

Analysis of Price-Clearing in the Generation Bidding Competition

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Abstract - As deregulation evolves, pricing electricity becomes a major issue in the electric power industry. Participants in the competitive marketplace are able to improve their profits substantially by effectively pricing the electricity. In this paper, game theory is applied to analyze price-clearing in the generation bidding competition with the competition modeled as the non-cooperative and complete information game. The result of this analysis can be useful in understanding spot price-clearing of electricity as well as GENCOs' strategic behavior in the competitive electricity market.

Keywords: Game theory, generation bidding competition, maximum capacity constraints, price clearing, strategic behavior

1. Introduction

Recently, conventional electric power industries integrated vertically have been transformed to fit into the deregulated and competitive market environments. As a result, the dispatch scheduling process is carried out by competitive market participants rather than a sole system operator [1, 2]. Moreover, since each participant seeks its own profit in the competitive marketplace, the conventional least-cost dispatch scheme is inadequate for this deregulated situation [3].

In the competitive electricity market, price is determined by interactions among market participants and not regulations. Moreover, each participant's profit is not a target of regulation and considerably depends on this price. Therefore, competitive market participants are interested in optimal decision making procedures for electricity pricing so as to maximize their own profit. Although various electricity market models have been proposed and conducted, the PoolCo model is commonly adopted throughout the world. In the PoolCo model, GENCOs offer a set generation price and amount to the Independent System Operator (ISO) and the ISO determines the spot price at a point where system load demand intersects the supply curve formed by all GENCOs' bidding sets. Therefore, the spot price is equivalent to the bidding price of the last-dispatched generator in merit-order and all GENCOs admitted to supply electricity are compensated for their generation output with the same market

clearing price (MCP) [6].

The objective of GENCOs' bidding is strictly to maximize their own profit and they have no regard for the PoolCo's benefit. To maximize profit through generation bidding, a GENCO needs to plan an optimal bidding strategy considering its own and the opponents' bidding price and amount simultaneously. However, although adjusting bidding amount affects MCP, spot pricing makes only a faint impact in the electricity markets having similar market power among the GENCOs. Therefore, in this case, it can be considered that each GENCO takes part in the generation bidding by utilizing only his bidding price as a strategy. Alternatively, if some of the GENCOs' bidding amount is limited by others, i.e. some of the GENCOs have considerable market power; the bidding competition may induce an alternate result. In this case where there is an imbalanced market power in generation, the electricity price may be distorted by the unfair bidding strategies of certain GENCOs, possibly resulting in a serious loss to customers.

As a consequence, since pricing of electricity becomes a major issue, it is expected that analysis on GENCOs' strategic behaviors can provide basic and useful information related with the price of electricity and GENCOs' bidding. In this paper, we analyze the price-clearing procedure according to GENCOs' optimal bidding strategies using game theory. In the competitive electricity market, although each GENCO seeks its own profit in consideration for interaction with the opponents, any GENCO cannot dominate generation bidding results. This is modeled as a game situation and consequently GENCOs' strategic behavior for optimal generation bidding can be analyzed using game theory. For this analysis, we assume that all information including the cost function of each generator should be revealed. Therefore, the generation bidding competition is modeled as a complete information game. This assumption is impractical. However, since each

The Transactions of the KIEE, Vol.53A, No.1, JAN. 2004, pp.56-66 : A paper recommended and approved by the Editorial Board of the KIEE Power Engineering Society for translation for the KIEE International Transaction on PE.

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Received February 10, 2003 ; Accepted November 28, 2003

GENCO can predict the opponents' information through repeated generation bidding more accurately, we adopt this assumption to analyze GENCOs' strategic behavior and price electricity.

2. Generation Bidding Game

In the PoolCo model, a GENCO offers a set of generation prices and amounts for his individual generator and the spot price is determined at a point where system load demand intersects the supply curve formed by all GENCOs' bidding sets. This spot price is determined by the bidding price of the last-dispatched generator in merit-order i.e. the marginal generator [2]. Therefore, GENCOs can utilize these bidding sets as strategies of the generation bidding game. However, to maximize profit, since GENCOs participating in unconstrained generation bidding must offer a generation amount for their individual generator so that the marginal production cost of each generator is equivalent to the spot price [6], GENCOs' practical strategies are bidding prices for their individual generators. Therefore, the generation bidding competition is modeled as a game selecting a GENCO's bidding price as the spot price. As a result, the generation bidding game can be represented as "the price-setting game", and the bidding amount of each generator can be regarded as a sub-condition (i.e. dependent variable) for this game. In practice, since the electricity market is oligopoly and each market participant can directly affect the price-clearing procedure, the price-setting game may be a rational model for the generation bidding competition.

To analyze this price-setting model using game theory, we introduce some assumptions as follows:

- All GENCOs are rational players, i.e. they formulate their own profit function and always choose one bidding strategy to maximize this function value.
 - Total bidding generation amount is sufficient to supply system load demand and consequently there is not insufficiency in generation. Therefore, we do not consider the case that insufficiency in generation causes price-spike.
 - All GENCOs have the same information on system load demand. In practice, since the TWBP market provides the forecasted system load demand to all market participants in advance of real-time dispatch, this assumption may be rational.
 - For the simplicity, the demand elasticity is not taken into consideration, i.e. demand bidding is ignored. To consider demand bidding, the price-elastic demand function can replace the constant system load demand without difficulty.
 - Transmission loss and network constrains are ignored,
- i.e. only unconstrained generation bidding is considered. In the TWBP market, each generator submits bidding for an unconstrained dispatch, and ISO compensates constrained-off generators for limited generation amount based on (unconstrained) MCP if the approved dispatch schedule is altered due to network constrains.
- All generators have a quadratic cost function. This implies that the marginal production cost of a generator is proportional to its generation output. Applying constant marginal cost has the potential to cause impractical results [14].
 - All information including the cost function of each generator is revealed. This assumes that each GENCO can accurately anticipate the opponents' information through an infinitely repeated bidding game. Therefore, the generation bidding competition can be implemented as a complete information game [6]. This assumption is somewhat impractical. However, since the objective of this study is to analyze the price-clearing procedure in unconstrained generation bidding, we adopt the complete information game model.
 - The supply curve of each generator is continuous, i.e. a generator should submit one pair of bidding generation price and amount. Since the expectation for opponents' strategies is also revealed under the complete information game situation, every GENCO considers ultimately only one pair of each opponent's bidding price and amount achievable from among multiple bidding sets.

3. Analysis of Generation Bidding Strategies

As stated above, the optimal bidding amount for each generator is equivalent to the generation output as long as its marginal production cost is equal to the forecasted spot price. Therefore, GENCOs' bidding strategies are to select one bidding price as the spot price. First, we will analyze the case in which two GENCOs having one generator respectively participate in the generation bidding competition, and then extend this analysis to a general case.

3.1 Determination of the Optimal Bidding Generation Amount for a Price-Taker

The optimal bidding generation amount for each GENCO taking electricity price is formulated by the same approach as described in [6], i.e. the production cost of a generator i is formulated by the following quadratic function,

$$C_i(P_i) = a_i + b_i P_i + c_i P_i^2, \quad (1)$$

and if the generator i supplies generation output P_i at a spot price λ_t corresponding to a bidding period t , then its profit function (i.e. payoff function) is modeled as follows:

$$\pi_i(\lambda_t, P_i) = \lambda_t P_i - C_i(P_i) = \lambda_t P_i - (a_i + b_i P_i + c_i P_i^2). \quad (2)$$

where,

P_i is the generation output of generator i .

a_i, b_i, c_i are the nonnegative coefficients of a production cost function for generator i .

λ_t is the spot price at a bidding period t .

$\pi_i(\lambda_t, P_i)$ is the profit function of generator i .

To maximize its profit, the generator i should satisfy the first-order necessary condition for its profit function (2) $\partial \pi_i / \partial P_i = 0$. As a result, the optimal bidding amount of generator i is

$$P_i^* = \frac{\lambda_t - b_i}{2c_i}. \quad (3)$$

It is noted that each GENCO forecasting the spot price λ_t can determine its optimal bidding amount by equation (3), and its profit depends on the spot price λ_t . Therefore, each GENCO's practical strategy is focused on how to set the spot price λ_t , i.e. the decision to select one GENCO's bidding price as the spot price.

3.2 Analysis of the Optimal Bidding Price for a Price-Setter

Each GENCO considers the following two options as its strategies: Will a GENCO set the spot price according to its bidding price? Or, will it set the spot price according to the opponent's bidding price? That is, each GENCO considers which GENCO's bidding price is his profit-maximizing spot price and according to this consideration, determines whether being price-setter or price-taker. Therefore, this generation bidding competition can be modeled as the game to select one GENCO's bidding price as the spot price.

For example, if GENCO A expects that the spot price is set by its bidding price, i.e. then that GENCO becomes the price-setter. The opponent GENCO B, i.e. the price-taker, will offer a price lower than GENCO A's and the optimal generation amount stated above. Therefore, price setter A's approved generation output is allocated by the difference

between the opponent's optimal bidding generation amount and total system load demand, i.e.

$$G_A = D_t - G_B. \quad (4)$$

D_t represents total system load demand at bidding period t . G_A and G_B represent GENCO A's and B's approved generation output respectively.

Moreover, GENCO A's profit function is formulated as follows:

$$\pi_A(\lambda_A, P_A) = \lambda_A D_t - \lambda_A P_B^*(\lambda_A) - C_A(D_t - P_B^*(\lambda_A)). \quad (5)$$

$P_B^*(\lambda_A)$ represents GENCO B's optimal bidding generation amount function corresponding to forecasted spot price λ_A .

To maximize its profit in consideration of the opponent's bidding amount, GENCO A needs to offer the optimal bidding price λ_A^* satisfying the following first order necessary condition:

$$\frac{\partial \pi_A(\lambda_A)}{\partial \lambda_A} = D_t - \left[\lambda_A \frac{\partial P_B(\lambda_A)}{\partial \lambda_A} + P_B(\lambda_A) \right] - \left[\frac{\partial P_B(\lambda_A)}{\partial \lambda_A} \frac{\partial C_A(D_t - P_B(\lambda_A))}{\partial P_B(\lambda_A)} \right] = 0. \quad (6)$$

Substituting equation (2) and (3), the price-setter's optimal bidding price λ_A^* is obtained as follows:

$$\lambda_A^* = \frac{b_B c_B + b_A c_B + 2c_A c_B D_t + b_B c_A + 2c_B^2 D_t}{2c_B + c_A}. \quad (7)$$

Since the generation bidding game is symmetric for all players, GENCO B's optimal bidding price is also obtained by the same procedure in case B is the price-setter.

$$\lambda_B^* = \frac{b_A c_A + b_B c_A + 2c_A c_B D_t + b_A c_B + 2c_A^2 D_t}{2c_A + c_B}. \quad (8)$$

3.3 Decision on the Optimal Bidding Strategy

Consequently, GENCO A and B have two candidate spot prices, i.e. λ_A^* and λ_B^* in the generation bidding game at period t . That is, their selectable bidding strategies can be reduced to the above two candidate spot prices. Each GENCO's strategic set can be expressed as $S_A = S_B = \{\lambda_A^*, \lambda_B^*\}$.

All GENCOs favor the spot price to maximize their profit between two candidate prices. If they agree on the preferred spot price, i.e. $\pi_A(\lambda_A^*) > \pi_A(\lambda_B^*)$ and $\pi_B(\lambda_A^*) > \pi_B(\lambda_B^*)$, or $\pi_A(\lambda_A^*) < \pi_A(\lambda_B^*)$ and $\pi_B(\lambda_A^*) < \pi_B(\lambda_B^*)$, then the spot price is set by λ_A^* or λ_B^* respectively.

However, if GENCOs do not agree on the preferred spot price, i.e. every GENCO prefers being the price-taker or price-setter, then they need to analyze their own and their opponent's bidding strategies simultaneously.

3.3.1 Price-Taking Competition

For the above strategic set, all GENCOs have the opportunity to obtain greater profit when they are the price-taker corresponding to the opponent's optimal bidding price. This situation is referred to as price-taking competition. Therefore, under the price-taking competition, each GENCO can maximize his/her profit by taking the spot price i.e. offering a generation price lower than their opponent's. This price-taking competition has a rule similar to the second-price auction. A successful bidder in the second-price auction makes payment for an article as the highest bidding price except their own. Similarly, a successful GENCO in the price-taking competition is also compensated for its approved dispatch as the highest price among the opponents' bidding except for their own. Under this price-taking competition, i.e. second-price auction rule, a GENCO's optimal strategy is to offer its actual production cost [12]. Therefore, a GENCO who wants to win the price-taking competition should offer the actual production cost corresponding to their optimal bidding amount. Since a GENCO offers generation price per MW and total production cost expressed as a quadratic function is proportional to its generation output nonlinearly, it is rational that a GENCO's actual production cost is defined as the unit cost of his optimal bidding amount.

Let $UC_i(\lambda_j^*)$ be a GENCO's per unit cost corresponding to the opponent's optimal bidding price as stated in equations (7) and (8). Then each GENCO's per unit cost is obtained as follows:

$$UC_A(\lambda_B^*) = \frac{C_A(P_A^*)}{P_A^*},$$

$$UC_B(\lambda_A^*) = \frac{C_B(P_B^*)}{P_B^*}.$$

Substituting equation (3), each GENCO's per unit cost can be rewritten as follows:

$$UC_A(\lambda_B^*) = \frac{(\lambda_B^*)^2 - b_A^2 + 4a_A c_A}{2(\lambda_B^* - b_A)}, \quad (9)$$

$$UC_B(\lambda_A^*) = \frac{(\lambda_A^*)^2 - b_B^2 + 4a_B c_B}{2(\lambda_A^* - b_B)}. \quad (10)$$

If $UC_A(\lambda_B^*) < UC_B(\lambda_A^*)$, then GENCO A will be the price-taker by offering the per unit cost $UC_A(\lambda_B^*)$. However, GENCO B expecting his per unit cost to be higher than the opponent's may make a comparison between the profit in the case of offering a generation price that is lower than the opponent's per unit cost $UC_A(\lambda_B^*)$ (i.e. setting the spot price as the opponent's per unit cost) and being the price-setter (i.e. setting the spot price as λ_B^*). If the profit in case of setting the spot price is greater, GENCO B maximizes its profit by offering generation price λ_B^* as the price-setter. Price-setter B may offer any generation price higher than $UC_A(\lambda_B^*)$. However, since GENCO B is aware that the profit-maximizing price is λ_B^* in case of setting the spot price by itself, B will choose this bidding price as its optimal strategy. As a result, this generation bidding game achieves strategic equilibrium at the spot price λ_B^* .

Differently, if GENCO B expects that its profit is larger in case of offering a generation price lower than the opponent's per unit cost $UC_A(\lambda_B^*)$, i.e. setting the spot price as $UC_A(\lambda_B^*)$ and not λ_B^* , B will revise her bidding strategy as offering a generation price lower than $UC_A(\lambda_B^*)$. GENCO A expecting this strategy from GENCO B will also make a comparison between its profit in case of taking the spot price as the opponent's and setting the spot price on its own. Repeating this procedure, if a GENCO expects that setting the spot price by his/her generation price is a superior bidding strategy, then this price-setter will offer its profit-maximizing price λ_i^* (where, $i=A$ or B) finally. Moreover, the price-taker expecting this will determine its own optimal bidding amount as $P_j(\lambda_i)$ (where, $j=B$ or A).

Since the generation bidding game is symmetric for all players, we can also analyze the case of $UC_A(\lambda_B^*) > UC_B(\lambda_A^*)$ by the same procedure.

3.3.2 Price-Setting Competition

For each GENCO's strategic set, all GENCOs may obtain greater profit when they are the price-setter. As a result, they may compete with each other to set the spot price by their bidding price. This situation is called price-setting competition. This price-setting competition is similar to the first-price auction. A successful bidder in the

first-price auction pays for an article as its bidding price. Since a successful GENCO in the price-setting competition, i.e. the price-setter, is also compensated by its own bidding price, the rule of price-setting can be regarded as similar to the rule of the first-price auction.

A GENCO expecting the spot price by its bidding price may offer the optimal bidding price as stated in equations (7) and (8). If $\lambda_A^* > \lambda_B^*$, GENCO A will set the spot price by offering λ_A^* . GENCO B expecting this can decide whether an offering price higher than λ_A^* can increase its profit or not. Moreover, each GENCO can anticipate that the opponent, i.e. the price-taker will adjust its bidding amount corresponding to the price-setter's bidding price using equation (3).

If GENCO B expects that offering a generation price higher than λ_A^* cannot increase its profit, B will determine its optimal bidding amount corresponding to λ_A^* by equation (4). Otherwise, to be the price-setter, GENCO B will revise its bidding strategy as offering any generation price $\lambda_B^> (\lambda_A^*)$. GENCO A anticipating this will also find whether offering a price higher than $\lambda_B^>$ can increase its profit or not. If not, as the price-taker, A determines its optimal bidding amount corresponding to the spot price $\lambda_B^>$ by equation (3). If it can increase its profit, GENCO A revises its bidding strategy again. By repeating this procedure, the price-setter can finally be determined, and this price-setter's bidding price will be the spot price at the relevant bidding period. Moreover, a GENCO expecting to be the price-taker will choose the optimal bidding amount to maximize its own profit corresponding to the spot price.

Since the generation bidding game is symmetric for all players, we can also derive the strategic equilibrium by the same procedure when $\lambda_A^* < \lambda_B^*$.

3.4 Analysis of the Equivalent Strategy

In this section, we will analyze which GENCO has an advantage in the generation bidding game for each condition, price-taking competition and price-setting competition. In the price-taking competition, a GENCO with the lower per unit production cost to supply its optimal bidding amount corresponding to the opponent's optimal bidding price has an advantage in this game. In equations (9) and (10), if two GENCOs have a similar production cost function, then their per unit production costs depend on expected spot price respectively, and a player who expects a lower spot price, i.e. the opponent's optimal bidding price has a lower per unit production cost.

That is, in this price-taking competition, a GENCO who has a higher optimal bidding price in case of being the price-setter has an advantage. As a result, there is an incentive to reduce the spot price in the generation bidding game with price-taking competition.

The same result can also be obtained during price-setting competition. In price-setting competition, each GENCO desires to be the price-setter, but the player with the higher optimal bidding price has priority to set the spot price. Therefore, a GENCO who has a lower optimal bidding price chooses a better strategy after making a comparison between its own profit in case of offering price being higher or lower than the opponent's. Contrary to price-taking competition, there is an incentive to increase the spot price in the generation bidding game with price-setting competition.

As a result, if GENCOs' expectations of profit-maximizing spot price are not identical with each other, a player who has a higher optimal bidding price to set the spot price has an advantage in generation bidding competition. For example, if GENCO A's optimal bidding price to set the spot price is higher than GENCO B's, i.e. $\lambda_A^* > \lambda_B^*$, then the following result can be obtained by substituting equations (7) and (8):

$$\begin{aligned} & \lambda_A^* > \lambda_B^* , \\ & \frac{b_B c_B + b_A c_B + 2c_A c_B D_t + b_B c_A + 2c_B^2 D_t}{2c_B + c_A} \\ & > \frac{b_A c_A + b_B c_A + 2c_A c_B D_t + b_A c_B + 2c_A^2 D_t}{2c_A + c_B} , \\ & b_B + 2c_B D_t > b_A + 2c_A D_t . \end{aligned}$$

Therefore,

$$MC_B(D_t) > MC_A(D_t) .$$

$MC_i(D_t)$ represents the marginal production cost of generator i supplying generation output D_t .

Consequently, by equation (11), a GENCO who has a lower marginal production cost corresponding to supplying system load demand at bidding period t has an advantage in the generation bidding game.

Moreover, the spot price through this generation bidding competition is set in a rational level without regulation on GENCOs' bidding price. That is, by the generation bidding competition, the spot price does not increase infinitely without a price-cap. This is because each GENCO determines its own bidding strategy in consideration of the opponent's.

3.5 N-Player Generation Bidding Game

The generation bidding game stated above can be extended to N-player game. In practical generation bidding, there are two or more GENCOs having multiple generators.

In this case, a GENCO expecting to be the price-setter at bidding period t can estimate its optimal bidding price corresponding to a specific generator, i.e. marginal generator through the following optimization problem:

$$\text{Maximize } \sum_{m=1}^M \pi_{im}(\lambda_{i,mag}^t)$$

subject to

$$P_{i,mag}^t = D^t - \sum_{j=1}^{N-1} \sum_{m=1}^{M_j} P_{jm}^t - \sum_{m=1}^{M_j-1} P_{im}^t \quad (13)$$

$$P_{im}^{\min} \leq P_{im}^t \leq P_{im}^{\max}$$

$$P_{im}^{t-1} - \Delta P_{im} \leq P_{im}^t \leq P_{im}^{t-1} + \Delta P_{im}.$$

where,

$\lambda_{i,mag}^t$ is the bidding price for GENCO i 's marginal generator.

$P_{i,mag}$ is the generation output for GENCO i 's marginal generator.

M_i is the number of GENCO i 's generators.

N is the number of GENCOs participating in generation bidding.

The first constraint in problem (13) represents system real power balance at bidding period t , and the second constraint represents generation output limit for each generator. Moreover, the third constraint implies ramp rate limit for each generator.

Using problem (13), each GENCO can estimate the optimal bidding price to set the spot price, then the N-candidate spot price can be obtained for generation bidding game at period t . For each candidate spot price, if there is only one GENCO that prefers being the price-setter, then the GENCO's bidding price will be the spot price, i.e. equivalent price at that period. In this case, other GENCOs that prefer taking the spot price offer an optimal bidding amount for their own generators by equation (4).

However, when there is a price-setting competition, the generator offering the highest price among each GENCO's marginal generators i.e. a GENCO having the marginal generator to be lowest marginal production cost has an advantage in this bidding game.

During price-taking competition, a similar result can also be obtained. That is, when there is a price-taking competition in the N-player generation bidding game, a

GENCO having the lowest marginal production cost generator corresponding to supplying system load demand at any bidding period can have an advantage in this price-taking competition.

4. Price-Clearing under Restricted Generation Capacity

In this section, we analyze the price-clearing procedure when a GENCO's bidding amount is restricted by its own generation capacity, i.e. when market power is concentrated on a specific GENCO.

4.1 A Case with Restriction on the Price-Taker's Bidding Amount

As shown in equation (3), price-taker (i.e. a GENCO having lower marginal production cost) A's optimal bidding amount is the generation output when A's marginal production cost is the same as price-setter (i.e. a GENCO having higher marginal production cost) B's bidding price. However, if the price-taker's maximum generation capacity P_A^{\max} is less than the optimal bidding amount P_A^* , i.e. its bidding amount is limited by the maximum generation capacity, then GENCO A should revise its optimal bidding strategy.

Price-setter B expecting this ensures at least its generation output as subtracting price-taker A's maximum generation capacity from system load demand at bidding period t . Therefore, price-setter B's minimum generation output $\min\{P_B^{bid}\}$ can be obtained as follows:

$$\min\{P_B^{bid}\} = D_t - P_A^{\max}. \quad (14)$$

As a result, price-setter B expecting fixed generation output as in equation (14) has an incentive to increase its bidding price so as to improve its profit. That is, for the following the price-setter B's profit function,

$$\begin{aligned} \pi_B(\lambda_B, P_B) &= \lambda_B P_B - C_B(P_B) \\ &= \lambda_B P_B - (a_B + b_B P_B + c_B P_B^2), \end{aligned}$$

if the bidding amount P_B is fixed as minimum generation output, GENCO B's profit will be proportional to its bidding price λ_B . Therefore, price-setter B has an incentive to increase the generation bidding price up to the price-cap so as to maximize its profit.

Moreover, since price-taker A is aware that price-setter

B will increase its bidding price up to price-cap, A should also determine its bidding amount to maximize its profit when the spot price is set as a price-cap. For the following GENCO A's profit function,

$$\begin{aligned}\pi_A(\lambda_B, P_A) &= \lambda_B P_A - C_A(P_A) \\ &= \lambda_B P_A - (a_A + b_A P_A + c_A P_A^2),\end{aligned}$$

the marginal profit corresponding to generation output is as follows:

$$\frac{\partial \pi_A}{\partial P_A} = \lambda_B - b_A - 2c_A P_A$$

For the spot price λ_B fixed by price-cap, since price-taker A's marginal profit decreases in proportion to his generation output, his total profit increases proportionally until the marginal profit is zero. The optimal bidding amount is the generation output having zero marginal profit. However, since the maximum generation capacity is less than the optimal bidding amount, the price-taker should offer maximum generation capacity to maximize its personal profit.

Consequently, if the price-taker's bidding amount is limited by its maximum generation capacity, the price-taker will offer maximum generation capacity to maximize its profit and the price-setter expecting this will increase bidding price up to price-cap so as to maximize profit corresponding to its fixed bidding amount $P_B^{bid} = D_t - P_A^{\max}$.

4.2 A Case with Restriction on the Price-Setter's Bidding Amount

As stated above, price-setter B chooses its optimal bidding amount and price by equations (5) and (9) in the generation bidding game. If price-setter B's bidding amount is limited by maximum generation capacity, i.e. maximum generation capacity P_B^{\max} is less than the bidding amount $P_B^{bid} = D_t - P_A^{\max}$, B cannot choose this bidding amount and price as its optimal bidding strategy. Therefore, price-taker A expecting this has an incentive to increase its bidding amount so that price-setter B's bidding price will increase. Equation (4) corresponding to price-taker A's optimal bidding amount can be rewritten in terms of the spot price, i.e. price-setter B's bidding price, as follows:

$$\lambda_B = b_A + 2c_A P_A.$$

By this equation, the price-setter's bidding price is proportional to the price-taker's bidding amount. Since to increase the price-taker's generation output P_A causes a reduction in the price-setter's $P_B = D_t - P_A$, the price-setter should increase its bidding price to compensate reduced profit by generation decrease.

Therefore, price-taker A can lead price-setter B to increase its bidding price (i.e. spot price) so that B can increase its profit. For the following, price-setter B's profit function is

$$\begin{aligned}\pi_B(\lambda_B, P_B) &= \lambda_B P_B - C_B(P_B) \\ &= \lambda_B P_B - (a_B + b_B P_B + c_B P_B^2).\end{aligned}$$

If price-setter B expects that reduced profit $\lambda_B \Delta P_B$ by generation decrement is larger than corresponding reduced generation cost $a_B + b_B \Delta P_B + c_B \Delta P_B^2$, then B has an incentive to increase its bidding price. As a result, the following is price-taker A's profit function,

$$\begin{aligned}\pi_A(\lambda_B, P_A) &= \lambda_B P_A - C_A(P_A) \\ &= \lambda_B P_A - (a_A + b_A P_A + c_A P_A^2).\end{aligned}$$

If price-taker A expects that its increased generation P_A and incremental profit $\lambda_B \Delta P_A$ by increase in the price-setter's bidding price is larger than incremental production cost $a_A + b_A \Delta P_A + c_A \Delta P_A^2$ by increased generation, then A has an incentive to raise the spot price by increasing its bidding amount.

Particularly, if price-taker A can supply the entire system load demand at any bidding period, it may monopolize the electric power supply by increasing his bidding amount to system load demand with a lowered price than the price-cap so that its profit will be maximized. As stated above, this is because price-setter B has an incentive to increase its bidding price to price-cap by generation decrease according to increase in price-taker A's bidding amount. Therefore, in proportion to the price-taker's generation capacity, i.e. the price-taker's market power, the price-setter's profit obtained by setting the spot price will decrease. Price-setter B expecting this may positively take the spot price rather than set the price.

Consequently, when the price-setter's bidding amount is limited by generation capacity, we can also obtain the same result, since the price-taker's bidding amount is limited. That is, in this case, the price-setter will maximize its profit by maximum generation capacity with very low bidding price (extremely, zero bidding), and the price-taker expecting this opponent's bidding strategy will also have an

incentive to increase its own bidding price to price-cap.

4.3 Analysis of the Price-Clearing Procedure under Restricted Generation Capacity

Based on the above analysis, we can obtain the following results when a GENCO's bidding amount is limited by his own generation capacity, i.e. the market power is concentrated on a specific GENCO.

If a GENCO's bidding amount is limited, its bidding amount is utilized as its own strategy rather than bidding price. As stated above, whether the market power is concentrated on the price-taker or the price-setter, a GENCO having a limited bidding amount (i.e. having less market power) will take the spot price by offering a lower price than the opponent's so that it will supply the maximum generation output. Moreover, the unrestricted GENCO (i.e. having greater market power) expecting this will increase the spot price by offering a bidding price as price-cap so as to maximize profit under its market power. That is, in the generation bidding market in which a specific participant has considerable market power, a GENCO having less market power prefers maximizing its generation output (i.e. market share) to setting the spot price by profit-maximizing the bidding price. Therefore, the GENCO only has an interest in increasing its generation output, and consequently transfers the opportunity for setting the spot price to the opponent having the greater market power. This means that a GENCO having less market power no longer considers its bidding price a bidding strategy. Moreover, although a GENCO having greater market power remains behind the order of priority in supplying generation output by setting the spot price, that GENCO has an incentive to increase the spot price for profit-maximization due to its guaranteed generation output.

Therefore, it is noted that sharing identical market power among GENCOs is an effective manner to stabilize the electricity price. As a result, imbalance of market power among GENCOs causes a price-spike in the electricity market, which conflicts with the objective of the competitive electricity market to induce reasonable electric charge.

5. Numerical Examples

In this section, we will demonstrate the above results on the price-clearing procedure through numerical examples. First, we analyze each GENCO's optimal bidding strategy and the corresponding equivalent strategy in generation bidding competition, then, based on this result, we observe the change in each GENCO's bidding strategy and

equivalent strategy when a GENCO's bidding amount is limited by its maximum generation capacity.

5.1 An Equivalent Strategy in Generation Bidding Competition

We assume that system load demand at a bidding period t is $D_t = 350\text{MW}$ and that two GENCOs have the following production cost function respectively:

$$C_A(P_A) = 12 + 7.3P_A + 0.23P_A^2,$$

$$C_B(P_B) = 5 + 5.5P_B + 0.35P_B^2.$$

Using equations (8) and (9), the optimal bidding price and each GENCO's corresponding profit is the price-setter i.e. the strategic set can be obtained as follows:

$$S = \left\{ \lambda_A^* = \$165.05 / \text{MW}, \lambda_B^* = \$158.31 / \text{MW} \right\},$$

$$\pi_A(\lambda_A^*) = \$14,485, \quad \pi_B(\lambda_A^*) = \$18,636,$$

$$\pi_A(\lambda_B^*) = \$17,804, \quad \pi_B(\lambda_B^*) = \$12,870.$$

As a result, since both GENCOs prefer taking the spot price, they participate in the price-taking competition.

When GENCO A sets the spot price, GENCO B's optimal bidding amount, total production cost and per unit cost corresponding to A's optimal bidding price can be calculated as follows:

$$P_B(\lambda_A^*) = 230.78\text{MW},$$

$$C_B[P_B(\lambda_A^*)] = \$19,916,$$

$$UC_B(\lambda_A^*) = \$86.30 / \text{MW}.$$

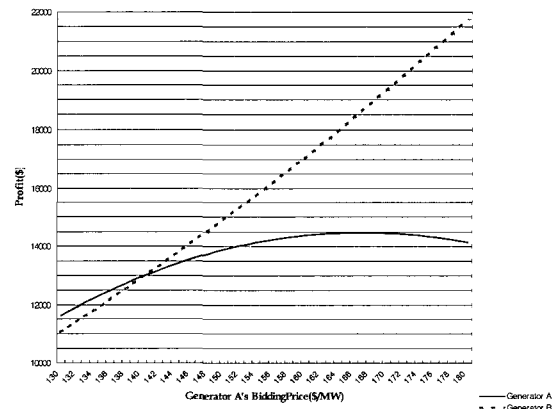


Fig. 1 Profit curve when player A is the price-setter

Using the same method, GENCO A's optimal bidding amount, total production cost and per unit cost

corresponding to how GENCO B sets the spot price also can be obtained as follows:

$$\begin{aligned}
 P_A(\lambda_B^*) &= 235.95MW , \\
 C_A[P_A(\lambda_B^*)] &= \$19,550 , \\
 UC_A(\lambda_B^*) &= \$82.90 / MW .
 \end{aligned}$$

Therefore, GENCO A having less per unit production cost under the expected spot price will have an advantage in this price-taking competition by offering his per unit production cost. Moreover, GENCO B expecting this will choose a better alternative after making a comparison between its profit in case of setting the spot price by its optimal bidding price and by the expected GENCO A's per unit production cost. This result is the same as follows:

$$\begin{aligned}
 \pi_B(\lambda_B^* = \$158.31 / MW) &= \$12,870 , \\
 \pi_B(\lambda_A^* = \$82.90 / MW) &= \$4,274 , \\
 P_B(\lambda_A^*) &= 115.43MW .
 \end{aligned}$$

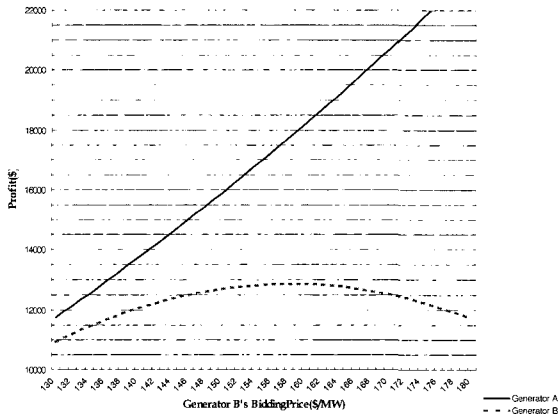


Fig. 2 Profit curve when player B is the price-setter

As a result, GENCO B can obtain more profit when it sets the spot price. Since GENCO B's optimal bidding price to maximize its profit is $\lambda_B^* = \$158.31/MW$, B chooses this price as its optimal bidding strategy.

Moreover, since price-taker A expects that the opponent will offer a profit-maximizing price λ_B^* in consideration of its strategy, A can derive its optimal bidding amount by equation (4). In the above example, price-taker A's optimal bidding amount is:

$$P_A^*(\lambda_B^*) = 235.95MW .$$

Therefore, the spot price in this generation bidding game is set as $\lambda^t = \lambda_B^* = \$158.31/MW$, and each GENCO's

generation output is approved as $P_A = 235.95MW$ and $P_B = 114.05MW$ respectively. Moreover, each GENCO's profit in this strategic equilibrium is $\pi_A(\lambda_B^*) = \$17,804$ and $\pi_B(\lambda_B^*) = \$12,870$ respectively.

5.2 Price-Clearing Procedure under Restricted Generation Capacity

Based on the above numerical result, we observe the change in price-clearing procedure when a GENCO's bidding amount is limited by its maximum generation capacity. For this observation, we add the assumptions that price-setter B's maximum generation capacity is 100MW and price-cap at a bidding period t is \$200/MW.

If GENCO B's bidding amount is unlimited, B can obtain a total profit of \$12,870 by approving generation output of 114.05MW with bidding price \$158.31/MW as shown in Fig. 2. However, since GENCO B's maximum bidding amount is limited to 100MW by its maximum generation capacity, B cannot offer this bidding strategy. Therefore, GENCO B should determine the appropriate bidding amount as the bidding strategy to maximize its profit. When GENCO B limited to 100MW sets the spot price, its profit according to generation output is shown in Fig. 3. As a result, GENCO B can obtain the maximum profit of \$12,675 by offering \$167.30/MW to supply its maximum generation output.

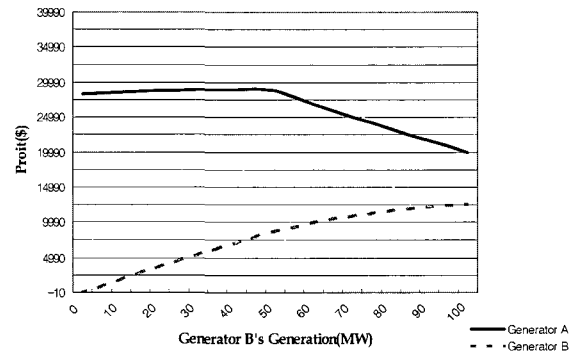


Fig. 3 Profit curve when player B is the price-setter under constraints on its bidding generation

Table 1 Maximum profits when player B is the price-setter

	GENCO A	GENCO B
Spot price (\$/MW)	167.3	
Generation output (MW)	250	100
Total Profit (\$)	19,988	12,675

If GENCO B increases its bidding price up to price-cap to increase its profit, then GENCO A expecting this will

revise its bidding amount to 301.09MW for profit-maximization according to equation (3). Therefore, GENCO B's profit will decrease to \$8742 when B offers price-cap as its bidding price.

When GENCO B sets the spot price, its maximum profit is \$12,675 obtained by supplying its maximum generation output of 100MW. Therefore, to obtain more profit with maximum generation output, GENCO B will deduce that GENCO A sets the spot price by price-cap. For this result, if GENCO B offers a zero bidding price with its maximum generation capacity, setting the spot price rather than competing to take the price is advantageous to GENCO A. This is because there is an incentive for each GENCO to decrease the bidding price in price-taking competition.

Since the opponent's bidding amount is limited, GENCO A can supply at least 250MW. If GENCO A's generation output is fixed, the profit increases in proportion to its bidding price, i.e. the spot price by equation (5). Therefore, GENCO A will increase its bidding price up to price-cap, and consequently A can increase its profit to \$28,163. GENCO B can also obtain its profit of \$19,410 by taking the spot price.

As a result, if GENCO B's bidding amount is limited by its maximum generation capacity, both GENCOs have an incentive to increase the spot price up to price-cap to maximize their own profits.

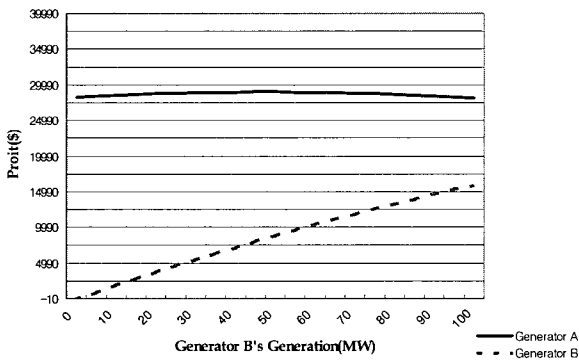


Fig. 4 Profit curve when player A is the price-setter under constraints on B's bidding generation

Table 2 Maximum profits when player A is the price-setter

	GENCO A	GENCO B
Spot price (\$/MW)	200.0	
Generation output (MW)	250	100
Total Profit (\$)	28,163	15,945

6. Conclusion

In this paper, we analyze each GENCO's bidding

strategy at a particular bidding period. Generation bidding competition is modeled as a static complete information game, and a GENCO participating in this generation bidding game can utilize bidding price as its strategy.

When the expectation with respect to each GENCO's preferred price is identical, i.e. when there is the price to maximize all GENCO's profit, this price is set as the spot price, and consequently competition among GENCOs does not exist. On the other hand, when the expectation with respect to each GENCO's preferred price is not identical, competition among GENCOs exists and consequently this competition is represented as the price-taking competition to take the spot price as the opponent's bidding or the price-setting competition to set the spot price by their own bidding. However, we demonstrate that a player having less marginal production cost corresponding to system load demand at a bidding period has an advantage over opponents without regard to the type of competition.

Moreover, when a GENCO's bidding amount is limited by its maximum generation capacity, i.e. when a specific GENCO has considerable market power, bidding amount rather than bidding price is utilized as its strategy. This means that a GENCO having less market power prefers maximizing its generation output, i.e. market share to setting the spot price by profit-maximizing bidding price. Therefore, although a GENCO having greater market power remains behind the order of priority in supplying generation output by setting the spot price, it has an incentive to increase the spot price for profit-maximization due to the guaranteed generation output.

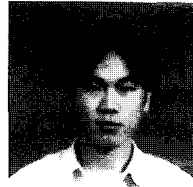
Although assumptions introduced in this paper are impractical, we can identify that reduction in GENCO's production cost is the essential incentive to maximize profit and the imbalance of market power causes distortion in the price signal for market participants. However, developing practical GENCOs' bidding strategies in consideration of various constraints and parameters is very difficult. Moreover, practical GENCOs' bidding strategies reflect uncertainties in the opponents' production cost function and system load demand and combine with 24-hour dispatch scheduling for each generator. However, we expect that our analysis on generation bidding competition as described can provide basic and useful information related with electricity price and GENCOs' strategic behavior.

Acknowledgements

This work was financially supported by MOCIE (I-2002-0-042-5-00) through the IERC program.

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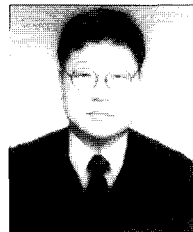
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