

# Reliability-Based Design Optimization of Slider Air Bearings

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This paper presents a design methodology for determining configurations of slider air bearings considering the randomness of the air-bearing surface (ABS) geometry by using the iSIGHT. A reliability-based design optimization (RBDO) problem is formulated to minimize the variations in the mean values of the flying heights from a target value while satisfying the desired probabilistic constraints keeping the pitch and roll angles within a suitable range. The reliability analysis is employed to estimate how the fabrication tolerances of individual slider parameters affect the final flying attitude tolerances. The proposed approach first solves the deterministic optimization problem. Then, beginning with this solution, the RBDO is continued with the reliability constraints affected by the random variables. Reliability constraints overriding the constraints of the deterministic optimization attempt to drive the design to a reliability solution with minimum increase in the objective. The simulation results of the RBDO are listed in comparison with the values of the initial design and the results of the deterministic optimization, respectively. To show the effectiveness of the proposed approach, the reliability analyses are simply carried out by using the mean value first-order second-moment (MVFO) method. The Monte Carlo simulation of the RBDO's results is also performed to estimate the efficiency of the proposed approach. Those results are demonstrated to satisfy all the desired probabilistic constraints, where the target reliability level for constraints is defined as 0.8.

**Key Words :** Slider Air Bearings, Optimization, Air-Bearing Surface, Reliability-Based Design

## 1. Introduction

The read/write head is supported by a slider carrying an air-bearing surface (ABS) whose profile is etched into the bottom surface of a slider. The head slider is a micro device fabricated using a semiconductor-like batch process, by which around 40,000 sliders are produced in a 6 inch ceramic wafer. It is not possible to fabricate the slider so that each parameter is exactly equal to its design value. The values of the parameters will vary statistically from the nominal design values, and thus the flying attitude of the resulting slider will also have a statistical distribution. For

mass production, especially, such lithographic processes result in manufacturing tolerances, which are responsible for the flying height variations. In fact, several parameters such as geometric dimensions of the slider, suspension characteristics, and operation conditions affect the slider's flying characteristics. It would be hard to control each one of these to get a tight control of the flying height in a high volume manufacturing environment. To overcome these variations and uncertainties, the safety factor has been introduced in practical design steps. In case of need, an expensive step has been additionally employed, that adjusts the flying height to the desired value after most of the manufacturing steps have been done. Thus, this leads to an extravagant design having a monotonic margin to account for unpredictable errors.

Considering design methodologies for slider air bearings, the conventional design optimization (i.

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e. the deterministic design optimization) has been successfully applied in many works (Kang et al., 2001, Yoon et. al., 2002, Zhu and Bogy, 2002, and Hanke and Talke, 2003) to systematically improve the desired flying performance of the slider. However, Stenberg et al. (1995) indicated that the spread in flying height of the individual sliders could happen seriously because the manufacturing tolerances may play an important role in control the flying characteristics. The designs determined by the conventional optimization could only provide a low level of confidence in practical products, and occasionally violate engineering requirements. The existence of uncertainties in either engineering simulations or manufacturing processes calls for a reliability-based approach to design optimization, which increases product or process quality by addressing randomness or stochastic properties of design problems.

The reliability-based design optimization (RBDO) provides safety assessments of the flying performance by taking into account uncertainties such as external loads, loading positions over the slider, and the geometry of the ABS. Then, they determine optimum solutions that are economical while satisfying engineering reliability requirements. Especially, for the design of slider air bearings whose manufacturing tolerances are relatively large to their geometric dimensions, it is necessary to employ a reliability-based design.

In this study, we chose a pico slider used in a commercialized product as an initial model. We first solved the deterministic optimization problem. Then, beginning with this solution, the RBDO was carried out with the probabilistic constraints affected by random variables. An optimized ABS shape, which meets a target flying height considering uncertainties of random variables, was automatically designed.

## 2. Reliability-Based Design Optimization

Uncertainties, which always exist in engineering design analyses, should be taken into consideration carefully in order to ensure that a design performs its function within a desired confidence limit without failure. The RBDO is to

design a reliable mechanical system which treats these uncertainties of design variables by introducing the probabilistic method into the optimization procedure (Chandu and Grandhi, 1995 and Tu et. al, 1999). The quantitative estimation of uncertainties has been recently addressed as hot issues. Up to now, techniques have been explored, which incorporate uncertainties during design optimizations at an affordable computational cost.

In general, the deterministic design optimization problem is formulated as a constrained minimization of the following form :

$$\begin{aligned} &\text{Find } \mathbf{x} \\ &\text{to minimize } f(\mathbf{x}) \\ &\text{satisfying } \begin{aligned} &g_j(\mathbf{x}) \geq 0, j=1, 2, \dots, m \\ &\mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{aligned} \end{aligned} \quad (1)$$

where  $\mathbf{x} \in R^n$  is a vector of design variables,  $f(\mathbf{x})$  is the objective function,  $g(\mathbf{x})$  is a constraint function,  $\mathbf{x}^L$  and  $\mathbf{x}^U$  and are the lower and upper limits of the vector of design variables, respectively.  $n$  and  $m$  are the number of design variables and constraints, respectively.

Consider a performance function of the system  $G(\mathbf{X})$ , where  $G(\mathbf{X}) < 0$  denotes failure. The probabilistic optimization problem can be generally defined as follows :

$$\begin{aligned} &\text{Find } \mathbf{d} \\ &\text{to minimize } f(\mathbf{d}) \\ &\text{satisfying } \begin{aligned} &\Pr(G_j(\mathbf{X}) < 0) \leq P_{fj}^t, \\ &j=1, 2, \dots, m \\ &\mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned} \end{aligned} \quad (2)$$

where  $\mathbf{d} = \boldsymbol{\mu}(\mathbf{X}) \in R^n$  is a design variable vector which denotes the mean values of the distributed random parameter vector  $\mathbf{X} \in R^{nr}$ .  $\Pr(G_j(\mathbf{X}) < 0)$  denotes the probability of failure at the  $j^{\text{th}}$  constraint as shown in Fig. 1.  $P_{fj}^t$  is the prescribed failure probability limit of the  $j^{\text{th}}$  constraint, which can be represented by the reliability target index (or the prescribed confidence level).

The statistical description of failure on the performance function is characterized by the cumulative distribution function (CDF). Since

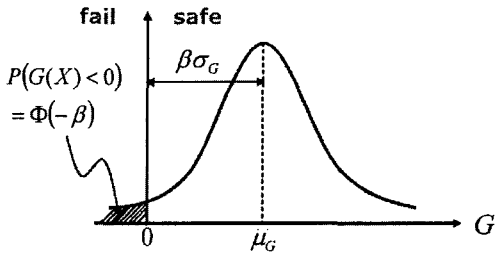


Fig. 1 Probabilistic concept of reliability index

the evaluation of the CDF required a complicated reliability analysis where multiple integrations are involved, some approximate probability integration methods have been developed to provide an efficient solution. As algorithms for most probable point searches, there are the first-order reliability method (FORM) and the asymptotic second-order reliability analysis (SORM). The evaluation of the probabilistic constraint, which is one of the most complicated elements to solve, can be done in two different ways, the reliability index approach (RIA) and the performance measure approach (PMA). For the RIA, the probabilistic constraint is viewed as a reliability index, which is originated from the reliability analysis concept. The PMA, which is originated from the RBDO concept, is proposed by converting the probability measure to a performance measure (Tu et. al, 1999 and Youn et. al., 2003).

To effectively treat the RBDO problem, many numerical methods have been developed and applied to various mechanical engineering systems. But, it is still hard to solve practical problems of which the performance function is implicitly formulated. That is because they increase the computational effort by orders of magnitudes in numbers of design variables and constraints. In this study, we efficiently solve the RBDO problem by using the single-loop-single-design-vector (SLSV: Chen et. al., 1997) method of the iSIGHT. This method can be implemented with only a single optimization loop and have only a single vector of design variables by efficiently using the constraint derivatives from previous design iterations. Hence, it requires only a modest increase in computational cost compared to that of the deterministic design optimization.

### 3. Problem Formulation for RBDO

The steady state distribution of pressure in the air bearing between the slider and the disk is described by the nonlinear generalized Reynolds equation, which can be written in dimensionless form as

$$\sigma \frac{\partial}{\partial T} (PH) + \text{div}(\Lambda PH) = \text{div}(\bar{Q}_p PH \text{ grad } P) \quad (3)$$

where

$$\Lambda = \frac{6\mu U l}{\rho a h_m^2}, \quad \sigma = \frac{12\mu \omega_0 l^2}{\rho a h_m^2}, \quad \bar{Q}_p = f(D_0 PH). \quad (4)$$

Here, H refers to the air film thickness, P is the pressure and  $\Lambda$  is the compressibility number.  $\bar{Q}_p$  is the flow rate coefficient considering high Knudsen numbers for the molecular gas film, which is obtained from the flow rate database corrected by Fukui and Kaneko (1990).

To achieve a reliable flying performance while preventing contact between the pads and disk, strictly confined flying attitudes over the recording band are required. From a practical point of view, the most fundamental performance index is to keep uniform flying heights, higher pitch angles and lower roll angles over the recording band. In other words, the flying height should be insensitive to the disk track, and the pitch and roll angles should be kept within a suitable range across the data zone.

Since the variations of flying height  $h$ , pitch angle  $\alpha$ , and roll angle  $\beta$  as a function of the radial position are known to be smooth and continuous over the recording band, we concentrate on the smallest and/or the largest values of these flying attitude parameters. Denoting the smallest and largest values by the subscripts min and max, respectively, we can formulate the multi-criteria deterministic design optimization as follows:

Find  $\mathbf{x}$   
to minimize  $f(1 - h_{\max}/h^*)^2 + (1 - h_{\min}/h^*)^2 \quad (5)$

satisfying  $\alpha_{\min}/\alpha^L - 1 \geq 0, \quad (6)$

$1 - \alpha_{\max}/\alpha^U \geq 0, \quad (7)$

$$\beta_{\min}/\beta^L - 1 \geq 0, \quad (8)$$

$$1 - \beta_{\max}/\beta^U \geq 0, \quad (9)$$

$$x_i^L \leq x_i \leq x_i^U, \quad i=1, \dots, n \quad (10)$$

If  $d_i$  is set to the mean values of the  $x_i$  respectively, its RBDO problem can be defined to find  $\mathbf{d}$

$$\text{to minimize } f(1 - h_{\max}/h^*)^2 + (1 - h_{\min}/h^*)^2 \quad (11)$$

$$\text{satisfying } \Pr(\alpha_{\min}/\alpha^L - 1 < 0) \leq P_{f1}^T, \quad (12)$$

$$\Pr(1 - \alpha_{\max}/\alpha^U < 0) \leq P_{f2}^T, \quad (13)$$

$$\Pr(\beta_{\min}/\beta^L - 1 < 0) \leq P_{f3}^T, \quad (14)$$

$$\Pr(1 - \beta_{\max}/\beta^U < 0) \leq P_{f4}^T, \quad (15)$$

$$d_i^L \leq d_i \leq d_i^U, \quad i=1, \dots, n \quad (16)$$

Here, the superscripts  $L$  and  $U$  denote lower and upper limit values, respectively.  $h^*$  denotes the target value for the flying height. The cost function of Eq. (11) represents the maximum variation of the normalized flying height according to the radial positions. Eqs. (12) ~ (16) mean that the probability of the failure of each constraint should be less than the limited failure probability.

As shown in Fig. 2, the design variables denote the mean values of recess depth  $d_1$ , shallow step depth  $d_2$  and each position of vertices on the ABS  $d_i$ , ( $i=3, \dots, 16$ ). This figure also indicates that each design variable is allowed to vary in direction of the arrow (i.e. x or y-direction) within the described broken-line box. Thus, the area of each box represents the side limit of the design variable. To find a more reasonable solution, the symmetry assumption about the center line of the slider is not considered.

To carry out the statistical optimization considering the reliability of slider air bearings, numerous endeavors are needed, which consist of mechanical response analysis, reliability analysis, optimization, and their integration programs. To solve the steady state characteristics of slider air bearings, the analyzer developed by Kang et. al. (2001) is used. The SLSV method as mentioned above is employed to effectively handle the RBDO problem. The modified method of feasible directions (MMFD) of the automated design synthesis (ADS: Vanderplaats, 1985) is used as the optimization technique during the RBDO process.

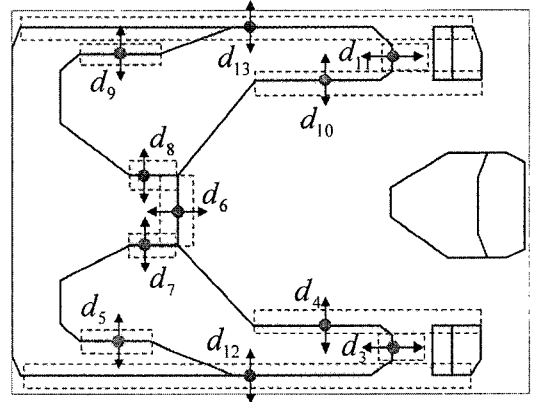
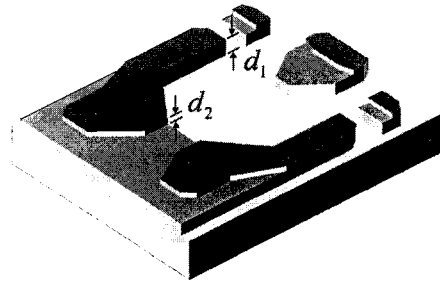


Fig. 2 Design variables and their limits

In our approach, the deterministic optimization is made first to determine the optimum design under conditions of no modeling uncertainties. Then, beginning with this solution, the RBDO is continued with the probabilistic constraints affected by random variables. Probabilistic constraints overriding the deterministic optimization constraints attempt to drive the design to a reliability solution with minimum increase in the objective.

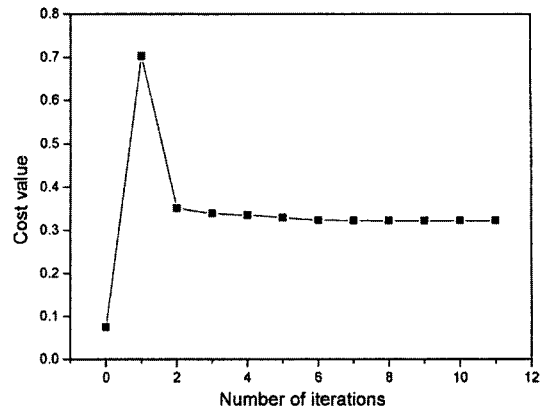
### 4. Computational Results

As the statistical properties of random variables, the standard deviations of the recess and shallow step depths are set to 0.1 and 0.06, respectively. All uncertainties about geometric variables are equally defined by the standard deviation 0.05. The statistical variation of each random variable is also approximated by normal distribution. Here, each failure probability limit  $P_{fi}^T$  ( $i=1, \dots, 4$ ) is set to 20%, which means the desired level of reliability is 80%. The target

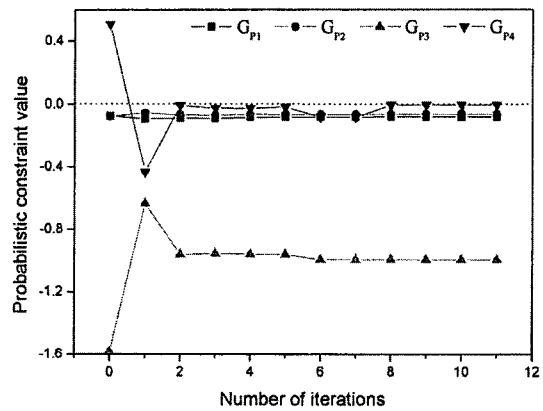
flying height  $h^*$  is set to 12nm. The lower and upper limits for the pitch angle are 170 and 250 $\mu$  rad, respectively, and for the roll angle, -10 and 10 $\mu$ rad, respectively. To solve the governing lubrication equation numerically, the control volume formulation based on the finite difference technique (Yoon et. al., 2002b) is employed likewise. More detailed operating conditions and specifications of the analysis model are the same as mentioned in Yoon et. al. (2002a).

Table 1 lists the simulation results of the RBDO in comparison with the initial design values and the results of the deterministic optimization, respectively. For the deterministic design optimization, the iteration number of the optimization process is 13 and the total number of function evaluations including the direction and line search is 214. On the other hand, the solution of the RBDO is obtained by optimization iterations of 11 and function calls of 2365, respectively. Since a real model installed in a commercialized HDD is used as the initial design, there are no significant changes from the initial design. But, it is found that the system of slider air bearings is very sensitive to geometric tolerances.

Figure 3 shows the convergence histories of the cost and probabilistic constraint values. Even though the cost value of the RBDO is somewhat



(a)



(b)

Fig. 3 Convergence history of the RBDO (a) Cost value ; and (b) Probabilistic constraint value

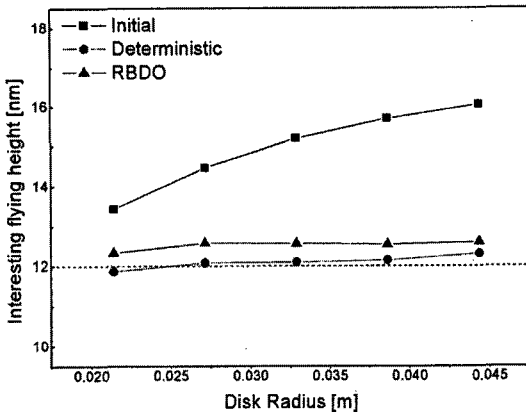
Table 1 Results of the deterministic and RBDO

Design variables	Initial	Deterministic	RBDO
$d_1[\times 10^{-6}\text{m}]$	1.524	1.764	1.782
$d_2[\times 10^{-7}\text{m}]$	1.524	1.523	1.522
$d_3[\times 10^{-4}\text{m}]$	9.083	9.084	9.084
$d_4[\times 10^{-4}\text{m}]$	1.774	1.775	1.771
$d_5[\times 10^{-4}\text{m}]$	1.376	1.376	1.375
$d_6[\times 10^{-4}\text{m}]$	3.963	3.960	3.955
$d_7[\times 10^{-4}\text{m}]$	3.871	4.065	4.066
$d_8[\times 10^{-4}\text{m}]$	5.699	5.705	5.699
$d_9[\times 10^{-4}\text{m}]$	8.871	8.874	8.884
$d_{10}[\times 10^{-4}\text{m}]$	8.172	8.340	8.298
$d_{11}[\times 10^{-4}\text{m}]$	9.083	9.083	9.083
$d_{12}[\times 10^{-4}\text{m}]$	0.484	0.508	0.508
$d_{13}[\times 10^{-4}\text{m}]$	9.570	9.500	9.515

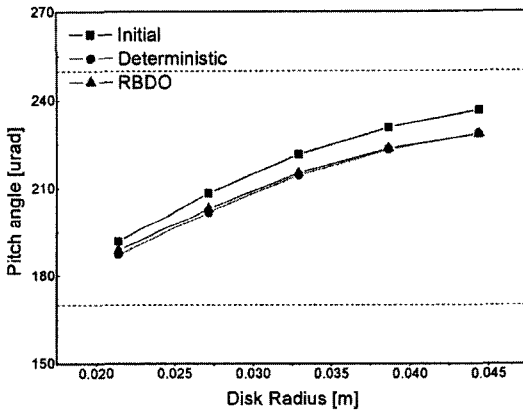
greater than that of the deterministic optimization, this result represents good convergence to the optimum solution without a constraint violation.

In order to compare the flying performances of the RBDO model with those of the initial and the deterministic ones, the variations of flying height, pitch angle, and roll angle along the disk radius are plotted in Fig. 4. Even though the largest deviation from the target flying height increase only from 0.3nm to 0.6nm, the result of the RBDO is believed to have higher confidence than that of the deterministic optimization when considering the variations of the pitch and roll angles with respect to random variables.

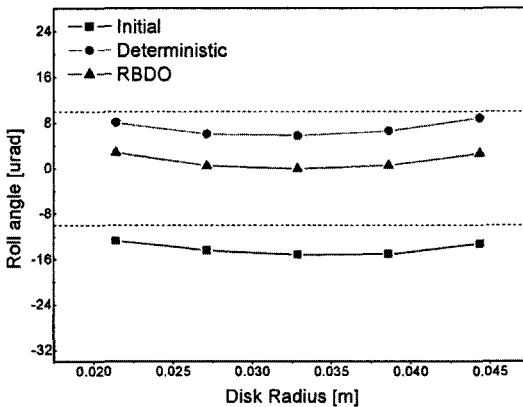
To evaluate the effectiveness of the proposed RBDO approach, the reliabilities of the con-



(a)



(b)



(c)

Fig. 4 Flying performances of the RBDO (a) Flying height; (b) Pitch angle; and (c) Roll angle

straints are directly compared with the initial and deterministic ones by using the mean value first-order second-moment (MVFO) method as listed

Table 2 Cost and reliability levels

	Initial	Deterministic	RBDO	
	MVFO	MVFO	MVFO	Monte Carlo
Cost	12.775	0.075	0.322	
Reliability of $G_{P1}$	0.9997	0.9873	0.9992	0.9990
Reliability of $G_{P2}$	0.9543	0.9821	0.9957	0.9897
Reliability of $G_{P3}$	0.2631	0.9133	0.9014	0.8862
Reliability of $G_{P4}$	0.9945	0.5299	0.8159	0.7856

in Table 2. For the deterministic optimization, the failure probability of the last constraint shows a very low confidence of 47%. That is because the maximum value of roll angles over the recording band is active in the optimization process as shown in Fig. 4(c). Even though the cost value of the RBDO increases to 0.322 from 0.075 of the deterministic optimization, the design determined by the RBDO guarantees higher confidence by considering the probable variations of design variables. The result shows that the failure probability is to be 18.4%, which means the proposed approach satisfies an 81.6% confidence level.

The Monte Carlo simulations are also performed to estimate the accuracy of our RBDO approach, where random variables are randomly generated from the 13 variables in Table 1 for each simulation cycle. The results are listed in Table 2 with those of the MVFO and their probability distributions are figured in Fig. 5. In this figure, the marked area of each figure denotes the failure probability of each probabilistic con-

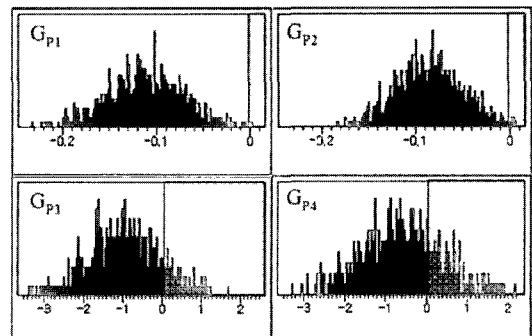


Fig. 5 Reliability distribution for probabilistic constraints

straint. Even though the last reliability level does not meet the target value exactly, it is demonstrated that the proposed approach is very efficient when considering the enormous computation time of the Monte Carlo method.

As a result, since the system of slider air bearings is very sensitive to geometric tolerances and its fabrication tolerance is relatively large to their geometric dimensions, it is necessary to apply the RBDO to the designs of head sliders like MEMS devices.

## 5. Conclusions

In modern disk drives, a recording transducer is positioned at the end of a small ceramic slider with carefully designed air bearings. This design is needed to reliably provide a physical spacing between the head's magnetic transducer and the disk down to almost 10 nanometers in today's drives. Since manufacturing tolerances of the ABS are relatively large to their geometry dimensions, it is necessary to apply the RBDO to this design problem.

In this study, the RBDO design is suggested to determine configurations of slider air bearings considering the randomness or uncertainty of the geometry of the ABS. The RBDO problem is formulated to minimize the variation in flying height from a target value while satisfying the desired probabilities keeping the pitch and roll angles within a suitable range. The proposed approach first solves the deterministic optimization problem. Then, beginning with this solution, the RBDO is continued with the probabilistic constraints affected by the random variables having a fixed standard deviation in normal distribution.

The RBDO results are directly compared with the values of the initial design and the results of the deterministic optimization, respectively. The reliability analyses are performed by the Monte Carlo method as well as the MVFO method to show the effectiveness and efficiency of the proposed approach. It is demonstrated that the proposed RBDO approach can effectively find an optimum solution satisfying all the desired

probabilistic constraints.

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