0&0&1&1&0\\0&0&0&0&1&0&1&1&1\\0&0&0&0&0&1&0&1&1\end{bmatrix}.$$

Then we have a linear system of equations to find $\mathbf{x}$.

$$A\mathbf{x} = -\mathbf{b} \Rightarrow A\begin{bmatrix}a\\b\\c\\d\\e\\f\\g\\h\\i\end{bmatrix} = -\mathbf{b} \text{ (or } \mathbf{j}-\mathbf{b}).$$

where $\mathbf{b}$ is a given (condition) matrix and $\mathbf{j}$ is a vector of all 1. Then $\text{RREF}(A) = I_9$ and $\text{rank}\, A = 9$. So the columns (rows) are linearly independent, and the system has a unique solution. (Furthermore all this process can be done in Modular 2 arithmetic.) For the example considered earlier we have

$$A\mathbf{x} = -\mathbf{b} \Rightarrow A\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \text{where } A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

Let

$$a\underbrace{\begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{bmatrix}}_{\text{given}} + a\underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{(1,1)} + b\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{(1,2)} + c\underbrace{\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{(1,3)} + d\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{(2,1)}$$

$$+e\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{(2,2)} + f\underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}_{(2,3)} + g\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}}_{(3,1)} + h\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}}_{(3,2)} + i\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}}_{(3,3)}$$

$$= \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{Goal 1}} + \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{\text{Goal 2}}.$$

We want to show that such a, b, c, d, e, f, g, h, and i exist for any B. Remember that we can represent 3×3 matrices by vectors and let

$$a\underbrace{\begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{bmatrix}}_{\text{given}} + a\underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{(1,1)} + b\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{(1,2)} + c\underbrace{\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{(1,3)} + d\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{(2,1)}$$

$$+e\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{(2,2)} + f\underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}_{(2,3)} + g\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}}_{(3,1)} + h\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}}_{(3,2)} + i\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}}_{(3,3)}$$

$$\equiv \mathbf{0}_{9\times 1} \ (\text{or } \mathbf{j})$$

Recall that $A\mathbf{x} = -\mathbf{b}$ is consistent if and only if $\operatorname{rank}(A) = \operatorname{rank}[A| - \mathbf{b}]$. Let

$$\mathbf{0} = [\,0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\,]^T, \quad \mathbf{j} = [\,1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\,]^T.$$

Case 1. If $\mathbf{b}$ is a zero matrix, it is done because $a, b, c, d, e, , g, h, i \equiv 0 \pmod 2$ give a solution.

Case 2. If $\mathbf{b}$ is not a zero matrix, then it is clear that

$$\text{rank}(A) = \text{rank}[A| - \mathbf{b}]$$

and

$$\text{rank}(A) = \text{rank}[A|\mathbf{j} - \mathbf{b}].$$

In any case, $A\mathbf{x} = -\mathbf{b}$ (or $A\mathbf{x} = \mathbf{j} - \mathbf{b}$) is consistent.

So, for given $A, \mathbf{x}, \mathbf{0}, \mathbf{j}$, and $\mathbf{b}$,

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix},$$

$$\mathbf{0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \mathbf{j} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T,$$

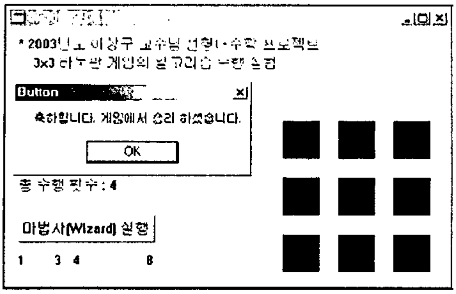
where the vector $\mathbf{b} = [b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9]^T$ comes from the given $(0,1)$-matrix $\mathbf{b}$, the system $A\mathbf{x} + \mathbf{b} = \mathbf{0}$ (or $A\mathbf{x} + \mathbf{b} = \mathbf{j}$) has a solution. Then $\mathbf{x}$ is obtained as

$$\mathbf{x} = 0 - A^{-1}\mathbf{b} \text{ (or } \mathbf{x} = \mathbf{j} - A^{-1}\mathbf{b})$$

where

$$A^{-1} = \frac{1}{7} \begin{bmatrix} -1 & 4 & -1 & 4 & -2 & -3 & -1 & -3 & 6 \\ 4 & -2 & 4 & -2 & 1 & -2 & -3 & 5 & -3 \\ -1 & 4 & -1 & -3 & -2 & 4 & 6 & -3 & -1 \\ 4 & -2 & -3 & -2 & 1 & 5 & 4 & -2 & -3 \\ -2 & 1 & -2 & 1 & 3 & 1 & -2 & 1 & -2 \\ -3 & -2 & 4 & 5 & 1 & -2 & -3 & -2 & 4 \\ -1 & -3 & 6 & 4 & -2 & -3 & -1 & 4 & -1 \\ -3 & 5 & -3 & -2 & 1 & -2 & 4 & -2 & 4 \\ 6 & -3 & -1 & -3 & -2 & 4 & -1 & 4 & -1 \end{bmatrix}$$

Let $\mathbf{x}' \equiv \mathbf{x} \mod 2$. Then $\mathbf{x}'$ is a real optimal winning strategy vector (matrix) which can be deduced from $\mathbf{x}$. $\square$



Now entries of $\mathbf{x}'$ are all 0 or 1 as is in real game situation and we can always find a $(0,1)$ matrix as a real optimal winning strategy vector(matrix). With this idea, one of my student made a computer program[4] in C^{++} based on this algorithm which tells us an optimal strategy to win. This software also verified our conjecture is right, and showed the proof was valid.

5. CONCLUSIONS AND FUTURE WORK

We had thought we only teach our students, but we now believe we can work better with creative students with this experience. Also this experience with a gifted student and leading teacher gave a stimulating mathematical activity for both. This process can be adapted to resolve other real world problems with basic mathematical knowledge.

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[4] We can download it and run from `http://matrix.skku.ac.kr/sglee/blackout_win.exe`