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Teaching Mathematics Based on Children's Cognition: Introduction to Cognitively Guided Instruction in U.S.

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Cognitively Guided Instruction (CGI) is one of the most successful professional development programs for elementary mathematics teachers in US. This article introduces its theoretical background, research-based framework of addition and subtraction work, and how the program has been disseminated. Carpenter and Fennema started CGI aiming to develop a professional development program that focused on research knowledge of children's thinking. Their goal was to bring a significant change in teaching by helping teachers understand how children think mathematically. This 3-year NSF funded project grew to be 11-year long, and a number of publications have reported consistent successful learning and teaching by CGI students and teachers compared to counterparts throughout US. CGI's success by focusing on improving teachers' knowledge of children's thinking offers possible opportunities for teacher educators to re-conceptualize teacher education in Korea.

In the last two decades, significant achievements of research on teaching and learning of mathematics can be summarized in two findings: one is learning of children's construction of mathematical knowledge, and the other is understanding of complexity of teaching processes.

A number of research programs in U.S. have initiated a reform movement of teaching and learning of mathematics based on the research findings of children's construction of knowledge and teachers' instructional activities (e.g. the Conceptually Based Instruction - Hiebert & Wearne, 1992; Hiebert & Wearne, 1996; the Cognitively Guided Instruction - Carpenter & Fennema, 1992; Carpenter, Fennema, & Franke, 1996; Carpenter, et al., 1999; the Problem Centered Mathematics Project - Cobb et al., 1991; Wood, Cobb, & Yackel, 1991; the Supporting Ten Structured Thinking Project - Fuson & Burghardt, 2003; the Summer Math for Teachers Project - Schifter & Fosnot, 1993; Schifter & Simon, 1992). The goal of this paper is to introduce the theoretical background and knowledge structure of one of the widely implemented successful programs focusing on elementary mathematics in U.S., *Cognitively Guided Instruction* (CGI).

Ⅰ. THEORETICAL BACKGROUND

1. Research on Children's Learning of Whole Number Operations

Recent research on children's mathematical

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learning has provided a substantial body of research on the development of children's informal or intuitive concepts of arithmetic. It shows that children both in and out of school can invent a wide range of solution strategies for arithmetic problems without direct instruction on specific strategies to follow (Carpenter & Fennema, 1992; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Carpenter & Moser, 1984; Carraher, Carraher, & Schliemann, 1987; Cobb & Wheatley, 1988; Hiebert & Wearne, 1996; Labinowicz, 1985; Saxe, 1988). Children's invented strategies for adding, subtracting, multiplying, and dividing single-digit numbers have shown remarkably consistent and coherent portraits of the development of children's understanding of whole number operations (Carpenter, 1985; Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Fuson, 1992; Gutstein & Romberg, 1995; Verschaffel & DeCorte, 1993). Based on the findings of children's invented strategies for arithmetic problems, several research projects have strived to facilitate children's informal knowledge of mathematics as a basis for developing a module for mathematics instruction for understanding.

The research findings of children's informal knowledge provided a basis for shared assumption that students construct knowledge rather than simply absorb rules or procedures that they were taught (Cobb, 1994; Davis, Maher, & Noddings, 1990). In addition, large-scaled assessments revealed American children's lack of conceptual understanding of fundamental mathematics (e.g. International Association for the Evaluation of Educational Achievement, 1987; Mullis et al., 2000; Silver & Kenny, 2000). Based on these findings, the mathematics education community in U.S. recommended curricular and instructional changes focusing on children's mathematical understanding (National Council of Teachers of Mathematics, 1989, 2000; National Research Council, 1989).

The research programs are based on the assumption about children's construction of their own knowledge, but they have taken different approaches to use the information about children's knowledge construction in supporting teachers' instructional activities. In the CGI program, Carpenter and his colleagues have focused more directly on helping teachers gain knowledge about children's thinking in well defined content domains in primary grades. CGI provides a knowledge framework that teachers learn to understand their students' thinking with, so that they construct their own instructional practices based on students' thinking. Teachers spend significant time to analyze problem types, children's invented strategies, and investigate how to use the knowledge of students' thinking for instructional decision making processes.

2. Research on Teachers' Instructional Change

Teachers are key figures in changing the ways in which mathematics is taught and learned (NCTM, 1991). Recent research on professional development argued that the kind of instruction envisioned in current reform recommendations require professional development more than showing teachers how a good teaching would look like. By the same token, understanding of complicated processes of teaching cannot be done only by observing the instructional activities that other teachers engage in.

Researchers share a consensus that teachers' knowledge is one of the determining factors of the actions taken in classrooms and the analysis of teachers' knowledge is one of the keys to understand the process of teaching and to bring fundamental changes in instruction (Carter, 1990; Fennema & Franke, 1992; Festermacher, 1994; Peterson, 1988). In other words, teachers need a structured support to engage in intellectual processes of reinventing their practices so that they can reexamine their epistemological perspectives and conceptions of teaching and learning (Fullan, 1991; Lieberman & Miller, 1990; Little, 1993; Lord, 1994; Sparks & Loucks-Horsley, 1990).

Initial attempts of researchers to relate teachers' knowledge of mathematics to children's learning focused on limitations of teachers' mathematical content knowledge (Ball, 1990; Even, 1993; Leinhardt & Smith, 1985). The findings of these studies, however, were limited that they failed to find a strong relationship between the global measures of teachers' knowledge to instruction in teachers' classrooms or students' achievement. After Shulman (1986, 1987) proposed a framework for analyzing teacher's knowledge, many studies on teaching found that teachers' pedagogical content knowledge is one of the influential factors that bring the changes in their teaching (Ball, 2000; Carpenter, Fennema, Peterson, & Carey, 1988; Sowder et al., 1998). They argued that teachers with rich pedagogical content knowledge provide a higher quality of instruction because the knowledge allows them to understand students' mathematical thinking, ways to respond to them, what problem to pose next, and ways to manage mathematical discussions.

Several research projects on professional development engaged teachers in building their pedagogical content knowledge of children's mathematical thinking (Schifter, 1998; Lehrer & Schauble, 1998). CGI has been one of the leading research programs that focused on students' arithmetical thinking to provide a framework for teachers to engage in inquiry that can serve as a basis for their ongoing learning (Fennema, et al., 1996; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Knapp & Peterson, 1995).

II. COGNITIVELY GUIDED INSTRUCTION

The research project, CGI, was created by Carpenter and Fennema at University of Wisconsin - Madison in 1985 and the funding continued until 1996, sponsored by National Science Foundation. Fennema is a renowned scholar in teacher education and gender equity issues in mathematics education, and Carpenter is a renowned scholar in children's mathematical thinking. In an interview, Fennema described the origin of the project as an attempt to integrate research on teaching and learning (Foster, 2000). Fennema said that many researchers agreed with "the importance of research on learning" but "we have had decades and decades of research on learning that has not made much of an impact on what goes on in the schools." This consensus led them to initiate the research project focusing on teaching along with learning, aiming to bring a significant change in the classroom by helping teachers understand children's mathematical thinking.

The researchers of CGI share a perspective that significant changes in practice depend on teachers' fundamentally altering their epistemological perspectives so that teachers appreciate that students construct knowledge (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Schifter & Simon, 1992). Their theory of teachers' growth aligns with the thesis of children's learning of mathematics. Teachers have intuitive knowledge about students' mathematical understanding, but this knowledge is not well structured and their instructional decisions are generally not based on the knowledge. Using CGI, Carpenter and Fennema have provided teachers with research-based knowledge of children's mathematical understanding so that teachers can make their instructional decisions based on their emerging knowledge of their students' understanding. Because no one else can have immediate knowledge about the students, the researchers recognize that teachers should make their own instructional decisions and research should support them to build and reconstruct their pedagogical content knowledge (Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999).

The CGI professional development program consists of a series of activities that provide the analyses of types of arithmetic problems, children's strategies for different types of problems, and a developmental trajectory of children's understanding of arithmetic. The main focus of CGI has been on whole number operations involving single- and multi-digit numbers in primary grades (K-3). More recently, the research has expanded to include analyses of the development of children's understanding of multi-digit multiplication and division (Ambrose, Baek, & Carpenter, 2003; Baek, 1998, 2002), their knowledge of fractions (Empson, 1995, 1999), the knowledge of geometry and measurement (Jacobson, & Lehrer, 2000; Lehrer & Chazan, 1998), and children's early algebraic reasoning (Carpenter, Franke, & Levi, 2001).

In the following, the analytical structure of CGI in the domain of addition and subtraction for primary grades is introduced as an example. The researchers of CGI define the operations in terms of semantic structures of problems and children's intuitive solution strategies. They provide a framework to help teachers understand problem types, strategy types, and a developmental trajectory of the operations and numbers. The following is an attempt to introduce the main structure of CGI as it appeared in recent publications by Carpenter and his colleagues (see publications in references and suggested readings).

1. Addition and Subtraction

For addition and subtraction problems, four basic classes of problems are identified: *Join, Separate, Part-Part-Whole,* and *Compare.* Join and Separate problems involve actions and Part-Part-Whole and Compare problems involve relationships. Within each class, several distinct types of problems are identified depending upon which quantity is the unknown. <TableⅡ-1> provides examples of each problem type.

Students use a variety of solution strategies to solve different addition and subtraction problems. Carpenter and his colleagues (1999) described children's different kinds of solution strategies using episodes of a first grader, Rachel (Video clips of Rachel available on the CD-Rom included in Carpenter et al., 1999). Rachel uses three different strategies for the following three problems:

TJ had 13 chocolate chip cookies. At lunch he ate 5 of them. How many cookies did TJ have left?
Rachel puts out 13 counters, removes 5 of them, and counts the remaining counters.
Janelle has 7 trolls in her collection. How many more trolls does she have to buy to have 11 trolls?
Rachel first puts out a set of 7 counters and adds counters until there is a total of 11. She then counts the counters she added to the initial set to find the answer.
Willy has 12 crayons. Lucy has 7 crayons. How many more crayons does Willy have than Lucy?
Rachel makes two sets of counters, one containing 12 counters and the other containing 7. She lines up the two sets in rows so that the set of 7 matches 7 counters in the set of 12 and counts the unmatched counter in the row of 12.

Rachel's solutions to the three problems provide a good example of how children perceive different types of addition and subtraction problems, which adults usually solve with subtraction. It illustrates that the distinctions among CGI problem types are reflected in children's ways of solving the problems.

Although Rachel's strategies for the three problems are different, there is a common thread in the strategies. She used counters to *directly model* the action or relationship described in each problem. Carpenter (1985) found that many young children use direct modeling strategies for a

<TableⅡ-1> Classification of Addition and Subtraction Word Problems (Carpenter et al., 1999, p. 12)

Problem Type			
Join	(Result Unknown) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?	(Change Unknown) Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?	(Start Unknown) Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	(Result Unknown) Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?	(Change Unknown) Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	(Start Unknown) Connie had some marbles. She gave 5 to Juan. Now she has 8 left. How many marbles did Connie have to start with?
Part-Part-Whole	(Whole Unknown) Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?	(Part Unknown) Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?	
Compare	(Difference Unknown) Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	(Compare Quantity Unknown) Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have?	(Referent Unknown) Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

variety of problems and over time they are replaced by more abstract and efficient strategies.

Counting strategy is an abstraction of the direct modeling strategy. Children recognize that it is not necessary to physically represent and count both sets in a problem and abbreviate the construction of two sets to only one. Carpenter et al. (1999) used an example of Vanessa, a first grader, solving a Join Result Unknown problem to illustrate the counting strategy.

> *James had 5 clay animals. During art he made 9 more clay animals. How many clay animals does James have now?*
> Vanessa counts "Nine [pause], 10, 11, 12, 13, 14. He has 14." With each count, Vanessa extends a finger. When she has five fingers up, she stops counting.

Research shows that children learn number facts in and out of the school even without specific drills of memorizing number facts (for reviews Fuson, 1992). Children learn certain number combinations before others, and often use a small set of memorized facts to derive solutions for problems involving other numbers.

American children are known to learn doubles (e.g., 3+3, 4+4) before others and sums of ten (e.g., 3+7, 6+4) are often learned relatively early. Carpenter et al. (1999) illustrates children's use of *Derived Facts* using the following example.

> *6 frogs were sitting on lily pads. 8 more frogs joined them. How many frogs were there then?*
> Ruby answers 14 almost immediately, explaining "6 and 6 is 12, and 2 more is 14."

Professional development programs of CGI facilitate teachers' understanding of problems types, children's construction of different strategies, and how they develop over time. Teachers are encouraged to provide a wide range of problems, instead of focusing only on Join or Separate Result Unknown problems. In professional development programs, teachers discuss how to use various problem types to facilitate children to make sense of situations and to construct their own strategies. They work to develop instructional activities based on children's development of numbers and strategies. However, they are not trained in specific techniques or curriculum to follow.

Beside addition and subtraction with single-digit numbers, CGI focuses on mathematical concepts for primary grades, providing teachers with structured knowledge of semantic differences of problems and underlying conceptual issues in areas of place value, and multi-digit addition and subtraction, multiplication and division, and geometry. Teachers learn about children's invented strategies for those problems, and explore instructional practices that foster learning environment where children discuss similarities and differences among their strategies, so that they could learn from each other. More recently CGI has evolved to include research on teaching and learning more complex domains of mathematics (more information is available in the publications listed in the references and suggested readings).

2. Dissemination of CGI

As discussed earlier, CGI was originated in Madison, Wisconsin and a number of teachers in Madison and surrounding areas participated in professional development workshops offered by researchers and experienced teachers of CGI. As the project developed, CGI produced a number of publications about teacher changes and students' achievements (Carpenter & Fennema, 1992; Carpenter, Fennema, & Franke, 1996, Carpenter et al., 1993; Carpenter et al., 1998; Fennema et al., 1996; Franke & Carpenter, Fennema, Ansell, & Behrend, 1998) and the demands for the workshops and training from other states as well as other countries have significantly grown. Because of the growing demands of the workshops, Carpenter and his colleagues published a CGI book with a supplementary CD for teachers (Carpenter et al., 1999) and a book for workshop leaders (Fennema et al., 1999) along with 7 videotapes containing individual children solving problems and classroom episodes (information available in Fennema, et al., 1999).

As an effort to implement CGI for systematic changes at elementary schools, Comprehensive Center in Madison, Wisconsin sponsored by U.S. Department of Education hosts annual CGI National Institutes for teachers and for leader teachers. CGI leaders throughout U.S. also meet and share research on children's thinking and professional development at biannual CGI conferences. National Science Foundation sponsored a large scale of CGI professional development in Phoenix, AZ as an Urban Systematic Initiative.

Among several professional development programs in U.S., CGI has been cited as one of the most exemplary programs at the elementary level based on several reasons (e.g. National Research Council, 2001). First of all, it is known for its strong research-based knowledge of children's mathematical thinking by Carpenter his colleagues (see research publications cited here and the suggested readings). CGI has also been recognized for its applicability for students in all ability levels and students with special needs (Hankes, 1996; Peterson, et al., 1991) and language barrier issues (Secada & Brendefur, 2000; Secada & Carey, 1990). Several research studies have documented that CGI students exceeds in developing mathematically rich conceptual understanding as well as number facts (e.g. Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). In addition, successful changes of CGI teachers in instruction, knowledge, and beliefs have been well documented (e.g. Fennema, et al., 1996; Franke, Carpenter, Fennema, Ansell, & Behrent, 1998; Knapp & Peterson, 1995).

Unlike many other professional development programs, CGI does not focus on dictating how teachers should teach. It is not like a typical training that outside experts come and lecture on math and/or exemplary teaching without effective follow-up. CGI assumes teachers as intellectual professionals who construct their knowledge and understanding. Its goal is to empower teachers by assisting them in their own knowledge construction of children's thinking, rather than prescribing them how to teach. It focuses on providing teachers access to structured knowledge of children's mathematical thinking, expects teachers to implement CGI in their ways based on their acquired knowledge of students' thinking, and work to develop their own instructional activities. A number of teachers report that this process made them truly professional and reflective on their instruction (Carpenter et al., 1999; Foster, 2000).

The applicability of CGI in Korea has not been tested. Its success in US can provide a model to consider for teacher education in Korea.

I believe that CGI's focus on teachers' knowledge of children's mathematical thinking offers great opportunities for Korean teacher educators and teachers to re-conceptualize teacher education. CGI's approach of focusing on teachers' knowledge could facilitate teachers to bring fundamental changes in classrooms and to provide students with eye opening experiences to learn what it means to make sense of mathematics.

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아동들의 인지를 바탕으로 한 수학 교수: 미국의 Cognitively Guided Instruction의 소개

백재민 (아리조나 주립 대학)

Cognitively Guided Instruction (CGI)는 미국 내의 초등 수학 교사들을 위한 교사 전문성 개발 프로그램 중의 가장 성공적인 프로그램의 하나이다. 이 논문은 CGI 프로그램의 이론적 배경과, 덧셈 뺄셈의 연구 중심 연수 내용, 그리고 이 프로그램의 확산 과정을 소개한다. Carpenter와 Fennema 교수가 시작한 이 프로그램은 아동들의 수학적 사고를 중심으로 한 교사 전문성 개발을 목적으로 했다. 그들의 목적은 아동들이 어떻게 수학적으로 사고하는지 관한 교사들의 이해를 도움으로써 수학 교수에 의미있는 변화를 가져오고자 했다. NSF 3년 지원 연구 프로젝트로 시작한 이 프로그램은 11년 동안의 연구로 확대되었고, 그동안 게재된 많은 연구 논문들은 미국 많은 지역의 CGI 학생들과 교사들의 지속적인 성공을 보고해 왔다. 교사들의 아동들의 수학적 사고에 관한 지식을 증진하는데 역점을 둔 CGI의 성공은 한국에서의 교사 연수를 재조명하는 계기를 제공한다.

* **Key words**: Teacher education(교사교육), Teacher knowledge(교사지식), Children's mathematical thinking(아동들의 수학적 사고), Cognition(인지), Elementary mathematics(초등수학).

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