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STRONG LAWS OF LARGE NUMBERS FOR ASYMPTOTICALLY QUADRANT INDEPENDENT RANDOM FIELDS

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ABSTRACT. In this paper we define the notion of asymptotically quadrant independent random field and derive the strong laws of large numbers for this random field.

1. Introduction

Let d be a positive integer, $\mathbb{R}^d$ the d-dimensional Euclidean space equipped with the coordinate-wise partial order $\leq$ and $\mathbb{Z}_+^d \subset \mathbb{R}^d$ the d-dimensional lattice. The notation $\underline{m} \leq \underline{n}$, where $\underline{m} = (m_1, m_2, \cdots, m_d)$ and $\underline{n} = (n_1, n_2, \cdots, n_d)$, thus means that $m_k \leq n_k$, for $k = 1, 2, \cdots, d$. We also use $|\underline{n}|$ for $n_1 \times n_2 \times \cdots \times n_d$, $\underline{n} \longrightarrow \infty$ is to be interpreted as $n_k \longrightarrow \infty$, for $k = 1, 2, \cdots, d$ and $\| \underline{n} \|$ for $\max\limits_{1 \leq k \leq n} |n_k|$.

Let $\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_+^d\}$ be a field of d-dimensional random variables on some probability space (Ω, K, P) with $EX_{\underline{j}} = 0$. The field $\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_+^d\}$ fulfills the strong law of large number if as $\underline{n} \longrightarrow \infty$,

$$|\underline{n}|^{-1} \sum_{\underline{1} \leqslant \underline{j} \leqslant \underline{n}} X_{\underline{j}} \longrightarrow 0 \ a.\ s.,$$

where $\underline{n} = (n_1, n_2, \cdots, n_d)$ and $|\underline{n}| = n_1 \times n_2 \times \cdots \times n_d$.

The purpose of the paper is to establish the strong laws of large numbers for the fields of d-dimensional random variables which are not necessarily independent. Evidently, we have to impose certain restrictions on the fields, that means we require the weak dependence. A field

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$\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_+^d\}$ is said to be pairwise positive quadrant dependent (PQD) if for every $\underline{i}$ and $\underline{j}$, and for every real s,t,

$$(1.1a) \qquad P\{X_{\underline{i}} > s, X_{\underline{j}} > t\} - P\{X_{\underline{i}} > s\}P\{X_{\underline{j}} > t\} \geq 0$$

or, as an equivalent condition,

$$(1.1b) \qquad P\{X_{\underline{i}} < s, X_{\underline{j}} < t\} - P\{X_{\underline{i}} < s\}P\{X_{\underline{j}} < t\} \geq 0.$$

This concept was introduced by Lehmann [4]. In the following we will drop the assumption of positive dependence, but use the magnitude of the left hand sides in (1.1a) and (1.1b) as a measure of dependence.

For this reason we introduce the notion of asymptotic quadrant independence : A field $\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_+^d\}$ is said to be pairwise asymptotically quadrant independent (AQI), if there exists a sequence $\{q(\| \underline{n} \|) : \underline{n} \in \mathbb{Z}_+^d\}$ with $q(\| \underline{n} \|) \longrightarrow 0$ as $\underline{n} \longrightarrow \infty$, such that for real s,t and $\underline{i} \neq \underline{j}$, there holds

$$(1.2a) \qquad \begin{aligned} &|P\{X_{\underline{i}} > s, X_{\underline{j}} > t\} - P\{X_{\underline{i}} > s\}P\{X_{\underline{j}} > t\}| \\ &\leq q(\| \underline{j} - \underline{i} \|) \cdot \alpha_{\underline{ij}}\ (s,t), \end{aligned}$$

$$(1.2b) \qquad \begin{aligned} &|P\{X_{\underline{i}} < s, X_{\underline{j}} < t\} - P\{X_{\underline{i}} < s\}P\{X_{\underline{j}} < t\}| \\ &\leq q(\| \underline{j} - \underline{i} \|) \cdot \beta_{\underline{ij}}\ (s,t), \end{aligned}$$

where $\alpha_{\underline{ij}}\ (s,t)$ and $\beta_{\underline{ij}}\ (s,t)$ are nonnegative numbers which may depend on $\underline{i}, \underline{j}, s, t$. Since the left hand sides in (1.2a) and (1.2b) converge to zero as $|s| \longrightarrow \infty$ and $|t| \longrightarrow \infty$, it seems natural to impose additional restrictions on $\alpha_{\underline{ij}}$ and $\beta_{\underline{ij}}$.

In the case of $d = 1$ Birkel [2] introduced the concept of AQI and obtained the strong law of large numbers for AQI random variables. In this paper we extend the results obtained by Birkel [2] to the case of $d \geq 2$.

In Section 2 we consider some preliminary results which will be important role to prove the strong law of large numbers and we will derive conditions on pairwise AQI field $\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_+^d\}$ which quarantees that the finiteness of $\sum_{\underline{n} \in \mathbb{Z}_+^d} |\underline{n}|^{-1} q(\| \underline{n} \|)$ implies the strong law of large number in Section 3.

In the following statement, C stands for a constant whose value may vary from line to line.

2. Preliminaries

The following theorems will be important roles to prove the strong laws of large for AQI field.

THEOREM 2.1. *Let* $\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_+^d\}$ *be a field of nonnegative random variables with* $EX_{\underline{j}}^2 < \infty$ *such that*

$$(i) \quad \sup_{\underline{j} \in \mathbb{Z}_+^d} EX_{\underline{j}} < \infty,$$

$$(ii) \quad \sum_{\underline{j} \geq \underline{1}} \sum_{|\underline{j}| \geq |\underline{i}| \geq 1} Cov^+(X_{\underline{i}}, X_{\underline{j}})/|\underline{j}|^2 < \infty.$$

Then, as $\underline{n} \longrightarrow \infty$,

$$|\underline{n}|^{-1} \sum_{\underline{1} \leq \underline{j} \leq \underline{n}} (X_{\underline{j}} - EX_{\underline{j}}) \longrightarrow 0 \ a.\ s. \tag{2.1}$$

PROOF. It is enough to prove it for the 2-dimensional case. Let $a > 1, b > 1$ and $n_{\underline{k}} = ([a^{k_1}], [b^{k_2}])$ for $\underline{k} = (k_1, k_2)$. By Chebyshev's inequality for every $\varepsilon > 0$

$$\begin{aligned}
&\sum_{\underline{k} \geq \underline{1}} P\{|S_{n_{\underline{k}}} - ES_{n_{\underline{k}}}| \geq \varepsilon |n_{\underline{k}}|\} \\
&\leq C \sum_{\underline{k} \geq \underline{1}} Var(S_{n_{\underline{k}}}) \ / \ |n_{\underline{k}}|^2 \\
&\leq C \sum_{\underline{k} \geq \underline{1}} \sum_{\underline{1} \leq \underline{j} \leq n_{\underline{k}}} \sum_{\underline{1} \leq \underline{i} \leq n_{\underline{k}}} Cov(X_{\underline{i}}, X_{\underline{j}}) \ / \ |n_{\underline{k}}|^2 \\
&\leq C \sum_{\underline{k} \geq \underline{1}} \sum_{\underline{1} \leq \underline{j} \leq n_{\underline{k}}} \sum_{\underline{1} \leq \underline{i} \leq n_{\underline{k}}} Cov^+(X_{\underline{i}}, X_{\underline{j}})/|n_{\underline{k}}|^2 \\
&= C \sum_{\underline{k} \geq \underline{1}} \frac{1}{|n_{\underline{k}}|^2} \sum_{\underline{1} \leq \underline{j} \leq n_{\underline{k}}} \sum_{1 \leq |\underline{i}| \leq |\underline{j}|} Cov^+(X_{\underline{i}}, X_{\underline{j}}) \\
&= C \sum_{\underline{j} \geq \underline{1}} \sum_{1 \leq |\underline{i}| \leq |\underline{j}|} Cov^+(X_{\underline{i}}, X_{\underline{j}}) \sum_{\{\underline{k} : n_{\underline{k}} \geq \underline{j}\}} \frac{1}{|n_{\underline{k}}|^2} \\
&\leq C \sum_{\underline{j} \geq \underline{1}} \sum_{1 \leq |\underline{i}| \leq |\underline{j}|} Cov^+(X_{\underline{i}}, X_{\underline{j}}) \ / \ |\underline{j}|^2 < \infty.
\end{aligned} \tag{2.2}$$

The last inequality of (2.2) follows from the following : First note that

$$\sum_{\{\underline{k}:n_{\underline{k}}\geq \underline{j}\}} \frac{1}{|n_{\underline{k}}|^2} = \sum_{\underline{k}\geq \underline{k_0}} \frac{1}{|n_{\underline{k}}|^2} \tag{2.3}$$

where $\underline{k_0} = \min\{\underline{k} : n_{\underline{k}} \geq \underline{j}\}$. Then the right-hand side of (2.3) yields

$$\begin{aligned}
\sum_{\underline{k}\geq \underline{k_0}} \frac{1}{|n_{\underline{k}}|^2} &\leq C \sum_{\underline{k}\geq \underline{k_0}} \frac{1}{a^{2k_1}b^{2k_2}} \\
&= C \sum_{k_2\geq k_0''} \sum_{k_1\geq k_0'} \frac{1}{a^{2k_1}b^{2k_2}} \\
&\leq \frac{D}{a^{2k_0'}b^{2k_0''}} \\
&\leq \frac{D}{|n_{\underline{k_0}}|^2} \leq \frac{E}{|\underline{j}|^2}
\end{aligned}$$

where $\underline{k_0} = (k_0', k_0'')$ and C, D and E are some positive constants. Thus by the Borel-Cantelli lemma it follows from (2.2) that

$$(S_{n_{\underline{k}}} - ES_{n_{\underline{k}}}) \;/\; |n_{\underline{k}}| \rightarrow 0 \ a.\ s. \tag{2.4}$$

Now given $\underline{k} = (k_1, k_2)$, positive integers k_1, k_2 for $n_{\underline{k}} \leq \underline{n} \leq n_{\underline{k+1}}$ we have

$$\left|\frac{S_{\underline{n}} - ES_{\underline{n}}}{|\underline{n}|}\right| \leq \left|\frac{S_{n_{\underline{k+1}}} - ES_{n_{\underline{k+1}}}}{|n_{\underline{k+1}}|}\right| \frac{|n_{\underline{k+1}}|}{|n_{\underline{k}}|} + \frac{ES_{n_{\underline{k+1}}} - ES_{n_{\underline{k}}}}{|n_{\underline{k}}|} \tag{2.5}$$

by the monotonicity of $S_{\underline{n}}$. Let $a > 1, b > 1$ and for each $n_{\underline{k}} = (n_{k_1}, n_{k_2})$ set $n_{k_1} = [a^{k_1}], n_{k_2} = [b^{k_2}]$. Then from (i), (2.4) and (2.5) one can easily verify that

$$\lim_{\underline{n}\geq \underline{1}} \sup\ (|S_{\underline{n}} - ES_{\underline{n}}| \;/\; |\underline{n}|) \ \leq \sup_{\underline{j}\geq \underline{1}} EX_{\underline{j}}(ab-1),$$

for every $a > 1$ and $b > 1$ which concludes the proof since both a and b may arbitrary close to 1. □

THEOREM 2.2. *Let $\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_+^d\}$ be a field of nonnegative d-dimensional random variable such that*

(i) $\sup_{\underline{j}\in \mathbb{Z}_+^d} EX_{\underline{j}} < \infty$,

(ii) $\sum_{\underline{j}\geq \underline{1}} P(X_{\underline{j}} > |\underline{j}|) < \infty,\ |\underline{n}|^{-1} \sum_{\underline{1}\leq \underline{j}\leq \underline{n}} EX_{\underline{j}} 1_{\{X_{\underline{j}}\geq |\underline{j}|\}} \longrightarrow 0$ *as* $\underline{n} \longrightarrow \infty$, (*iii*) $\sum_{\underline{j}\geq\underline{1}} \sum_{|\underline{j}|\geq|\underline{i}|\geq 1} |\underline{j}|^{-2}\, Cov^{+}(X_{\underline{i}}1_{\{X_{\underline{i}}\leq|\underline{i}|\}},\ X_{\underline{j}}1_{\{X_{\underline{j}}\leq|\underline{j}|\}}) < \infty.$

Then, as $\underline{n} \longrightarrow \infty$,

$$|\underline{n}|^{-1} \sum_{\underline{1}\leq\underline{j}\leq\underline{n}} (X_{\underline{j}} - EX_{\underline{j}}) \longrightarrow 0 \ a.\ s..$$

PROOF. For $\underline{j} \in \mathbb{Z}_{+}^{d}$ let $Y_{\underline{j}} = X_{\underline{j}}1_{\{X_{\underline{j}}\leq|\underline{j}|\}}$. Clearly, the random field $\{Y_{\underline{j}} : \underline{j} \in \mathbb{Z}_{+}^{d}\}$ satisfies the assumptions of Theorem 2.1. Hence as $\underline{n} \longrightarrow \infty$,

$$|\underline{n}|^{-1} \sum_{\underline{1}\leq\underline{j}\leq\underline{n}} (Y_{\underline{j}} - EY_{\underline{j}}) \longrightarrow 0 \ a.\ s. \tag{2.6}$$

Assumption (*ii*) of Theorem 2.2 takes care of the difference between $\sum_{\underline{1}\leq\underline{j}\leq\underline{n}} Y_{\underline{j}}$ and $\sum_{\underline{1}\leq\underline{j}\leq\underline{n}} X_{\underline{j}}$, and $\sum_{\underline{1}\leq\underline{j}\leq\underline{n}} EY_{\underline{j}}$ and $\sum_{\underline{1}\leq\underline{j}\leq\underline{n}} EX_{\underline{j}}$, which is sufficient to get the desired result. □

LEMMA 2.3. (Newman [5]) *Let* X_1, X_2 *be random variables with* $EX_j^2 < \infty$ *for* $j = \underline{1}, \underline{2}$. *Then*

$$\begin{aligned} &Cov(X_{\underline{1}}, X_{\underline{2}}) \\ = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (P\{X_{\underline{1}} > s, X_{\underline{2}} > t\} - P\{X_{\underline{1}} > s\}P\{X_{\underline{2}} > t\})\, dsdt. \end{aligned}$$

LEMMA 2.4. (Birkel [2]) *Let* X *be a random variable with* $EX^2 < \infty$. *Then*

$$\begin{aligned} (i)\quad & Var(X^{+}) \leq Var(X) \\ (ii)\quad & Var(X^{-}) \leq Var(X). \end{aligned}$$

3. Main results

In this section we extend some results in Birkel [2] to the d-dimensional case.

THEOREM 3.1. *Let* $\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_{+}^{d}\}$ *be an AQI field of uniformly bounded random variables with* $EX_{\underline{j}} = 0$ *such that*

$$\begin{aligned} \alpha_{\underline{i},\underline{j}}\ (s,t) &\leq C_{\alpha} < \infty, \\ \beta_{\underline{i},\underline{j}}\ (s,t) &\leq C_{\beta} < \infty. \end{aligned}$$

If $\{q(\| \underline{n} \|) : \underline{n} \geq \underline{1}\}$ *satisfies* $\sum_{\underline{n}\geq\underline{1}} |\underline{n}|^{-1}q(\| \underline{n} \|) < \infty$, *then* $\{X_{\underline{j}} : \underline{j} \in \mathbb{Z}_+^d\}$ *fulfills the strong law of large numbers.*

PROOF. Let $M < \infty$ be such that $|X_{\underline{j}}| \leq M$ for $\underline{j} \in \mathbb{Z}_+^d$. We will verify that

$$|\underline{n}|^{-1} \sum_{\underline{1}\leq\underline{j}\leq\underline{n}} (X_{\underline{j}}^+ - EX_{\underline{j}}^+) \longrightarrow 0 \ a.\ s. \tag{3.1}$$

and

$$|\underline{n}|^{-1} \sum_{\underline{1}\leq\underline{j}\leq\underline{n}} (X_{\underline{j}}^- - EX_{\underline{j}}^-) \longrightarrow 0 \ a.\ s.. \tag{3.2}$$

As $\sup_{\underline{j}\in\mathbb{Z}_+^d} EX_{\underline{j}}^+ \leq M < \infty$, by using Theorem 2.1 it suffices to prove

$$\sum_{\underline{j}\geq\underline{1}} \sum_{|\underline{j}|\geq|\underline{j}|\geq|\underline{1}|} |\underline{j}|^{-2}\, Cov^+(X_{\underline{i}}^+, X_{\underline{j}}^+) < \infty. \tag{3.3}$$

Applying Lemma 2.3 and the pairwise AQI property (1.2a), we obtain for $\underline{i} \neq \underline{j}$,

$$\begin{aligned}
& Cov^+(X_{\underline{i}}^+, X_{\underline{j}}^+) \\
\leq\ & \left|\int_0^M \int_0^M (P\{X_{\underline{i}}^+ > s, X_{\underline{j}}^+ > t\} - P\{X_{\underline{i}}^+ > s\}P\{X_{\underline{j}}^+ > t\})dsdt\right| \\
\leq\ & \int_0^M \int_0^M |P\{X_{\underline{i}}^+ > s, X_{\underline{j}}^+ > t\} - P\{X_{\underline{i}}^+ > s\}P\{X_{\underline{j}}^+ > t\}|dsdt \\
\leq\ & q(\| \underline{j} - \underline{i} \|) \int_0^M \int_0^M \alpha_{\underline{i},\underline{j}}(s,t)dsdt \\
\leq\ & C_\alpha \cdot M^2 q(\| \underline{j} - \underline{i} \|).
\end{aligned}$$

Hence

$$\begin{aligned}
&\sum_{\underline{j}\geq\underline{1}}\sum_{1\leq|\underline{i}|\leq|\underline{j}|}|\underline{j}|^{-2}cov^+(X_{\underline{i}}^+,X_{\underline{j}}^+)\\
\leq\ & C_\alpha\sum_{\underline{j}\geq\underline{1}}\sum_{1\leq|\underline{i}|\leq|\underline{j}|}|\underline{j}|^{-2}M^2q(\|\underline{j}-\underline{i}\|)+\sum_{\underline{j}\geq\underline{1}}|\underline{j}|^{-2}Var(X_{\underline{j}}^+)\\
\leq\ & C_\alpha M^2\sum_{|\underline{n}|\geq\underline{1}}q(\|\underline{n}\|)\sum_{|\underline{l}|\geq|\underline{n}|+1}(\|\underline{l}\|)^{-2}+M^2\sum_{\underline{j}\geq\underline{1}}|\underline{j}|^{-2}\\
\leq\ & C_1M^2\sum_{|\underline{n}|\geq\underline{1}}|\underline{n}|^{-1}q(\|\underline{n}\|)+C_2M^2<\infty.
\end{aligned}$$

This proves (3.3). Thus, by Theorem 2.1 we have as $\underline{n}\longrightarrow\infty$,

$$|\underline{n}|^{-1}\sum_{\underline{1}\leq\underline{j}\leq\underline{n}}|X_{\underline{j}}^+-EX_{\underline{j}}^+|\longrightarrow 0\ a.\ s..$$

Using that for $s,t\geq 0$,

$$\begin{aligned}
&P\{X_{\underline{i}}^->s,X_{\underline{j}}^->t\}-P\{X_{\underline{i}}^->s\}P\{X_{\underline{j}}^->t\}\\
=\ &P\{X_{\underline{i}}^-<-s,X_{\underline{j}}^-<-t\}-P\{X_{\underline{i}}^-<-s\}P\{X_{\underline{j}}^-<-t\}
\end{aligned}$$

and applying the pairwise AQI property (1.2b), (3.2) is verified in the same way. Since $X_{\underline{j}}^+-X_{\underline{j}}^-=X_{\underline{j}}$, $EX_{\underline{j}}^+-EX_{\underline{j}}^-=0$, this gives us the desired result. □

If the random variables are not uniformly bounded, $\alpha_{\underline{i},\underline{j}}$ and $\beta_{\underline{i},\underline{j}}$ have to satisfy additional assumptions.

THEOREM 3.2. *Let $\{X_{\underline{j}}:\underline{j}\in\mathbb{Z}_+^d\}$ be a field of AQI d-dimensional random variables with $EX_{\underline{j}}=0$ such that*

$$\sup_{\underline{i}\neq\underline{j}}\int_0^\infty\int_0^\infty\alpha_{\underline{i},\underline{j}}(s,t)\,dsdt<\infty,\ \sup_{\underline{i}\neq\underline{j}}\int_0^\infty\int_0^\infty\beta_{\underline{i},\underline{j}}(-s,-t)\,dsdt<\infty.$$

Assume

$$(i)\quad \sup_{\underline{j}\in\mathbb{Z}_+^d}E|X_{\underline{j}}|<\infty,$$

$$(ii)\quad EX_{\underline{j}}^2<\infty,\ \sum_{\underline{j}\geq\underline{1}}|\underline{j}|^{-2}\,Var(X_{\underline{j}})<\infty.$$

If $\{q(\|\underline{n}\|):\underline{n}\geq\underline{1}\}$ satisfies $\sum_{|\underline{n}|\geq\underline{1}}|\underline{n}|^{-1}\,q(\|\underline{n}\|)<\infty$ then $\{X_{\underline{j}}:\underline{j}\in\mathbb{Z}_+^d\}$ fulfills the strong law of large numbers.

PROOF. We proceed as in the proof of Theorem 3.1. Using the integrability of $\alpha_{\underline{i},\underline{j}}$, we get for $\underline{i} \neq \underline{j}$

$$\begin{aligned} Cov^+(X_{\underline{i}}^+, X_{\underline{j}}^+) &\leq q(\| \underline{j} - \underline{i} \|) \int_0^\infty \int_0^\infty \alpha_{\underline{i},\underline{j}}(s,t)\, dsdt \\ &\leq C_\alpha \cdot q(\| \underline{j} - \underline{i} \|), \end{aligned}$$

and hence,

$$\begin{aligned} &\sum_{\underline{j} \geq \underline{1}} \sum_{|\underline{j}| \geq |\underline{i}| \geq |\underline{1}|} |\underline{j}|^{-2}\, Cov^+(X_{\underline{i}}^+, X_{\underline{j}}^+) \\ \leq\ & C \sum_{|\underline{n}| \geq \underline{1}} |\underline{n}|^{-1} q(\| \underline{n} \|) + \sum_{\underline{j} \geq \underline{1}} |\underline{j}|^{-2}\, Var(X_{\underline{j}}) < \infty, \end{aligned}$$

by Lemma 2.4 and assumption (ii). Now the proof of Theorem 3.2 follows along the lines of the proof of Theorem 3.1. □

REMARK. The strong law of large numbers, resulting from Theorem 3.2, shows the possibility that Theorem 1 of Birkel [1] and Etemadi's strong law of large numbers ([3], Corollary 1) can be extended to the case of $d \geq 2$.

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