한국공작기계학회논문집 Vol.13 No.5 2004. 10.
Transactions of the Korean Society of Machine Tool Engineers

Development of the Automated Calculation System for Air-Bearing Spindle

Y.A. Chernopyatov+, W.J. Chung++, K.S. Dolotov+++, D.S. Kim++++, C.M. Lee*
(논문접수일 2004. 1. 16, 심사완료일 2004. 7. 1)

공기 베어링 주축의 자동설계시스템 개발

Y.A. Chernopyatov+, 정원지++, K.S. Dolotov+++, 김대성++++, 이춘만*

Abstract

Recently the use of high-speed equipment in machine-tool industry has greatly increased, which requires the development of prognostics and prediction methods on the design stage. Conversion of the test/experiments stage from real to virtual reality will not only significantly reduce the design and manufacturing cost, but will also increase design quality. This paper shows how it is possible to develop the automated system for the design calculations of the air-bearing spindles. First, the general calculation method is introduced. It contains several steps, namely, geometry identification, pressure calculation, stiffness calculation, dynamics characteristics calculation. For geometry identification reducing spindle shaft to rings was proposed, which helps to automate the calculation process. For pressure calculation the Peshti method was implemented. For stiffness calculation the analysis was made, which shown the necessity of correct calculation step selection. Then the system of ordinary differential equations containing influence coefficients was evolved, which is used for trajectories calculation. The graphical representation of the calculation results shows the dynamic behavior of the spindle unit concerning various working conditions. Finally, this automated system is illustrated by an example of the air-bearing spindle calculation.

Key Words : Air-Bearing Spindle(공압베어링 주축), Dynamic Quality(동적 품질), Characteristics Prognostics(특성 조짐), CAE

1. Introduction

The optimal design of complex technical objects is impossible without the prognostics and prediction of their efficiency characteristics. The multi-criteria optimization methods used for this purpose are directly using

* 교신저자, 창원대학교 기계설계공학과 (cmlee@sarim.changwon.ac.kr)
주소: 641-773 경남 창원시 사림동 9번지
\+ 창원대학교 RRC 연구원
++ 창원대학교 기계설계공학과 교수
+++ Ph.D., TTM Dept., MSUT STANKIN, Moscow, Russ
++++ 창원대학교 기계설계공학과 대학원

the spindle output dynamic characteristics as a criterion. Thus we can conclude that the problem of prognostics and prediction of these spindle dynamic quality characteristics defines the quality of design.

All above-mentioned circumstances make the prognostics stages of air-bearing spindle working characteristics to be very important. A number of well-known researches investigated the prediction of machine tools and units characteristics, among them we can note Push[1~3], Pronikov[4], studied the general issues of machine tool design; Zverev, Levina[5], developed the methods of computer aided calculation of spindle units on ball bearings using the deterministic approach; Sokolov and Figathner, proposed the deterministic methods of calculation for various kind of spindle characteristics.

It could be seen, that the aspect of output properties and design parameters optimization plays a significant role. During this optimization a designer selects the best one amongst other compromising variants considering different contradictory requirements. In optimization tasks two main directions prevail: the research of exploitation capabilities of designed units considering lots of local criteria, and the determination of the feasible set of design variants.

Nowadays there are a lot of methods and algorithms of optimization. In most cases the problem of determining design parameters can be considered as a problem of non-linear programming: one criterion(considered to be most important from the designer's point of view) is selected for optimization, while constraints and bounds are applied to others. However, due to the objective reasons, it is almost impossible to prove the criteria constraints on the initial design stage. As the implication of this fact we can consider that the task of optimization is also unclear in this approach. As a result the efficiency of single-criteria optimization is quite low[6] and multi-criteria optimization could be suggested instead.

Summarizing all above-mentioned questions of spindle unit's quality, we can assume that the studying methods of prognostics and prediction of spindle characteristics during the design stage is a very important problem. Moreover, the specifics of air-bearings, namely the high speed and high precision of rotation strongly require the high accuracy of calculations.

In this paper we will show how it is possible to develop the automated system for the design calculations of the air-bearing spindles. First, the theoretical methods are implemented into the algorithms, suitable for computer programming. Then they are employed into the computer aided engineering(CAE) system. This CAE incorporates the static and dynamics parameter calculation together with the visualization means of results. Finally, this automated system is illustrated with an example of the air-bearing spindle calculation

2. Background

The quality prognostic of production is a quite difficult problem. It is connected with the variety and complexity of physical processes taking place in the spindle unit, and complexity of their interaction. Thus the modeling requires the deep analysis of these processes and revealation of most important characteristics from the exploitation quality point of view. However, despite of the exploitation modeling complexity, transition of the test/experiments stage from real to virtual reality will not only significantly reduce the design and manufacturing cost, but will also increase design quality.

Modern industry requires the power tools for designers to help them make research and engineering process more efficient and productive. Open architecture system based on prognostic complex of output spindle exploitation characteristics can highly improve the design process.

In this case the entire design process is performed under constant control of prognostics unit, which apply the feedback to the input parameters of the design process. At the same time these parameters are affected by the local feedbacks of component optimization by criteria, not directly connected with the quality properties(see Fig. 1). Using the information provided by prognostication unit, it is possible to use any design methods(including heuristics)[7].

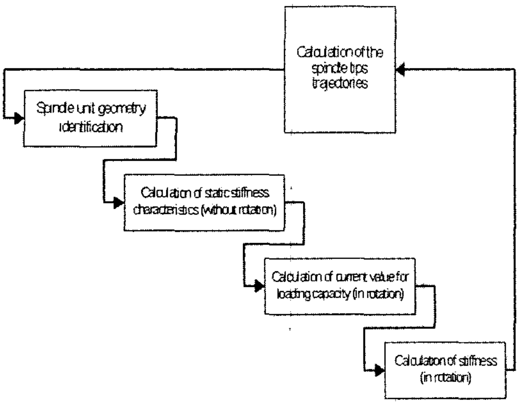


Fig. 1 Monitoring of step-by-step optimization of spindle unit design parameters

The most complex problem to be solved is the development of the mathematical model, adequately describing the spindle unit behavior during exploitation cycle. Processes in the lubricant layer can be considered as the most difficult to model. On the one hand, the lubricant is compressible, on the other hand it is flowing along the bearing. The model testing(physical effects bank) is possible only after their integration into the system "spindle-bearings" and comprehensive tests for its conformity to the real objects(by using the whole data of experiment).

A prognostic of dynamic quality of spindle unit in dynamic processes is performed based upon spindle tips trajectory. Also frequency characteristics are used[1]. Analysis of system "spindle-bearings" is based on the joined equations, which describe the bearings and spindle behavior.

Meanwhile a designer is limited in parameters set one can vary to affect the target function. At every stage of modeling problem it is necessary to find a number of design parameters, by varying that we can significantly change the main characteristics. However it could be assumed, that the optimal values of design parameters should be refined on the stage of spindle tips trajectories prediction and analysis.

The main task of trajectories prediction includes the

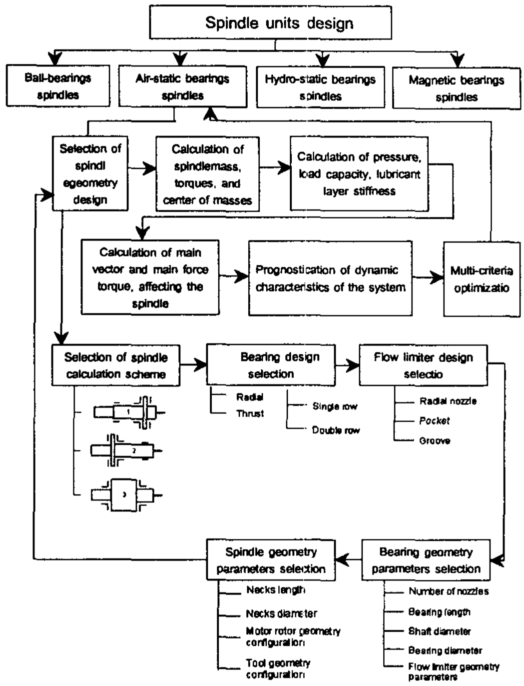


Fig. 2 The structure of the air-bearing spindle design problem(considering the stage of spindle geometry identification)

following stages(see Fig. 2):

1. Spindle geometry identification(this stands for determination of spindle and bearings design parameters)
2. Calculation of the current values of disturbing effects
3. Calculation of the current values of static component of pressure in the clearance
4. Determining the distribution law of static component of pressure in the clearance in axial and radial direction
5. Calculation of the static stiffness characteristics
6. Calculation of the current values of dynamic component of pressure in the clearance
7. Determining the common distribution law of pressure in the clearance
8. Calculation of current loading capacity value
9. Stiffness and influence coefficients calculation
10. Calculation of the center of masses position and angles of spindle axes

11. Calculation of the spindle tips trajectories

On the one hand, the steps 5, 8 and 9 can be treated as the independent problems of determining basic characteristics of spindle unit. They are the components of prognostics process. Thus based on the above mentioned steps, we will show how it is possible to implement all necessary mathematics components on CAE systems.

3. Calculation of geometry-related parameters

IIn our study we regard the spindle as a homogenous stepped shaft. Thus to make calculations of masses, torques, etc., we should input the spindle geometry design data. In general case we can separate the whole shaft into the simplest possible segments rings(with or without inner diameters). These segments will have their own masses and centers of masses(see Fig. 3)

Considering that the spindle is a body of revolution, and has the main axis of symmetry, its center of masses can be evaluated as the center of masses of the planar geometrical figure:

$$x_c = \frac{\sum_{i=1}^{n} x_i (S_{outer_i} - S_{inner_i})}{\sum_{i=1}^{n} (S_{outer_i} - S_{inner_i})} \tag{1}$$

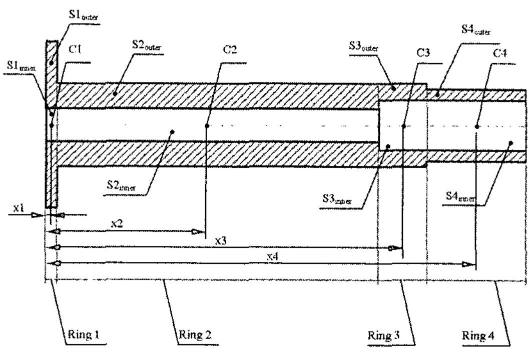


Fig. 3 Example of calculation scheme for geometry-related parameters calculation

where S_{outer_i} is the square of i-th ring, S_{outer_i} is the square of i-th ring's inner rectangle(cross-section of the hole), x_i is the distance to the center of masses of the i-th ring. As far as we know the density of shaft material ρ and we can calculate the volume V from the given geometry dimensions, we can calculate the mass m using formula $m=V\rho$. For each i-th ring the mass can be evaluated as $m_i = \pi\left(R_i^2 - r_i^2\right)H_i\rho$, where H_i is the ring width, R is the outer diameter and r is the inner diameter. The steel density is ρ=7874kg/m^3, however spindle is not made from steel completely. For example, some parts of rotor can contain copper with ρ_r=8960 kg/m^3(in the cage area) with ratio to steel 0.5:0.5. For this case it is necessary to use the average value of density.

Calculation of other necessary parameters, such as torques, is also straightforward and could be easily implemented in software.

The moment of inertia in axial direction is:

$$I_z = \sum_{i=1}^{n} \frac{m_i (R_i^2 - r_i^2)}{2} \tag{2}$$

The moment of inertia in radial direction is:

$$I_x = I_y = \left(\frac{R_1^2 - r_1^2}{4} + \frac{h_1^2}{12} + z_{c_1}^2\right) m_1 + \dots + \left(\frac{R_i^2 - r_i^2}{4} + \frac{h_i^2}{12} + z_{ci}^2\right) m_i \tag{3}$$

where z_{c_1}, z_{ci} are distances from spindle center of masses C to centers of masses of each ring, R_i, r_i, h_i, m_i are outer radius, inner radius, ring width, ring weight correspondingly.

4. Pressure calculation

For pressure calculation we propose to use the Peshti method[8], which is very straightforward and thus easy to be implemented on CAE software. The brief description of theory background is necessary here.

The air at some pressure p_s comes into air-bearing spindle, located in atmosphere(environment) with pressure p_a. Air-static bearing of any type can be represented as a sum of element-model, containing one(at a single row pressure charging) or two(at a double row pressure charging) nozzles with constant clearance. Fig. 4 shows such an element of radial bearing.

A nozzle is located in the center of this element. If e=0 the flow-over is absent, and air is flowing in parallel currents. The envelope curve of pressure distribution diagram is a parabola; however it can be reduced to the linear distribution law, though it will add some minor errors. Thus the air flow could be considered as laminar

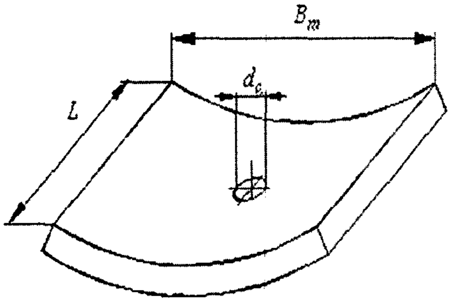


Fig. 4 Geometry parameters of the bearing segment

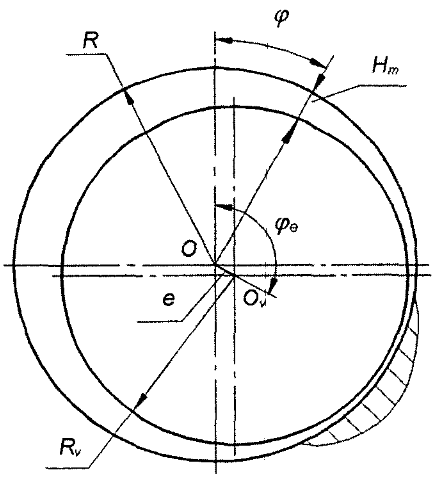


Fig. 5 Location of the shaft neck inside the bearing

and isothermal.

The radial clearance in the nozzle plane(see Fig. 5) can be calculated using formula(4):

$$H_m = \frac{D}{2} - \frac{D_v}{2}\left(\cos\left(\arcsin\left(\frac{2e \cdot \sin(\varphi - \varphi_e)}{D_v}\right)\right)\right) - e \cdot \cos(\varphi - \varphi_e) \tag{4}$$

where D is bearing diameter, D_v is diameter of the shaft neck, e is absolute eccentricity, φ is angle of pressure line, φ_e is angle of eccentricity.

At pressure calculation we can assume that coefficient of air expense through nozzle ξ is depending on Reynolds number Re. This assumption makes us introduce the following empirical system of conditions:

$$\xi \Rightarrow \begin{cases} \mathrm{Re} \le 1000;\ \xi = 470 \cdot \left(\dfrac{\mathrm{Re}}{9 \cdot 10^7}\right)^{0.6001} \\ \mathrm{Re} \le 10000;\ \xi = 0.5 \cdot \left(\dfrac{\mathrm{Re} - 765}{2}\right)^{0.0899} - 0.2673 \\ \mathrm{Re} > 10000;\ \xi = 0.801 \end{cases} \tag{5}$$

This system of conditions is clearly reflected on the algorithm of pressure calculation(see Fig. 6).

The pressure is calculated for each segment of bearing. The number of iterations thus is equal to N (where N is the number of nozzles) for the first and the second bearing. According to Peshti[8] method the pressure values are calculated for the minimal and maximal clearances. The pressures are assumed to be distributed along the clearance according to cosine law. However, to increase the precision of pressure calculation we propose to calculate the clearance according to the position of shaft at the current moment of time, for each segment and considering the parameters of the "spindle-bearing" system.

The carrying capacity of lubricant layer in the general case represents the main vector of pressure forces, which equalize the applied external load. In this paper we suppose that the pressure is distributed along the clearance according to the triangular law. This gives a possibility to reduce general carrying capacity equation

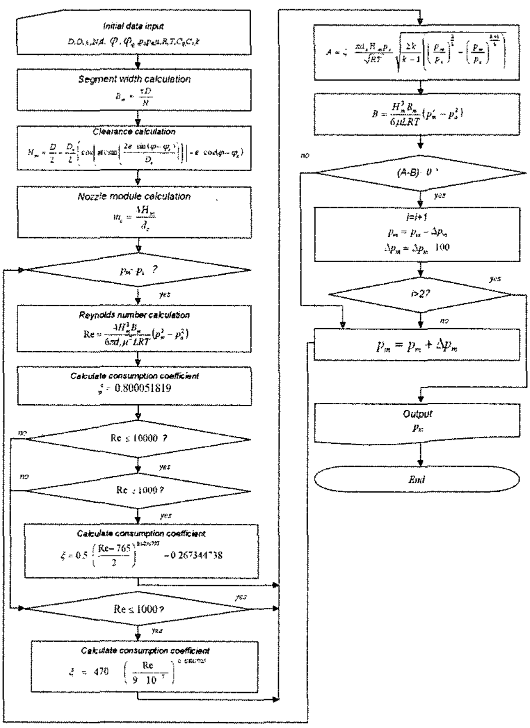


Fig. 6 Pressure calculation algorithm

for simpler particular case:

$$P_i = \frac{p_{m_i} - p_a}{2} B_m L \tag{6}$$

where B_m is bearing width, L is bearing height, P_{m_i}, P_a are pressure in the clearance and atmosphere pressure respectively. Therefore the total loading capacity will be:

$$\overline{P}_N = \sum_{i=1}^{N} \overline{P}_i = \sum_{i=1}^{N} P_i \cos \varphi_i \tag{7}$$

where

$$\varphi_i = \frac{2\pi i}{N} \tag{8}$$

Thus we have the system of converging forces, the main vector of which is directed from the minimal clearance to the center of bearing.

Since the clearance depends only on the diameter of shaft and the diameter of the bearing ring(in case of radial bearing) or effective clearance(for thrust bearing), it means that the proposed method of pressure and loading capacity calculation is defined for all the types of flow limiters and bearings. This allows concluding, that using proposed method and following the algorithm shown in Fig. 6 it is possible to completely implement the pressure loading capacity calculation unit.

However, to increase precision it is possible to introduce the different kinds of pressure distribution laws for different types of flow limiters and bearings, but this question is a subject of further investigation.

5. Stiffness calculation

Stiffness of lubricant layer can be regarded in general case as a tangent of tilting angle of load capacity curve *vs.* clearance fluctuating. The well-designed spindle unit should possess the necessary static stiffness for safe tool changing and making shaft "float" under existing air-supply without shaft rotation.

Since it is not possible to change the clearance in bearing during the experiments, and considering the fact that the bearing is the body of rotation, the static stiffness characteristics can be calculated using clearance, bearing geometrical dimensions and pressure only once, and then used during numerical experiments without recalculating. However, if the pressure is variable parameter(for example, if pressure is controlled during acceleration stage), then it is necessary to calculate whole characteristics once again, using the new parameters of pressure.

In a general form, the static stiffness could be calculated as the following:

$$C = \frac{dP_N}{de} \approx \frac{\Delta P_N}{\Delta e} = \frac{P_{N_i} - P_{N_{i-1}}}{\Delta x} \tag{9}$$

The main calculation algorithm is shown in Fig. 7.

A very important moment in the static stiffness calculation is the selection of correct eccentricity increment Δe. As a rule, the radial clearance in the

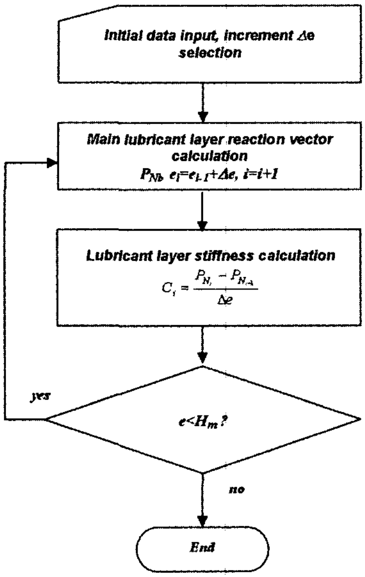


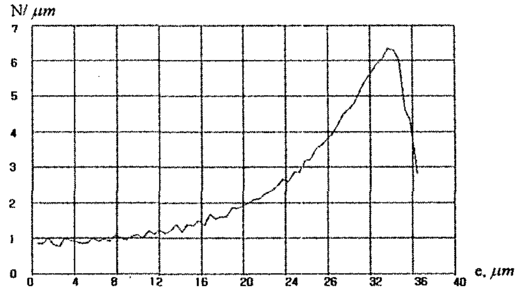
Fig. 7 Stiffness calculation

air-bearing is a value of tenths of microns. During the dynamics experiment the shaft neck shift is measured with the step about 0.1 μm step. However, at calculation of static stiffness characteristics the eccentricity increment step could be rather large. Intermediate values could be obtained by linear interpolation, which considerably increase the speed of calculations. On the Fig. 8(a, b) there is an example of two characteristics.

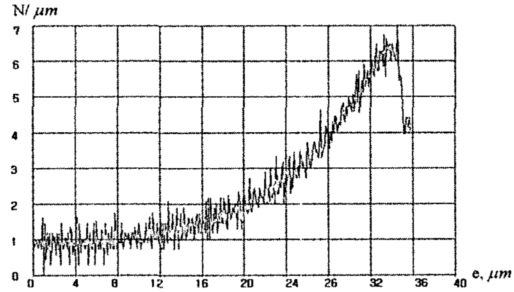
The first(Fig. 8a)is calculated using the step eccentricity increment Δe=0.5 μm, while the second(Fig. 8b) is calculated at Δe=0.1 μm. It could be easily seen, that in this example the calculation precision does not allow to choose the smaller Δe, because the iteration error becomes comparable to stiffness value.

6. Dynamic characteristics of the "spindle-bearing" system

After obtaining the initial design data and performing all necessary calculations we can make the calculation of spindle's characteristic points spatial motion. This will give us the visual representation of spindle behavior under the given working conditions.



(a)



(b)

Fig. 8 Static stiffness characteristics at various Δe

The radial stiffness of the bearings is considerably less than the bending stiffness of a spindle itself, thus it will oscillate as a rigid solid body. The spindle rotation axis Z (its symmetry axis at the same time) will describe the certain spatial conical surface. It means that the spindle will perform some angular oscillations about the X and Y axis. Due to these angular oscillations the angular inertial torques appear, which are affecting the spindle. Using the influence coefficients ij, we may evaluate the displacements and turning angles through the applied forces. Force and torque are regarded as applied to rotor shaft center of masses. Thus the system of equations of shaft forced oscillations will be given as follows:

$$\left.\begin{aligned} x &= -m\ddot{x}\delta_{11} - \left(J\ddot{\Theta}_y - J_z\Omega\dot{\Theta}_x\right)\delta_{12} + \left(P_0\cos\Omega t\right)\delta_{11} + \left(M_0\cos\Omega t\right)\delta_{12} \\ \Theta_y &= -m\ddot{x}\delta_{12} - \left(J\ddot{\Theta}_y - J_z\Omega\dot{\Theta}_x\right)\delta_{22} + \left(P_0\cos\Omega t\right)\delta_{12} + \left(M_0\cos\Omega t\right)\delta_{22} \\ y &= -m\ddot{y}\delta_{11} + \left(J\ddot{\Theta}_x + J_z\Omega\dot{\Theta}_y\right)\delta_{12} + \left(P_0\sin\Omega t\right)\delta_{11} + \left(M_0\sin\Omega t\right)\delta_{12} \\ \Theta_x &= m\ddot{y}\delta_{12} - \left(J\ddot{\Theta}_x + J_z\Omega\dot{\Theta}_y\right)\delta_{22} + \left(P_0\sin\Omega t\right)\delta_{12} + \left(M_0\sin\Omega t\right)\delta_{22} \end{aligned}\right\} \quad (10)$$

where θx and θy are spindle rotation angles about X an Y axes, $Jx=Jy=J$, Jz inertia moment about X, Y, Z axes correspondingly, δ_{11} is displacement of the spindle center of mass under unit force application, δ_{12} is the angle of shaft rotation around the axes X or Y under unit force application, δ_{22} is the angle of shaft rotation around the axes X or Y under the unit force torque application, Ω is the spindle rotation frequency, Po, Mo are main vector of force and torque correspondingly.

The system of differential equations (10) describes the spatial spindle oscillations as a solid body.

Taking into account that the influence coefficients δ_{11}, δ_{12}, δ_{22} are the complex functions of the turning angle of the spindle around Z axis, and the radial spindle displacements in air-bearing[1], it is advisable to use numerical methods when integrating system of equations (10), such as Runge-Kutta method, the further improvement of the Euler's method.

· The Runge-Kutta's method is easy to implement as a software algorithm. The high accuracy and relative easiness of evaluation makes it a very popular method.

7. Illustrative example

An example is given here to illustrate the incorporation of the proposed methods in CAE system. Since the CAE system have been developed considering the sequence shown in Fig. 1(excluding the optimization stage), it was reasonable to implement the "wizard" style application. See the Fig. 9 for the block diagram of CAE operating.

In this style the sequence of actions is predefined, and an user inputs necessary data and execute all operations in strict order. There are 5 main steps(and therefore5 screens in this application):

1) "Welcome" step. It welcomes the user and lets him load previous stored data.
2) Next is "Geometry identification" step. It is intended for input of all necessary information about spindle dimensions and material properties. Also geometry-related parameters(such as torques) are calculated there. Fig. 10 shows the spindle drawing, becoming to terms of "segments" using the CAE system means.
3) The third step is the "Spindle design data input". An user inputs all information related to the spindle design, i.e. information about necks, bearings, imbalance, etc.
4) After the information is entered, a user proceeds to "Calculation" step. During this procedure characteristic points trajectories are calculated on selected time and frequency range. This data are stored in temporary arrays for further investigation.
5) The results analysis step. A user may analyze calculated results using various kinds of charts - 2D and 3D. See the following figures for examples.

On the Fig. 14 subfigures represent various compari-

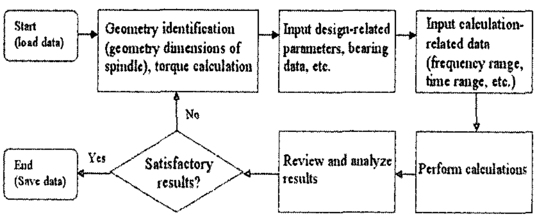


Fig. 9 The block diagram of CAE operation

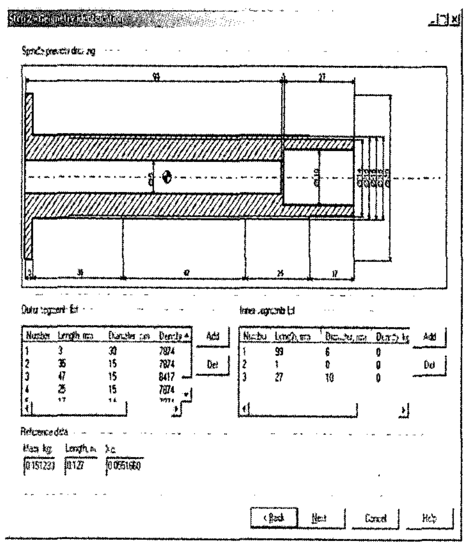


Fig. 10 Spindle drawing in CAE system

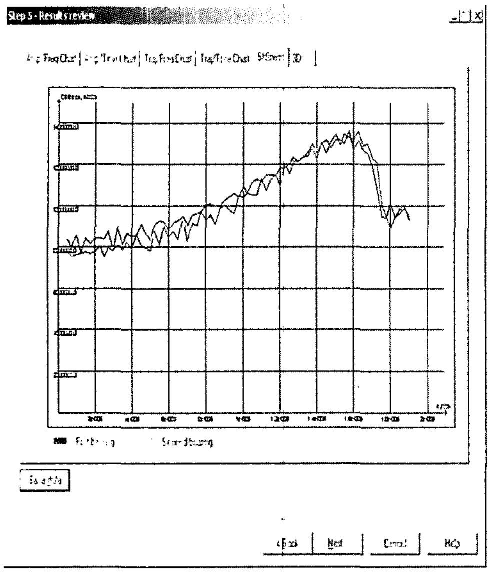


Fig. 11 Radial stiffness calculated for both bearings

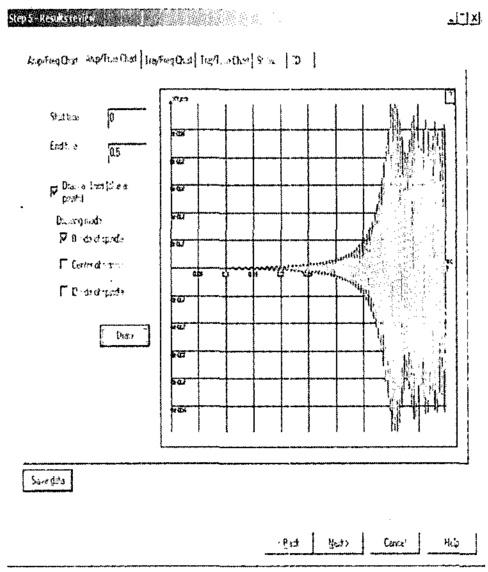


Fig. 13 Amplitude-time characteristics of the first tip of the exemplary spindle on the time range from 0 to 0.5. The initiation of oscillation process can be thoroughly investigated

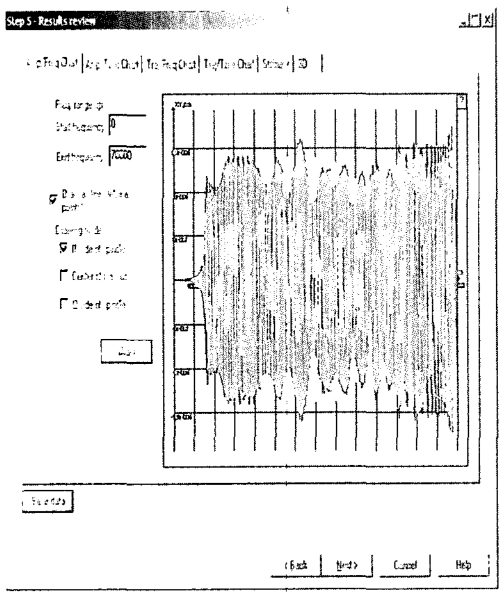


Fig. 12 Amplitude-frequency characteristics of the first tip of the exemplary spindle on the frequency range from 0 to 7000rpm

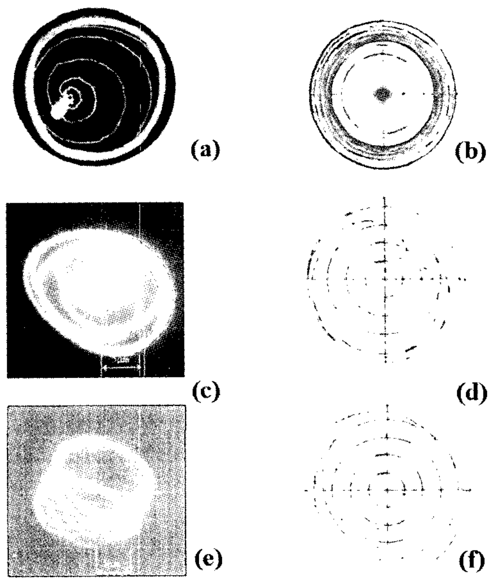
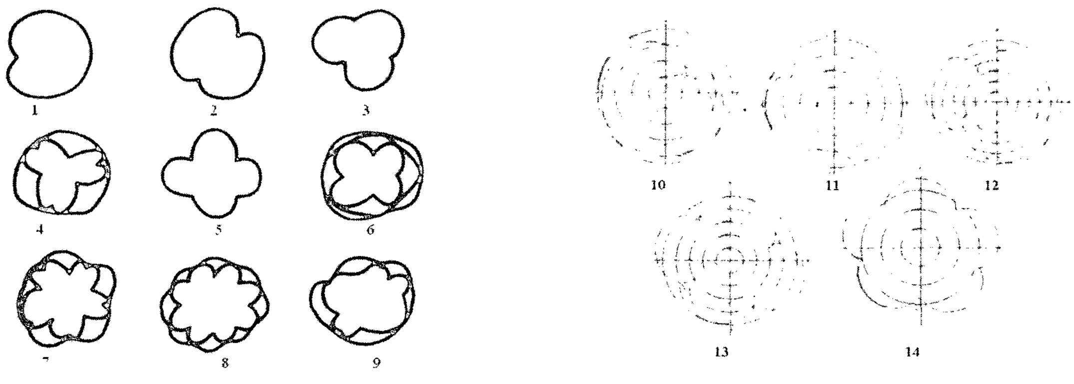


Fig. 14 The experimental (a) [10], (c,e) [11] and the calculated (b,d,f) spindle trajectories



(a) (b)

Fig. 15 Fragment of Peshti[8] characteristics trajectories (a) and conformable calculated trajectories (b)

sons between real and numeric experiments. The Fig. 14 (a, b) represents example of acceleration of the spindle from 0 to 5000rpm. The Fig. 14 (c, d) shows the example of phenomena known as "half-speed whirl". It was described, for example, in[6]. The Fig. 14 (e, f) shows another example of spindle behavior modeled using proposed methods.

The Fig. 15 (a) shows characteristics trajectories of shaft pivot at the edge of stability zone (1 is cardioid, 2-9 are epicycloids). Within the speed range near W_{lim} the shaft axis is oscillating with trajectory, which form is close to the cardioid or epicycloids[8]. At the same time the axis of rotor pivot self-excitationaly oscillates with Ω precession speed. These oscillations are superimposed with synchronic oscillations of rotor, caused by manufacturing errors and rotor imbalance. The Fig. 15 (b) shows the calculated trajectories, conformable to Peshti's trajectories.

8. Conclusion

In this paper we showed that it is possible to provide the required precision and speed of air-spindle units during the design stage by the prognosis of dynamic quality.

The mathematics dependencies between design parameters and load capacity and stiffness were revealed and described. Developed mathematical models allow to estimate static characteristics(such as stiffness and load capacity) of spindle unit, which then can be used for the calculation of dynamic characteristics. Evaluated mathematics dependencies for calculation of air expense coefficient allow increasing the precision of calculation comparing with other methods, where this coefficient is constant. The accuracy is about 15% higher, comparing to the simplified models (not considering the physical processes, for example, used linear stiffness)[10]. Also the system of mathematics equations is derived, which describes the spindle axis motion trajectory, considering the gyroscopic effect. This effect appears on high speed spindle rotation. This allows the estimation of the dynamic characteristics of air-bearing spindle on the design stage. The comparison of the experimental result with the theoretical one has shown the high congruence, which proves the correctness of proposed algorithms. Finally, the CAE system is developed to illustrate the suggested methods. It is hoped that this automated system can enable engineers to refine the air-bearing spindles design more readily.

Acknowledgement

This work was supported by the Korea Science and Engineering Foundation(KOSEF) through the Machine Tool Research Center at Changwon National University.

Nomenclature

Ω = Spindle rotation frequency
δ_{11} = Displacement of the spindle center of mass under unit force application
δ_{12} = Angle of shaft rotation around the axes X or Y under the unit force application
δ_{22} = Angle of shaft rotation around the axes X or Y under the unit force torque application
θ_x = Spindle rotation angle about X axis
θ_y = Spindle rotation angle about Y axis
B_m = Bearing width
C = Lubricant layer stiffness
D = Bearing diameter
Dv = Diameter of the shaft neck
e = Eccentricity
H_m = Radial clearance
I_x = X component of inertia moment in radial direction
I_y = Y component of inertia moment in radial direction
I_z = Inertia moment inaxial direction
L = Bearing height
M_0 = Main vector of torque
N = Number of nozzles
P_a = Atmospheric pressure
P_i = Loading capacity of i-th bearing segment with nozzle
P_{mi} = Pressure in the clearance
P_o = Main vector of force
R_e = Reynolds' number
S_{inneri} = Square of the cross-section of the i-th ring inner hole
S_{outeri} = Full square of the cross-section of the i-th ring
X_c = Distance to the center of masses of the spindle
X_i = Distance to the center of masses of the i-th ring
Δe = Eccentricity increment
ξ = Air expense through a nozzle
ϕ = Angle of pressure line
ϕ_e = Angle of eccentricity

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