韓國經營科學會誌
第29卷 第3號
2004年 9月

공급자 주도의 동적 재고 통제와 정보 공유의 수혜적 효과 분석에 대한 연구*

김은갑** · 박찬권*** · 신기태****

Dynamic Supplier-Managed Inventory Control and the Beneficial Effect of Information Sharing*

Eun-gab Kim** · Chan-kwon Park*** · Ki-tae Shin****

■ Abstract ■

This paper deals with a supplier-managed inventory(SMI) control for a two-echelon supply chain model with a service facility and a single supplier. The service facility is allocated to customers and provides a service using items of inventory that are purchased from the supplier.

Assuming that the supplier knows the information of customer queue length as well as inventory position in the service facility at the time when it makes a replenishment decision, we identify an optimal replenishment policy which minimizes the total supply chain costs by reflecting these information into the replenishment decision.

Numerical analysis demonstrates that the SMI strategy can be more cost-effective when the information of both customer queue length and inventory position is shared than when the information of inventory position only is shared.

Keyword : Supplier-Managed Inventory, Information Technology, SCM, Markov Decision Processes

논문접수일 : 2003년 10월 19일 논문게재확정일 : 2004년 7월 20일
* 본 연구는 한국과학재단 지정 인천대학교 동북아전자물류연구센터의 지원에 의한 것임.
** 이화여자대학교 경영대학 경영학전공
*** 한양사이버대학교 e-비즈니스학과
**** 대진대학교 산업시스템공학과

1. Introduction

We consider a two-echelon supply chain that consists of a service facility and a single supplier. The service facility is allocated to customers and provides a service using items of inventory that are replenished from the outside supplier. The model presented here falls into the category of supply chain management that procures raw materials, transforms those materials into finished goods, and delivers those goods into the hands of the customer.

This paper presents an inventory management model in which the supplier takes the responsibility of the inventory at the facility and makes a replenishment decision, based on the information provided by the facility. This feature of our model is also known as vendor-managed inventory(VMI) in the literature and no VMI program can be successful without some form of information sharing between participants.

The goal of this paper is to characterize optimal properties of the supplier-managed inventory policy assuming that the supply chain considered here is a coordinated channel, that is, it aims at achieving the minimum of the total supply chain costs. The problem presented here contains a difficult, fundamental trade-off. From the viewpoint of customer service, large order replenishments might be scheduled to minimize customer service delay cost. From the viewpoint of inventory management, however, large inventories should be avoided due to inventory holding cost.

The replenishment management considered in this paper is limited not only by the rate of service process and its inherent randomness, but also by the availability of items in inventory and the replenishment process from the supplier. If inventory is depleted, service is not provided until the facility receives an inventory replenishment. To provide an incentive to process customers in the system, we include a linear cost for each unit of time customers wait in the system. By Little's law [15], this cost can be viewed as providing an incentive to minimize the weighted flow times of customers, which is one of the key factors for the performance evaluation of service facilities. We also include inventory holding cost for each unit of time that items are kept in inventory in order to avoid large inventory, and a replenishment setup cost is incurred at every instant the supplier initiates a replenishment process.

The variants of the model described above are found in Berman et al. [2], Berman and Kim [3], Berman and Sapna [4], He and Jewkes [9], He et al. [10, 11]. Unlike our model, all the paper assumes that the replenishment lead time is zero. Given that the demand, service, and replenishment processes are deterministic, Berman et al. [2] found an optimal replenishment quantity. Berman and Kim [3] studied the stochastic version of Berman et al. [2] for both the infinite and finite queueing systems. Under the condition of the instantaneous replenishment, Poisson customer arrival, and exponential service processes, they showed that the optimal policy is never to replenish either when the system is empty (no customers and no items in the system) or when the inventory level is positive. They also proved that there exists a threshold type of replenishment policy when inventory is empty. He and Jewkes [9] and He et al. [10, 11] also studied the same problem as in Berman and Kim [3] except that they did not consider a cost incurred when customers are waiting for service in queue. Assuming replenishment orders are placed when inventory becomes empty, they focused on developing an algorithm that finds an optimal replenishment quantity which is varied depending on the queue of outstanding customer orders. Berman and Sapna [4] discussed the problem of finding the optimal stocking level that minimizes the long-run expected cost of the system for the problem with Poisson arrivals and general service times with a finite waiting space and zero replenishment times.

In this paper, we consider the same problem as in Berman and Kim [3] except that replenishment processes are assumed to be probabilistic. It is important to note that this is not a minor extension since the uncertainty in order replenishment process and in demand process significantly complicates the analysis. For the sake of mathematical tractability and simplicity, we assume that replenishment times are exponentially distributed. Thus, we have a problem of scheduling the order replenishment in a *M/M/1* queueing system with unscheduled server vacations due to the stockout. Although the assumption of exponentially distributed replenishment times may be unrealistic in practice, it is our hope that the insights gained here will remain useful in addressing other systems for which exponential distributions are inappropriate.

The optimality of monotonic control policy for different problem settings has been well established from certain properties of the value function [3, 7, 8, 12, 13, 17, 18]. In this paper, we follow a similar approach, that is, the optimal replenishment policy is characterized by investigating the properties of value function.

The model presented in this paper can be applied to a work center that performs an assembly operation fed by two work streams in which one stream is continuous with Poisson arrivals and the other one is an intermittent stream originated from batch production. We can regard the replenishment quantity as the size of batch production, the replenishment lead time as batch processing time, and the replenishment setup cost as the batch production initiation cost. Obviously, the scheduling issue becomes when to initiate a batch production.

Our model can be viewed as a special case of two echelon inventory system composed of a warehouse and a single retailer [1]. Unlike the classical two echelon models, however, the service facility does not serve as a stocking location. Rather, the demand at the warehouse depends on the service process as well as the demand process at the service facility.

The paper is organized as follows. The next section provides the formulation of our Markov decision model. In Section 3, we analyze the model using dynamic programming and characterize the optimal replenishment policy. Section 4 provides the results of computational study and the last section contains conclusions.

2. Problem formulation

A service facility faces customers arriving according to a Poisson process with rate $\lambda > 0$. Each customer requires exactly one item in inventory for service. Service times provided by the service facility are independent and identically distributed (i.i.d.) exponential random variables with mean μ^{-1} and are independent of all else. Denote the service facility capacity utilization by $\rho (\rho < 1)$.

A replenishment process invoked by the supplier takes an exponential time with mean d^{-1}. A setup cost, K, is incurred at each instant a replenishment process is activated. A waiting cost is assessed at rate c_1 for each customer in queue while a holding cost is incurred at rate c_2 for each item in inventory.

A policy specifies, at each decision epoch, whether or not the supplier should replenish the service facility. Without any loss of optimality, the class of admissible strategies, is taken to be the set of non-anticipative, stationary, non-randomized, Markov policies. The set of decision epochs is assumed to be the set of customer arrival, service completion, and replenishment arrival epochs. We assume that if a replenishment process is not terminated the supplier never starts new ones.

The problem considered is a continuous time Markov decision problem (MDP). The sum of the transition rates at every state is bounded by $\gamma \equiv \lambda + \mu + d$. By following the same uniformization process [5], however, this continuous MDP can be converted into an equivalent discrete time MDP. Fictitious self-loop transitions allow each state to have the same transition rate equal to γ. In other words, the state transitions can be regarded as if they occur at a uniform rate of γ. That is, at each state, the expected transition time is constant and equals $1/\gamma$. The discount factor corresponding to this discrete time MDP is given by $\beta \equiv \gamma/(\gamma+\alpha)$ where α is a discount parameter for the continuous MDP. Without any loss of generality, it is assumed that $\gamma+\beta=1$.

We denote the customer queue length and inventory level at time t by x_1 and x_2, respectively (for convenience, index t is dropped from x_1 and x_2). A state at a decision epoch t is described by the following vector : (x_1, x_2, δ) where δ is an indicator variable which describes whether or not the supplier completes a replenishment process: if $\delta=0$, no replenishment orders are in process; if $\delta=1$, a replenishment order is in process. The state space is denoted by $\Gamma = Z^+ \times Z^+ \times \{0,1\}$. At a decision epoch t, there are two admissible actions for each state $(x_1, x_2, 0)$: *Non-replenishment* and *Replenishment*. The objective is, given the replenishment quantity Q, to identify a replenishment policy which minimizes the total expected discounted customer waiting, inventory holding, and replenishment setup cost over a horizon T.

Let $r_t(x_1, x_2, \delta)$ $(s_t(x_1, x_2, \delta))$ and $V_t(x_1, x_2, \delta)$ denote the expected discounted cost in state (x_1, x_2, δ) given that the *Non-replenishment (Replenishment)* and the optimal action, respectively, is taken at t. Define

$$D(x, x_2) = \begin{cases} (x_1-1, x_2-1) & \text{if } x_1>0 \text{ and } x_2>0 \\ (x_1, x_2) & \textit{otherwise.} \end{cases}$$

With the initial condition $V_0(x_1, x_2, \delta)=0$, the dynamic programming(DP) recursive equation of the discrete time model over the horizon of T periods is given as follows : For $t=1,\cdots,T$,

$$V_t(x_1, x_2, \delta) = \begin{cases} \min r_t(x_1, x_2, \delta), s_t(x_1, x_2, \delta) & \text{if } \delta=0 \\ r_t(x_1, x_2, \delta) & \text{if} \delta=1 \end{cases}$$

where

$$\begin{aligned} r_t(x_1, x_2, 0) &= c_1x_1 + c_2x_2 + \lambda V_t(x_1+1, x_2, 0) \\ &\quad + \mu V_t(D(x_1, x_2), 0) + dV_t(x_1, x_2, 0), \\ r_t(x_1, x_2, 1) &= c_1x_1 + c_2x_2 + \lambda V_t(x_1+1, x_2, 1) \\ &\quad + \mu V_t(D(x_1, x_2), 1) + dV_t(x_1, x_2+Q, 0), \\ s_t(x_1, x_2, 0) &= K + r_t(x_1, x_2, 1). \end{aligned}$$

Note that since Replenishment action is allowed in only states with $\delta=0$, $s_t(x_1, x_2, 1)$, is not considered.

3. The optimal replenishment policy

In this section, we characterize the optimal replenishment policy for minimizing discounted costs over a horizon T. In particular, we show that there exists a monotonic threshold reorder point for the optimal replenishment decision. Define

$$\Delta_1 V_t(x_1, x_2, \delta) = V_t(x_1+1, x_2, \delta) - V_t(x_1, x_2, \delta)$$
$$\Delta_2 V_t(x_1, x_2, \delta) = V_t(x_1, x_2+1, \delta) - V_t(x_1, x_2, \delta),$$
$$\Delta_{11} V_t(x_1, x_2, \delta) = V_t(x_1+1, x_2+1, \delta) - V_t(x_1, x_2, \delta)$$

In a similar way, we define $\Delta_1 r_t(x_1, x_2, \delta)$, $\Delta_1 s_t(x_1, x_2, \delta)$, $\Delta_2 r_t(x_1, x_2, \delta)$, $\Delta_2 s_t(x_1, x_2, \delta)$, $\Delta_{11} r_t(x_1, x_2, \delta)$, and $\Delta_{11} s_t(x_1, x_2, \delta)$. The operator Δ_1 is the marginal cost of holding one more customer in queue while the operator Δ_2 is the marginal cost of holding one more item in inventory. The operator Δ_{11} is the marginal cost of holding both one more item in inventory and one more customer in queue.

Numerical investigation shows that $\Delta_1 V_t(x_1, x_2, \delta)$ is not increasing in x_1 and $\Delta_2 V_t(x_1, x_2, \delta)$ is not increasing in x_2 In other words, $V_t(x_1, x_2, \delta)$ is neither convex in x_1 nor convex in x_2. We also found that $\Delta_1 V_t(x_1, x_2, \delta)$ is not decreasing in x_2, that is, $V_t(x_1, x_2, \delta)$ is not submodular. Therefore, general results about minimization of submodular functions on the lattice([16]) cannot be applied here to obtain the optimality of the monotonic threshold replenishment policy.

The following theorem presents properties of $\Delta_{11} V_t$. The first property says that V_t is increasing in x_1 and x_2. That is, it is cost effective to serve customers when items are available in inventory. The second property states that the marginal cost incurred when holding one more customer and one more item in the system is larger when a replenishment order is in process than when no replenishment orders are in process. In particular, if the supplier does not place a replenishment order at state $(x_1, x_2, 0)$, Property (ii) states that

$$r_t(x_1+1, x_2+1, 0) - s_t(x_1+1, x_2+1, 0) \le r_t(x_1, x_2, 0) - s_t(x_1, x_2, 0) < 0$$

which means that if it is optimal not to replenish at state $(x_1, x_2, 0)$, it is also optimal not to replenish at state $(x_1+1, x_2+1, 0)$. The third property states that the marginal cost $\Delta_{11} V_t$ is larger when the inventory has Q more items.

Theorem1 : For $t=0, \cdots, T$ and $x_1 \ge 0$, $x_2 \ge 0$,

(i) $\Delta_{11} V_t(x_1, x_2, \delta) \ge 0, \delta = 0, 1,$

(ii) $\Delta_{11} V_t(x_1, x_2, 0) \le \Delta_{11} V_t(x_1, x_2, 1),$

(iii) $\Delta_{11} V_t(x_1, x_2, \delta) \le \Delta_{11} V_t(x_1, x_2+Q, \delta).$

Proof : See the Appendix.

The following theorem analyzes the marginal cost of the t-stage optimal value function V_t.

Theorem 2 : For $t=0, \cdots, T$, $x_1 \ge 0$, $x_2 \ge 0$, and $\delta = 0, 1$,

(i) $\Delta_1 r_t(x_1, x_2, 0) \ge \Delta_1 s_t(x_1, x_2, 0),$

(ii) $\Delta_2 r_t(x_1, x_2, 0) \le \Delta_2 s_t(x_1, x_2, 0),$

(iii) $\Delta_1 V_t(x_1, x_2, 0) \ge \Delta_1 V_t(x_1, x_2, 1),$

(iv) $\Delta_2 V_t(x_1, x_2, 0) \le \Delta_2 V_t(x_1, x_2, 1)$,

(v) $\Delta_1 V_t(x_1, x_2, \delta) \ge \Delta_1 V_t(x_1, x_2 + Q, \delta)$,

(vi) $\Delta_2 V_t(x_1, x_2, \delta) \le \Delta_2 V_t(x_1, x_2 + Q, \delta)$,

(vii) $\Delta_1 V_t(x_1, x_2, \delta) \le \Delta_1 V_t(x_1 + 1, x_2 + 1, \delta)$,

(viii) $V_t(x_1 + 1, x_2, 1) - V_t(x_1, x_2, 0) \le$
$V_t(x_1 + 2, x_2 + 1, 1) - V_t(x_1 + 1, x - 2 + 1, 0)$.

Proof : See the Appendix.

Property (i) of Theorem 2 states that the marginal cost of holding an additional customer in queue at time t is smaller for action *Replenishment* than for action *Non-replenishment*. If the supplier places a replenishment order at state $(x_1, x_2, 0)$, then Property (i) gives

$$s_t(x_1 + 1, x_2, 0) - r_t(x_1 + 1, x_2, 0) \\ \le s_t(x_1, x_2, 0) - r_t(x_1, x_2, 0) < 0.$$

Thus, it is also optimal to replenish with $x_1 + 1$ customers, which guarantees that a threshold function exists and is optimal.

Property (ii) of Theorem 2 states that the marginal cost of holding an additional item in inventory is smaller for the action *Non-replenishment* than for the action *Replenishment*. If the supplier does not places a replenishment order at state $(x_1, x_2, 0)$, then Property (ii) implies

$$s_t(x_1, x_2 + 1, 0) - r_t(x_1, x_2 + 1, 0) \\ \ge s_t(x_1, x_2, 0) - r_t(x_1, x_2, 0) > 0.$$

Therefore, if it is optimal not to replenish with x_2 items in inventory, then it is also optimal not to replenish with $x_2 + 1$ items, which establishes that the threshold function is monotonically increasing in x_2.

Properties (iii) and (v) say that the incremental cost of holding one more customer in queue is greater when a replenishment order is in process than when no replenishment orders are placed, and greater when a replenishment order arrives at the service facility than when a replenishment order is in process, respectively. Properties (iv) and (vi) state that the incremental cost of holding one more item in inventory is smaller when a replenishment order is in process than when no replenishment orders are placed, and smaller when a replenishment order arrives at the service facility than when a replenishment order is in process, respectively. These properties imply that as the replenishment process evolves, $\Delta_1 V_t$ is increasing but $\Delta_2 V_t$ is decreasing. These results are straightforward because the larger inventory means less customer waiting and production delays but extra payment of holding costs.

Property (vii) can be rewritten as

$$V_t(x_1 + 2, x_2 + 1, \delta) + V_t(x_1, x_2, \delta) \\ \ge V_t(x_1+1, x_2+1, \delta) + V_t(x_1+1, x_2, \delta),$$

which can be interpreted as a diagonal dominance (see Ha(1997) for terminology).

Now we can state the main result for the problem.

Theorem 3 : A T-stage optimal replenishment policy can be characterized by the threshold function $\Theta_t(x_2)$, increasing in x_2, such that the supplier should replenish the service facility if $x_1 \ge \Theta_t(x_2)$ where

$$\Theta_t(x_2) := \min\{x_1 \in \{0, 1, \cdots, \infty\} : \\ s_t(x_1, x_2, 0) < r_t(x_1, x_2, 0)\}$$

Proof : We first show the optimality of a threshold property. Suppose $r_t(\Theta_t(x_2) + 1, x_2, 0) < s_t(\Theta$

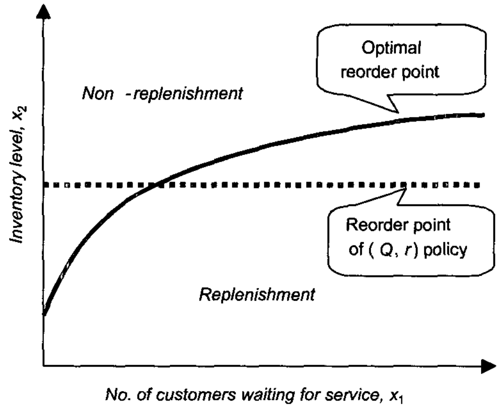
$t(x_2)+1, x_2, 0)$. From the definition of $\Theta_t(x_2)$, we have $r_t(\Theta_t(x_2), x_2, 0) > s_t(\Theta_t(x_2), x_2, 0)$. It follows that $\Delta_1 s_t(\Theta_t(x_2), x_2, 0) > \Delta_1 r_t(\Theta_t(x_2), x_2, 0)$. However, this is a contradiction by (i), Theorem 2.

To show $\Theta_t(x_2)$ is increasing in x_2, suppose $\Theta_t(x_2) > \Theta_t(x_2+1)$. Then, we have $s_t(\Theta_t(x_2+1), x_2, 0) > r_t(\Theta_t(x_2+1), x_2, 0)$ and $s_t(\Theta_t(x_2+1), x_2+1, 0) < r_t(\Theta_t(x_2+1), x_2+1, 0)$. It follows that $\Delta_2 s_t(\Theta_t(x_2+1), x_2, 0) < \Delta_2 r_t(\Theta_t(x_2+1), x_2, 0)$. This is a contradiction by (ii), Theorem 2.

Theorem 4 : The optimality of a monotonic threshold ordering policy holds for the infinite horizon problem.

Proof : Let $V^*(x_1, x_2, \delta)$ be the optimal discounted cost over the infinite horizon, given state (x_1, x_2, δ). Then, the T-stage optimal cost function, V_T, converges to V^*, that is, $V^*(x_1, x_2, \delta) = \lim T \to \infty V_T(x_1, x_2, \delta)$, because the number of actions admissible for every $(x_1, x_2, \delta) \in \Gamma$ is two (see Propositions 12 and 13, Chapter 5 of [5]). Therefore, Theorems 1 and 2 hold for the infinite hozrizon.

The optimality of a threshold replenishment policy holds for the average cost problem. This can be proven by showing that the Cavazos-Cadena axioms [6] are satisfied here. This is not difficult to prove but rather messy and thus omitted from the paper. Using [16], the expected average cost optimal stationary policy has the same properties as the discounted cost optimal stationary policy. [Figure 1] graphically compares the optimal reorder function defined in Theorem 3 with the reorder point of (Q, r) policy.



[Figure 1] Comparison of optimal reorder function and reorder point of (Q, r) policy

4. Numerical results

In this section we evaluate the performance of the optimal policy. The magnitude of the state space, however, may make it prohibitive to find the optimal policy using the successive approximation method [5]. One way to resolve this problem is simply to truncate the customer queue length and inventory level. Even though the average cost computed by this truncation is not optimal, it can be proven that it is a lower bound on the optimal performance, using a similar argument as in Theorem 1 of Kim and Van Oyen [14].

In order to show the benefit of information sharing between the service facility and supplier, we also consider a SMI strategy which uses the information of inventory position only, that is, (Q, r) policy. The average costs of (Q, r) policy are computed by successive approximation with the same truncation level as in evaluating the optimal policy. When implementing (Q, r) policy, the reorder point r is set to the sum of the average demand during replenishment process, λ / d, and the safety stock.

For the numerical test, the size of customers waiting for service, x_1 and inventory level, x_2, are truncated to 30 and 60, respectively. In the following tables, *Diff* is defined as the change in percent of the cost given by the (Q, r) policy to the optimal cost. The successive approximation code is written in C. We use the stopping rule given by Proposition 7, Ch 7 of Bertsekas([5]) and the termination criterion, ϵ, is set to 10^{-2}.

In the following tables, Q^* and J^* represent the optimal replenishment quantity and optimal average cost corresponding to Q^*. To find Q^* and J^*, we first compute the optimal average cost $J(Q)$ given Q using value iteration. Then, varying Q, we find Q^* which minimizes $J(Q)$. Numerical investigation for a variety of examples indicates that $J(Q)$ may be convex with respect to Q even though we could not prove it. Similarly, Q^h and J^h represent the replenishment quantity that achieves the minimum cost under (Q, r) policy and the average cost corresponding to Q^h.

Examples 1-12 in <Table 1> are designed to investigate the impact of server utilization on the performance of the optimal policy. Each of 3 example sets has 4 different arrival rates : λ = 0.3, 0.5, 0.7, 0.9, For each example set, three types of replenishment time rates are tested : d = 0.1, 0.3, 1.0.

It is intractable to find the optimal reorder point in (Q, r) policy for our model. In implementing (Q, r) policy, we tested it varying the safety stock level and report test results of when it is set to 20% of λ / d for $\rho = 0.3, 0.5$ and 40% of λ / d for $\rho = 0.7, 0.9$. We observed a similar result for cases with the different safety stock levels other than this. For the accurate comparison of the optimal policy and (Q, r) policy, the numerical search of Q and r which guarantee the best performance of (Q, r) policy is necessary for each example, which is computationally very time consuming. For this reason, we note that the performance of the optimal policy over (Q, r) policy might be overestimated in our numerical test which finds the optimal quantity given the safety stock level in implementing (Q, r) policy.

⟨Table 1⟩ Performance evaluation of the optimal policy

Example	K	c_1	c_2	λ	μ	d	Q^*	J^*	Q^h	J^h	*Diff*
1	100	4	1	0.3	1	0.1	12	12.981	12	13.345	2.80
2				0.5			17	22.243	19	22.993	3.37
3				0.7			21	36.609	29	38.772	5.91
4				0.9			25	70.113	45	74.447	6.18
5				0.3		0.3	9	9.949	8	10.587	6.41
6				0.5			12	15.894	12	16.407	3.23
7				0.7			15	25.713	16	26.745	4.01
8				0.9			17	53.999	28	59.207	9.64
9				0.3		1	8	9.337	8	9.857	5.57
10				0.5			11	14.344	11	14.706	2.52
11				0.7			13	22.513	15	23.579	4.74
12				0.9			15	47.011	20	50.828	8.12

Examples 1-12 in <Table 2> investigates the impact of the inventory holding cost on the optimal performance. Each of 3 example sets has 4 different ratios of $c_1/c_2 = 1, 2, 3, 4$. For each example set, 4 types of replenishment time rates are tested : d = 0.1, 0.2, 0.3, 0.4. In <Table 3>, we test how the optimal performance is changed as a function of the replenishment setup cost. Each of 3 example sets has 4 different replenishment setup costs : K = 50, 100, 200, 400.

〈Table 2〉 Effect of inventory holding cost on the optimal performance

Example	K	c_1	c_2	λ	μ	d	Q^*	J^*	Q^h	J^h	*Diff*
1	100	1	1	0.6	1	0.1	18	17.208	16	18.235	5.97
2		2					18	21.773	19	22.590	3.75
3		3					19	25.339	22	26.349	3.99
4		4					19	28.410	24	29.795	4.88
5		1				0.3	13	13.118	12	14.008	6.78
6		2					13	15.688	13	16.248	3.57
7		3					13	17.900	14	18.430	2.96
8		4					13	19.964	14	20.564	3.01
9		1				1	12	12.248	11	12.819	4.66
10		2					12	14.245	12	14.609	2.56
11		3					12	16.005	12	16.387	2.39
12		4					12	17.702	12	18.165	2.62

〈Table 3〉 Effect of replenishment cost on the optimal performance

Example	K	c_1	c_2	λ	μ	d	Q^*	J^*	Q^h	J^h	*Diff*
1	50	3	1	0.5	1	0.1	15	18.333	16	19.164	4.53
2	100						16	19.943	17	20.650	3.55
3	200						20	22.671	20	23.357	3.03
4	400						25	27.037	24	27.983	3.50
5	50					0.3	9	12.077	9	12.594	4.28
6	100						12	14.458	11	15.074	4.26
7	200						16	17.966	15	18.878	5.08
8	400						22	23.214	21	24.497	5.53
9	50					1	8	10.472	8	10.834	3.46
10	100						11	13.205	11	13.566	2.73
11	200						15	17.014	15	17.555	3.18
12	400						21	22.530	21	23.292	3.38

The average sub-optimality of (Q, r) policy for 36 examples in <Table 1>~<Tales 3> is 4.34%. This result clearly implies that incorporating customer processing data, x_1, into a replenishment decision contributed to cutting costs. <Table 1> shows that the worst performance of (Q, r) policy compared to the optimal policy is found in $\lambda = 0.9$, i.e., the highest capacity utilization. It also says that the best and second best performance are found in $\lambda = 0.5$ and $\lambda = 0.7$, respectively. This result indicates that the value of information sharing is more obvious when the capacity utilization is within a moderate range.

The results in <Table 2> state that when $c_1 = c_2$, the optimal policy performs much better than (Q, r) policy. The intuition behind this is as follows. When $c_1 = c_2$, the optimal reorder point curve shifts right to avoid frequent replenishment orders (see [Figure 1]) because the service delay is not expensive. That is, unless enough customers are accumulated, the optimal policy restricts the action of Replenishment while (Q, r) policy allows it below the fixed reorder point r regardless of the number of customers waiting for service. Therefore, when $c_1 = c_2$, the dynamic reorder point given by the optimal policy greatly contribute to reducing system costs in the region below the fixed reorder point r.

From <Table 3>, the sub-optimality of (Q, r) policy seems to be not affected by the replenishment setup cost. It is also interesting to see from <Table 1>~<Table 3> that even though (Q, r) policy is implemented, the order quantity Q and reorder point r should be chosen depending on the system parameters in order to achieve the best performance, which means (Q, r) policy requires almost the same effort as the optimal policy in terms of computational complexity.

Based on the computational results, we also observe the following monotonic behavior of the optimal performance with respect to system parameters :

- If other system parameters are the same, the optimal order quantity Q^* and the optimal cost J^* are non-decreasing in λ, that is, ρ.
- If other system parameters are the same, the optimal order quantity Q^* and the optimal cost J^* are non-increasing in d.
- If other system parameters are the same, Q^* and J^* are non-decreasing in c_1.
- If other system parameters are the same, Q^* and J^* are non-decreasing in K.

5. Conclusion

In this paper, we studied a supplier managed inventory management problem at a service facility that provide a service for customers using items purchased from the supplier. This paper focused on the issue of how the information shared between these two companies should be reflected into the inventory control for SMI to be a more cost-effective inventory management strategy.

We assumed that the supplier knows the information of the customer queue length as well as the inventory position in the service facility at the time when it makes a replenishment decision. Using the Markov decision theory, we proved that the replenishment decision at the supplier is justified only when the inventory level at the service facility exceeds a threshold value which is increasing as the number of customers waiting for service increases. In this context our SMI policy can be viewed as a dynamic version of (Q, r) policy.

We implemented a numerical analysis for two types of the SMI strategy : the one uses the information of both customer queue length and inventory position, dynamic SMI, and the other uses the information of inventory position only, that is, (Q, r) policy.

Compared to (Q, r) policy, the dynamic SMI policy has a varying reorder point depending on the number of customers waiting for service. Hence, it is a reasonable thought that the dynamic SMI policy can be more adaptive with respect to the change in system parameters, which leads to the cost savings to both the service facility and the supplier. Numerical investigation with designed examples confirmed this conjecture. The dynamic SMI policy outperformed (Q, r) policy by 4.34% on the average.

For the effective inventory management in the supply chain, the information sharing among participants through the information system or strategical alignment is still an important issue. In the other direction, the result of this paper suggests that the appropriate use of the shared information into the inventory control should be another important consideration.

REFERENCES

[1] Axsäter, A., "Continuous review policies for multi-level inventory systems with stochastic demand," *Handbooks in OR & MS*, Vol.4(1993), pp.175-197.

[2] Berman, O., E. Kaplan, and D. G. Shimshak, "Deterministic approximations for inventory management at service facilities," *IIE Transactions*, Vol.25, No.5(1993), pp.98-104.

[3] Berman, O. and E. Kim, "Stochastic scheduling policies for inventory management at service facilities," *Stochastic Models*, Vol. 15, No.4(1999), pp.695-718.

[4] Berman, O. and K.P. Sapna, "Inventory l

[5] Bertsekas, D.P., *Dynamic Programming : Deterministic and Stochastic Models*, Prentice-Hall, Englewood Cliffs, 1987.

[6] Cavazos-Cadena, R. and L.I. Sennott, "Comparing recent assumptions for the existence of average optimal stationary policies," *Operations Research Letters*, Vol.11, No.1(1992), pp.33-37.

[7] Glasserman, P. and D. Yao, "Monotone optimal control of permutable GSMPs," *Mathematics of Operations Research*, Vol.19 (1994), pp.449-476.

[8] Ha, A., "Optimal Dynamic Scheduling Policy for a Make-to-Stock Production System," *Operations Research*, Vol.45, No.1 (1997), pp.42-53.

[9] He, Q. and E. Jewkes, "Performance Measures of a Make-to-Order Inventory Production System," *IIE Transactions*, Vol.32, No.5(2000), pp.409-419.

[10] He, Q., E.M. Jewkes and J. Buzacott, "Optimal and near-optimal inventory control policies for a make-to-order inventory-production system," *European Journal of Operational Research*, Vol.141(2002), pp.113-132.

[11] He, Q., E.M. Jewkes, and J. Buzacott, "The value of information used in inventory control of a make-to-order inventory-production system," *IIE Transactions*, Vol.34 (2002), pp.999-1013.

[12] Kim, E. and M.P. Van Oyen, "Beyond the $c\mu$ rule : Dynamic scheduling of a two-class loss queue," *Mathematical Methods of Operations Research*, Vol.48, No.1(1998a), pp.17-36.

[13] Kim, E. and M.P. Van Oyen, "Dynamic

scheduling to minimize lost sales subject to set-up costs," *Queueing Systems - Theory and Applications*, Vol.29, No.2-4 (1998b), pp193-229.

[14] Kim, E. and M.P. Van Oyen, "Finite-capacity multi-class production scheduling with set-up times," *IIE transactions*, Vol.32, No.9(2000), pp.807-818.

[15] Ross, S.M., *Stochastic Processes*, Wiley, New York, 1983.

[16] Sennott, L.I., "Average cost optimal stationary policies in infinite state Markov decision processes with unbounded costs," *Operations Research*, Vol.37, No.4(1989), pp.626-633.

[17] Topkis, D.M., "Minimizing a submodular function on a lattice," *Operations Research*, Vol.26, No.2(1978), pp.305-321.

[18] Veatch, M. and L. Wein, "Monotonic Control of Queueing Networks," *Queueing Systems - Theory and Applications*, Vol.12(1992), pp.391-408.

APPENDIX

Proof of Theorem 1 : We use induction. The case for $t=0$ is trivial. Assume that (i)~(iii) hold up to t. Consider period $t+1$.

(i) $\Delta_{11}V_{t+1}(x_1, x_2, \delta) \geq 0$: The inequality easily follows by $c_1, c_2 \geq 0$ and induction.

(ii) $\Delta_{11}V_{t+1}(x_1, x_2, 0) \leq \Delta_{11}V_{t+1}(x_1, x_2, 1)$: We focus on combinations of actions in state $(x_1, x_2, 0)$ and $(x_1+1, x_2+1, 0)$. Denote by $(r/s, r/s)$ the optimal policy in these states where r and s represent *Non-replenishment* and *Replenishment* actions, respectively. By (ii) at t, case (r/s) is excluded. For (r/r),

$$\Delta_{11}r_{t+1}(x_1, x_2, 0) - \Delta_{11}V_{t+1}(x_1, x_2, 1) = \lambda[\Delta_{11}V_t(x_1+1, x_2, 0) - \Delta_{11}V_t(x_1+1, x_2, 1)]$$
$$+\mu[\Delta_{11}V_t(D(x_1, x_2), 0) - \Delta_{11}V_t(D(x_1, x_2), 1)]\,1\{x_1>0\ ,\ x_2>0\}$$
$$+d[\,V_t(\Delta_{11}V_t(x_1, x_2, 0) - \Delta_{11}V_t(x_1, x_2+Q, 0)] \leq 0$$

where $1\{A\}$ is the function that has a value of 1 if the event A occurs and 0 if not. The inequality holds by induction to terms corresponding to λ and μ and by (iii) to term corresponding to d. For (s, s), $\Delta_{11}s_{t+1}(x_1, x_2, 0) - \Delta_{11}V_{t+1}(x_1, x_2, 1) = 0$ by the definition of value functions. Case (s, r) follows by case (s, s) because $r_{t+1}(x_1+1, x_2+1, 0) - s_{t+1}(x_1, x_2, 0) \leq \Delta_{11}s_{t+1}(x_1, x_2, 0)$.

(iii) $\Delta_{11}V_{t+1}(x_1, x_2, \delta) \leq \Delta_{11}V_{t+1}(x_1, x_2+Q, \delta)$: By (ii), it is sufficient to show that

$$\Delta_{11}V_{t+1}(x_1, x_2, 1) - \Delta_{11}V_{t+1}(x_1, x_2+Q, 0) \leq 0.$$

Assume that the above equation holds at t. We focus on combinations of actions in state $(x_1, x_2+Q, 0)$ and $(x_1+1, x_2+Q+1, 0)$. By (ii), the case (r, s) is excluded. For (r, r),

$$\Delta_{11}V_{t+1}(x_1, x_2, 1) - \Delta_{11}r_{t+1}(x_1, x_2+Q, 0) = \lambda[\Delta_{11}V_t(x_1+1, x_2, 1) - \Delta_{11}V_t(x_1+1, x_2+Q, 0)]$$
$$+\mu[\Delta_{11}V_t(D(x_1, x_2), 1) - \Delta_{11}V_t(D(x_1, x_2+Q), 0)] \leq 0$$

The inequality of the term corresponding to λ follows by induction. The term corresponding to μ follows by induction if $x_1>0$ and $x_2>0$ and is zero if $x_1=0$ and $x_2 \geq 0$. If $x_1>0$ and $x_2=0$, it becomes $-V_t(x_1, Q, 0) + V_t(x_1-1, Q-1, 0)$ which is less than zero by (i). For (s, s), using (ii), the inequality of terms corresponding to λ and μ can be shown by the same argument as in case (r/r). The term corresponding to d becomes $\Delta_{11}V_{t+1}(x_1, x_2+Q, 0) - \Delta_{11}V_{t+1}(x_1, x_2+2Q, 0)$ which is less than zero by (ii) and induction. It can be easily checked that case (s, r) follows by case (r/r).

Proof of Theorem 2 : We use induction. The case for t = 0 is trivial. Assume that (i)~(viii) hold up to t. Consider period $t+1$.

(i) $\Delta_1 r_{t+1}(x_1, x_2, 0) \geq \Delta_1 s_{t+1}(x_1, x_2, 0)$:

$$\Delta_1 r_{t+1}(x_1, x_2, 0) - \Delta_1 s_{t+1}(x_1, x_2, 0)$$
$$= \lambda[\Delta_1 V_t(x_1+1, x_2, 0) - \Delta_1 V_t(x_1+1, x_2, 1)] + \mu[\Delta_1 V_t(D(x_1, x_2), 0) - \Delta_1 V_t(D(x_1, x_2), 1)]$$
$$+ d[\Delta_1 V_t(x_1, x_2, 0) - \Delta_1 V_t(x_1, x_2+Q, 0)] \geq 0.$$

The terms corresponding to λ and d follow by (iii) and (v), respectively. If $x_1 = 0$ and $x_2 > 0$, the term corresponding to μ becomes

$$V_t(0, x_2-1, 0) - V_t(0, x_2, 0) - (V_t(0, x_2-1, 1) - V_t(0, x_2, 1))$$
$$= \Delta_2 V_t(0, x_2-1, 1) - \Delta_2 V_t(0, x_2-1, 0) \geq 0 \qquad \text{(by (iv))}.$$

Otherwise, it follows by (iii).

(ii) $\Delta_2 r_{t+1}(x_1, x_2, 0) \leq \Delta_2 s_{t+1}(x_1, x_2, 0)$:

$$\Delta_2 r_{t+1}(x_1, x_2, 0) - \Delta_2 s_{t+1}(x_1, x_2, 0)$$
$$= \lambda[\Delta_2 V_t(x_1+1, x_2, 0) - \Delta_2 V_t(x_1+1, x_2, 1)] + \mu[\Delta_2 V_t(D(x_1, x_2), 0) - \Delta_2 V_t(D(x_1, x_2), 1)]$$
$$+ d[\Delta_2 V_t(x_1, x_2, 0) - \Delta_2 V_t(x_1, x_2+Q, 0)] \leq 0.$$

The terms corresponding to λ and d follow by (iv) and (vi), respectively. If $x_1 > 0$ and $x_2 = 0$, the term corresponding to μ becomes

$$V_t(x_1-1, 0, 0) - V_t(x_1, 0, 0) - (V_t(x_1-1, 0, 1) - V_t(x_1, 0, 1))$$
$$= \Delta_1 V_t(x_1-1, 0, 1) - \Delta_1 V_t(x_1-1, 0, 0) \leq 0 \qquad \text{(by (iii))}.$$

Otherwise, it follows by (iv).

(iii) $\Delta_1 V_{t+1}(x_1, x_2, 0) \geq \Delta_1 V_{t+1}(x_1, x_2, 1)$: We focus on combinations of actions in states $(x_1, x_2, 0)$ and $(x_1+1, x_2, 0)$. By (i), the combination (s, r) is excluded. For (r, r), the proof is the same as (i). If (s, s), $\Delta_1 s_{t+1}(x_1, x_2, 0) - \Delta_1 V_{t+1}(x_1, x_2, 1) = 0$ by the definition of value function. Case (r, s) holds by case (s, s) because $s_{t+1}(x_1+1, x_2, 0) - r_{t+1}(x_1, x_2, 0) \geq \Delta_1 s_{t+1}(x_1, x_2, 0)$.

(iv) $\Delta_2 V_{t+1}(x_1, x_2, 0) \leq \Delta_2 V_{t+1}(x_1, x_2, 1)$: A similar argument as in (iii) can be applied here.

(v) $\Delta_1 V_{t+1}(x_1, x_2, \delta) \geq \Delta_1 V_{t+1}(x_1, x_2+Q, \delta)$: Using (iii), it is sufficient to show that

$$\Delta_1 V_{t+1}(x_1, x_2, 1) \geq \Delta_1 V_{t+1}(x_1, x_2+Q, 0).$$

Thus, we focus on the combinations of actions in $(x_1, x_2+Q, 0)$ and $(x_1+1, x_2+Q, 0)$. By (i), the case (s, r) is excluded. For (r, r),

$$\Delta_1 V_{t+1}(x_1, x_2, 1) - \Delta_1 r_{t+1}(x_1, x_2+Q, 0) = \lambda[\Delta_1 V_t(x_1+1, x_2, 1) - \Delta_1 V_t(x_1+1, x_2+Q, 0)]$$
$$+ \mu[\Delta_1 V_t(D(x_1, x_2), 1) - \Delta_1 V_t(D(x_1, x_2+Q), 0)] \geq 0.$$

The term corresponding to λ follows by induction. The term corresponding to μ follows by induction if $x_1 > 0$ and $x_2 > 0$. If $x_1 = 0$ and $x_2 > 0$.

$$V_t(0, x_2-1, 1) - V_t(0, x_2, 1) - (V_t(0, x_2+Q-1, 0) - V_t(0, x_2+Q, 0))$$
$$= \Delta_2 V_t(0, x_2+Q-1, 0) - \Delta_2 V_t(0, x_2-1, 1) \geq 0 \qquad \text{(by (vi))}.$$

If $x_1 > 0$ and $x_2 = 0$, it becomes

$$\Delta_1 V_t(x_1, 0, 1) - \Delta_1 V_t(x_1-1, Q-1, 0) \geq \Delta_1 V_t(x_1, 0, 1) - \Delta_1 V_t(x_1, Q, 0) \text{(by (vii))}.$$
$$\geq 0 \text{ (by induction)}.$$

Finally, if $x_1=0$ and $x_2=0$,

$$\Delta_1 V_t(0,0,1)-(V_t(0,Q-1,0)-V_t(0,Q,0))$$
$$\geq \Delta_1 V_t(0,0,1)-(V_t(1,Q,0)-V_t(0,Q,0))\text{(by (i) of Theorem 1)}$$
$$=\Delta_1 V_t(0,0,1)-\Delta_1 V_t(0,Q,0)\geq 0 \text{ (by induction)}$$

Case (s,s) follows by case (r,r) because

$$\Delta_1 V_{t+1}(x_1,x_2,1)-\Delta_1 s_{t+1}(x_1,x_2+Q,0)\geq \Delta_1 V_{t+1}(x_1,x_2,1)-\Delta_1 r_{t+1}(x_1,x_2+Q,0)\text{(by (i))}.$$

Case (r,s) holds by case (r,r) because

$$s_{t+1}(x_1+1,x_2+Q,0)-r_{t+1}(x_1,x_2+Q,0)\leq \Delta_1 r_{t+1}(x_1,x_2+Q,0).$$

(vi) $\Delta_2 V_{t+1}(x_1,x_2,\delta)\leq \Delta_2 V_{t+1}(x_1,x_2+Q,\delta)$: Using (iv), it is sufficient to show that

$$\Delta_2 V_{t+1}(x_1,x_2,1)\leq \Delta_2 V_{t+1}(x_1,x_2+Q,0).$$

Thus, we focus on the combinations of actions in $(x_1,x_2+Q,0)$ and $(x_1,x_2+Q+1,0)$. By (ii), case (r,s) is excluded. For (r,r),

$$\Delta_2 V_{t+1}(x_1,x_2,1)-\Delta_2 r_{t+1}(x_1,x_2+Q,0)$$
$$=\lambda[\Delta_2 V_t(x_1+1,x_2,1)-\Delta_2 V_t(x_1+1,x_2+Q,0)]$$
$$+\mu[\Delta_2 V_t(D(x_1,x_2),1)-\Delta_2 V_t(D(x_1,x_2+Q),0)]\leq 0.$$

The term corresponding to λ follows by induction and the term corresponding to d is zero. If $x_1>0$ and $x_2=0$, the term corresponding to μ becomes

$$V_t(x_1-1,0,1)-V_t(x_1,0,1)-\Delta_2 V_t(x_1-1,Q-1,0)$$
$$\leq V_t(x_1-1,0,1)-V_t(x_1,0,1)-(V_t(x_1-1,Q,0)-V_t(x_1,Q,0))\text{ (by (i) of Theorem 1)}$$
$$=\Delta_1 V_t(x_1-1,Q,0)-\Delta_1 V_t(x_1-1,0,1)\leq 0 \qquad \text{(by (v))}.$$

Otherwise, it follows by induction. Case (s,s) holds by case (r,r) because

$$\Delta_2 V_{t+1}(x_1,x_2,1)-\Delta_2 s_{t+1}(x_1,x_2+Q,0)\leq \Delta_2 V_{t+1}(x_1,x_2,1)-\Delta_2 r_{t+1}(x_1,x_2+Q,0)\text{ (by (ii))}.$$

Finally, case (s,r) holds by case (r,r) because

$$r_{t+1}(x_1,x_2+Q+1,0)-s_{t+1}(x_1,x_2+Q,0)\geq \Delta_2 r_{t+1}(x_1,x_2+Q,0).$$

(vii) $\Delta_1 V_{t+1}(x_1,x_2,\delta)\leq \Delta_1 V_{t+1}(x_1+1,x_2+1,\delta)$: Suppose that $\delta=1$.

$$\Delta_1 V_{t+1}(x_1,x_2,1)-\Delta_1 V_{t+1}(x_1+1,x_2+1,1)$$
$$=\lambda[\Delta_1 V_t(x_1+1,x_2,1)-\Delta_1 V_t(x_1+2,x_2+1,1)]+\mu[\Delta_1 V_t(D(x_1,x_2),1)-\Delta_1 V_t(x_1,x_2,1)]$$
$$+d[\Delta_1 V_t(x_1,x_2+Q,0)-\Delta_1 V_t(x_1+1,x_2+Q+1,0)]\leq 0.$$

The terms corresponding to λ and d follow by induction, respectively. The term of μ follows by induction if $x_1>0$ and $x_2>0$. If $x_1\geq 0$ and $x_2=0$, it is zero. Finally, if $x_1=0$ and $x_2>0$, it becomes zero because

$$V_t(0, x_2-1, 1) - V_t(0, x_2, 1) - \Delta_1 V_t(0, x_2, 1)$$
$$\le V_t(1, x_2, 1) - V_t(0, x_2, 1) - \Delta_1 V_t(0, x_2, 1) \text{ (by (i) of Theorem 1).}$$

Assume that $\delta = 0$. We focus on combinations of actions in states $(x_1+1, x_2, 0)$, $(x_1, x_2, 0)$, $(x_1+2, x_2+1, 0)$, and $(x_1+1, x_2+1, 0)$. Denote by $(r/s, r/s, r/s, r/s)$ the optimal actions in these states. Using (i), (ii), and (ii) of Theorem 1, only 6 cases are admissible among 16 combinations of actions : (r, r, r, r), (s, s, s, s), (s, s, r, r), (s, s, s, r), (s, r, r, r), (s, r, s, r).

The proof of cases (r, r, r, r) and (s, s, s, s) follows by the similar arguments as in the case $\delta = 1$. Case (s, s, r, r) follows by (r, r, r, r) because $\Delta_1 s_t + 1(x_1, x_2, 0) \le \Delta_1 r_t + 1(x_1, x_2, 0)$ (by (i)). Case (s, r, r, r) follows by (r, r, r, r) because $s_{t+1}(x_1+1, x_2, 0) - r_{t+1}(x_1, x_2, 0) \le \Delta_1 r_{t+1}(x_1, x_2, 0)$. Similarly, case (s, s, s, r) follows by (s, s, s, s). Now consider case (s, r, s, r). Then, we have

$$\Delta_1 V_{t+1}(x_1, x_2, 0) - \Delta_1 V_{t+1}(x_1+1, x_2+1, 0)$$
$$= \lambda[V_t(x_1+2, x_2, 1) - V_t(x_1+1, x_2, 0) - (V_t(x_1+3, x_2+1, 1) - V_t(x_1+2, x_2+1, 0))]$$
$$+ \mu[V_t(D(x_1+1, x_2), 1) - V_t(D(x_1, x_2), 0) - (V_t(x_1+1, x_2, 1) - V_t(x_1, x_2, 0))]$$
$$+ d[V_t(x_1+1, x_2+Q, 0) - V_t(x_1, x_2, 0) - (V_t(x_1+2, x_2+Q+1, 0) - V_t(x_1+1, x_2+1, 0))] \le 0$$

The term corresponding to λ follows by (viii). The term corresponding to d becomes

$$V_t(x_1+1, x_2+Q, 0) - V_t(x_1, x_2, 0) - (V_t(x_1+2, x_2+Q+1, 0) - V_t(x_1+1, x_2+1, 0))$$
$$\le V_t(x_1, x_2+Q, 0) - V_t(x_1, x_2, 0) - (V_t(x_1+1, x_2+Q+1, 0) - V_t(x_1+1, x_2+1, 0)) \text{ (by induction)}$$
$$\le 0 \text{ (by (iii) of Theorem 1).}$$

The term corresponding to μ follows by (viii) if $x_1 > 0$ and $x_2 > 0$. If $x_1 \ge 0$ and $x_2 = 0$, it is zero. Finally, if $x_1 = 0$ and $x_2 > 0$, it is less than zero by (i) of Theorem 1 because

$$V_t(0, x_2-1, 1) - V_t(0, x_2, 0) - (V_t(1, x_2, 1) - V_t(0, x_2, 0)) = V_t(0, x_2-1, 1) - V_t(1, x_2, 1)$$

(viii) $V_{t+1}(x_1+1, x_2, 1) - V_{t+1}(x_1, x_2, 0) \le V_{t+1}(x_1+2, x_2+1, 1) - V_{t+1}(x_1+1, x_2+1, 0)$:

$$V_{t+1}(x_1+1, x_2+1, 0) - V_{t+1}(x_1, x_2, 0)$$
$$\le V_{t+1}(x_1+2, x_2+1, 0) - V_{t+1}(x_1+1, x_2, 0) \quad \text{(by (vii))}$$
$$\le V_{t+1}(x_1+2, x_2+1, 1) - V_{t+1}(x_1+1, x_2, 1) \text{ (by (ii) of Theorem 1).}$$