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A Prescriptive Group Decision Making Method with Imprecise Preference Information*

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■ Abstract ■

This paper presents a prescriptive approach to group decision making with group members' imprecise preference information. This includes an alternative method to Salo's inventive approach for identifying group's preferred alternative when attribute weights, consequences, and possibly group members' importance weights are specified in imprecise ways. The imprecise additive group value function can be decomposed into individual group member's imprecise decision making problems, which are finally aggregated to identify group's preferred alternative. The proposed approach is intuitive and easy to implement, and has merits in a couple of points. First, it is possible to view individual group member's inclinations toward conflicting alternatives and the degree of discrepancies to each other. Second, we can observe how much previous decision results of individual decision maker are influenced during interaction since decisions usually are not made at a single step especially in presence of partial preference information. Finally, the individual group member's decision results can be utilized for further investigation of dominance relations among alternatives in a case that interactive questions and responses fail to give a convergent group consensus.

Keyword : Group Decision Making, Imprecise Preference, Dominance Rules

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1. Introduction

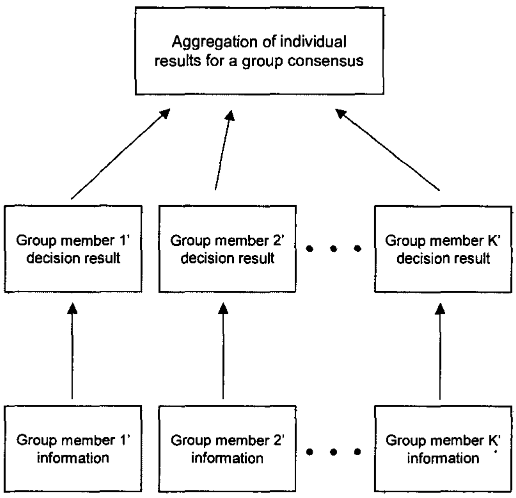
A decision making problem with multiple attributes consists of a multitude of subjects such as decision alternatives, criteria and preference information for consequences. Moving from single decision maker to multiple decision makers' setting introduces a great deal of complexity into the analysis. Each of decision makers involved in a group decision problem can have different preference judgments and hence it is usually understood that it is more difficult to aggregate them and form a satisfactory group consensus.

There have been research efforts to represent a group preference as additive value (utility) functions under some conditions. Harsanyi [6] presented the theory for an additive cardinal preference aggregation rule consistent with von Neumann and Morgenstern rationality axioms. The Bayesian rationality postulate (the group preferences satisfy the Bayesian rationality axiom) may not be appropriate for a group since they do not consider the equity of the outcomes. Keeney and Kirkwood [8] have specified sufficient conditions for a group cardinal social welfare function whose arguments are the individual utility functions of group members to have weighted additive form. The difficulties in the use of additive social welfare function lies in the assessment of group members' importance weights. The assessment involves addressing questions of trading-off utility to one individual against utility to another individual, so simultaneously trying to consider the inherent value of different measured utilities to each individual (i.e., interpersonal comparisons of utilities).

As a group extension of single decision maker's multiple criteria method for choosing among discrete alternatives [24, 25], Korhonen et al. [15] suggested an interactive approach to multiple criteria optimization with multiple decision makers. They assume that group members are able to make pairwise choices among competing alternatives according to its (implicit) utility function in one of their procedural steps. The difficulty in interactive multiple objective methodology is how to determine most preferred alternative (s) which is consistent with the information (usually through the holistic judgments between competing alternatives) given by decision makers interactively through adjusting attribute weights. Under multiple criteria, however, it is difficult for the group to build consensus between closely competing alternatives. If the group members are fairly in agreement, they can evaluate choices. Otherwise, the group will have to resolve a choice among pairs of solution. If not, a stalemate may result.

Imprecisely specified models are especially appropriate when there are multiple inputs to the analysis and group decision making is one of important areas of possible applications for the concept of incomplete information. To this end, Salo [18] has spearheaded a new thrust in the area of imprecise group decision making among others. He adopts the imprecise group additive value representation for aggregating group members' preference judgments when group decision parameters such as attribute weights, consequences, and group members' importance weights are specified in imprecise ways. He also provides a vast of literature surveys in the area of group decision analysis and group decision support system. The additive representation of imprecise group preference judgments was hereafter adopted and extended to treat such features as types of imprecision and interaction [10-12].

This paper deals with an alternative method for identifying group's preferred alternative to Salo's inventive approach when attribute weights, consequences, and possibly group members' importance weights are specified in imprecise ways. The imprecise additive group value function can be decomposed into individual group member's imprecise additive value functions, which are finally aggregated to identify group's preferred alternative [Figure 1]. This approach suggests an intuitive and easy procedure to implement. Meanwhile, information such as individual group member's preference strengths displays group members' disagreements on preferred alternatives.



[Figure 1] Three Stages of Group Decision Making

The rest of this paper is organized as follows. In Section 2, a proposed approach is compared with Salo's approach and group dominance rules for identifying group's preferred alternative are formally defined and their properties are exploited. In Section 3, a numerical example is illustrated. Finally concluding remarks follow.

2. A prescriptive group decision making method with imprecise information

In a single decision making context, it is usually understood that a decision situation is imprecise if and only if at least one of the parameters such as attribute weights or performance evaluations is not exactly specified. To be specific, imprecise information can be expressed in various ways, but we confine imprecise information within certain types of linear inequalities so that the developed linear programs can be used to solve the problem. With regard to the imprecise weight judgments, for example, if it is believed that i-th attribute is more important than j-th attribute, the imprecise preference information can be denoted as $w_i \geq w_j$, in which w_i and w_j signify i-th and j-th attribute weights respectively. Though no definite terminology has been developed for the general case, some authors describe "partial information," "imprecise information," "incomplete knowledge," "set inclusion," or "linear partial information (LPI)," interchangeably [5, 13, 14, 16, 17, 20, 22, 23].

Normally group members can be considered to have equal powers to each other in terms of their importance in reaching a group consensus. There are, however, more general situations in which the importance weights need not be equal due to the facts of the extent of domain knowledge, cognitive constraints and the like. A decision affecting the entire national economy such as energy policy decision is an example. In such a situation, the criteria to be considered are highly diverse and no single expert can be expected to have expertise to comment on all such relevant information. As one of possible ways to encompass these situations, we assume that group members' importance weights can be different and they can be specified in imprecise ways in case it is difficult to obtain precise evaluations about group members' importance weights. In doing so, there remains, however, a problem in that we use an additive representation of group preferences with imprecise group member's importance weights. The additive group representation requires interpersonal comparisons of utility in which either some external party, for instance 'a benevolent dictator', or the group members themselves must trade off utility between individuals [9]. Unfortunately, there are no entirely satisfactory procedures for making these tradeoffs and as a partial remedy to the situation, Sen [21] suggests that instead of aiming at precise importance weights, it is advisable to work with a reasonable range of importance weights even if the results from the imprecise model may be incomplete [18].

With K group members and a finite set of alternatives $A = \{x, y, z, \cdots\}$, the additive value representation of group members' preference judgments on alternative x can be denoted as

$$V(x) = \sum_{k=1}^{K} g^k \sum_{i=1}^{N} w_i^k v_i^k(x), \qquad (1)$$

where $\mathbf{g} = (g^k) \in G$ is the importance weight of k-th group member, $w_i^k \in W^k$ is an imprecisely specified weight of k-th group member with respect to i-th attribute, and $v_i^k(\cdot) \in V_i^k$ is an imprecisely specified value score of k-th group member with respect to i-th attribute. Under a certainty case, the required conditions for an additive representation are measurable individual rationality [3], measurable group rationality, and exchange independence. See Keeney and Raiffa [9] for risk and Dyer and Sarin [4] for certainty.

To identify group's most preferred alternative with the representation in (1), it is necessary to check dominance relations among considered alternatives. A few researches, however, have geared to a group decision making with imprecise information though group decision making is an important area of possible application for the concept of imprecise information [22]. One of those efforts can be found in Salo [18]. To resolve the imprecise additive group value problem, he argues that a major reason for working with unnormalized scores is that the minimization problem would become nonlinear and consequently exceedingly difficult to solve if the preference statements were expressed as separate constraints on the group weights, attribute weights, and normalized scores. He thus introduces variable transformations to deal with nonlinear optimization problem and here we introduce his approach in brevity.

2.1 Salo's approach

For a group with K members, the additive value representation can be denoted as follows :

$$V(x) = \sum_{k=1}^{K} g^k v^k(x) = \sum_{k=1}^{K} g^k \sum_{i=1}^{N} w_i^k v_i^k(x), \qquad (2)$$

where g^k is the importance weight of the k-th group member and $v^k(x)$ is the value that k-th group member attaches to consequence x characterized by N multiple attributes. By convention, the function $v^k(x)$ is normalized so that it maps the worst and the best consequences for the k-th group member to zero and one respectively. An alternative expression for this function is $\sum_{i=1}^{N} w_i^k v_i^k(x)$, where w_i^k is the k-th group member's weight for the i-th attribute and $v_i^k(x)$ is the normalized value function which takes the k-th group member's extreme achievement levels on the i-th attribute to zero and one.

According to Salo, to circumvent the nonlinearity contained in (2) caused by incomplete information on attribute weight, value function and group members' weight, the additive representation (2) can be converted into an equivalent linear program such as

$$V(x) = \sum_{k=1}^{K} v^{k'}(x) = \sum_{k=1}^{K}\sum_{i=1}^{N} v^{k'}(x). \qquad (3)$$

The expressions (3) is related with (2) through the change of variables such as $v^{k'}(x) = g^k v^k(x)$ and $v_i^{k'}(x_i) = g^k w_i^k v_i^k(x)$.

The importance weight for the k-th group member can thus be expressed as $g^k = \sum_{i=1}^{N} v_i^{k'}(x^*)$, where x^* denotes an alternative that attains a maximum value score with respect to i-th criterion. Then the k-th group member's weight for the i-th attribute becomes $w_i^k = \dfrac{v_i^k(x^*)}{\sum_{i=1}^{N} v_i^k(x^*)}$, and the normalized score $v_i^k(x)$ becomes equal to the ratio $v_i^{k'}(x)\, v_i^{k'}(x^*)$. Due to the relationships as shown above, imprecise preference judgments can be converted into linear constraints on the single-attribute scores of the additive model.

2.2 The Proposed approach

This transformation presents a good treatment for circumventing intractable nonlinear features contained in (2) and we can thus use linear programs with ease to discern group's preferred alternative. But some scrutiny reveals that the additive group representation in (1) or (2) can be resolved with no transformation for evading a nonlinearity when two conditions are met. In other words, the problem (1) is separable in an optimization procedure when preferential information is given as types of linear constraints as mentioned before and functional independence condition holds. The latter condition implies that it is allowed to specify imprecise value evaluations among alternatives not across attributes but within each of attributes. For example, when buying a car, the following preference expression is not allowed such as "the value of car x with respect to a *design* attribute is three times more valuable than that of car y with respect to a *horse power* attribute." The assumption of functional independence is appropriate in many realistic problems and it is operationally verifiable in practice.

With two conditions fulfilled, it is required to solve minimization or maximization problem of (1) for identifying dominant alternatives. In case of a minimization problem, for instance, the problem of (1) can be decomposed into $\min\{\sum_k g^k \cdot \min\{\sum_i w_i^k v_i^k(x)\}\}$ due to the fact that group members' weight is independent of product of attribute weight times consequence, i.e., $G \perp W \times V$. And the expression $\min\{\sum_i w_i^k v_i^k(x)\}$ can be further decomposed into $\min\{\sum_i w_i^k \cdot \min\{v_i^k(x)\}\}$ because the attribute weight is independent of attribute consequence, i.e., $W \perp V$. We thus only have to solve a series of small linear programs backwards from value to weight and then finally to group when we encounter seemingly complicated imprecise group decision problems. This decomposition process takes even simpler form when group's importance weight, attribute weight, and value scores are specified in bounded forms. This decomposition concept can also be applied to paired dominance checks between alternatives. For a paired dominance check between alternative x and y, we should take into account of the sign of the following problem,

$$\text{minimize } \{\Sigma_k g^k \Sigma_i w_i^k [v_i^k(x) - v_i^k(y)]\}$$
subject to imprecise information.

If a sign of the minimized objective value in a paired comparison is positive, it is said that alternative x is strictly pairwise dominant over alternative y. Otherwise, it is required to check a paired dominance in reverse order of alternatives. The expression for a paired dominance check can also be decomposed into

$$\min\{\Sigma_k g^k \cdot \min\{\Sigma_i w_i^k [\underline{v}^k(x) - \overline{v}^k(y)]\}\}$$
$$\underline{v}^k(x) = \min\{v_i^k(x)\} \text{ and } \overline{v}^k(y) = \max\{v_i^k(y)\}.$$

With this in mind, we further exploit group dominance rules and properties that are extensions of a single decision making context with imprecise information. Similar mathematics for a single decision making context can be found in several research works [16, 19, 23].

Definition 1(*GSD* : *Group Strict Dominance*) : alternative x strictly dominates alternative y from the viewpoint of a group if and only if $\zeta_{\min}(x) > \zeta_{\max}(y)$, where
$\zeta_{\min}(x) = \min\{\sum_k g^k \Sigma_i w_i^k v_i^k(x) \mid g^k \in G,$
$w_i^k \in W^k, v_i^k(x) \in V_i^k\}$ and
$\zeta_{\max}(y) = \max\{\Sigma_k g^k \Sigma_i w_i^k v_i^k(y) \mid g = (G^k) \in G,$
$w_i^k \in w^k,\ V_i^k(y) \in V_i^k\}$

Definition 2 (*GPSD* : *Group Pairwise Strict Dominance*) : alternative x is strictly pairwise dominant over alternative y from the viewpoint of a group if and only if $\zeta_{\min}(x, y) > 0$, where
$\zeta_{\min}(x, y) = \min\{\Sigma_k g^k \Sigma_i w_i^k [v_i^k(x) - v_i^k(y)] \mid$
$g^k \in G, w_i^k \in W^k, v_i^k(x), v_i^k(y) \in V_i^k\}.$

Definition 3 (*GPWD* : *Group Pairwise Weak Dominance*) : alternative x is weakly pairwise dominant over alternative y from the viewpoint of a group if and only if $\zeta_{\min}(x, y) > \zeta_{\min}(y, x)$.

Theorem 1 : Let Ω_i denote a of identified dominance relations among alternatives with dominance rules in Definition i = *GSD, GPSD, or GPWD*. Then it holds that

$$\Omega_{GSD} \subseteq \Omega_{GPSD} \subseteq \Omega_{GPWD}.$$

Proof : a) Let us prove the first assertion (i.e., $\Omega_{GSD} \subseteq \Omega_{GPSD}$). Suppose that a GSD relation holds between alternative x and y, that is, $\zeta_{\min}(x) > \zeta_{\max}(y)$ it means that $\zeta_{\min}(x) - \zeta_{\max}(y) = \zeta_{\min}(x) - (-\zeta_{\min}(-y)) > 0$ where $\zeta_{\min}(-y) = \min\{\Sigma_k g^k \Sigma_i - w_i^k v_i^k(y)\}$ and in turn, becomes $\zeta_{\min}(x) + \zeta_{\min}(-y) > 0$. The value of $\zeta_{\min}(x, y)$ is always greater than or equal to $\zeta_{\min}(x) + \zeta_{\min}(-y)$ since a minimized value that is attained with common weights is always greater than or equal to the sum of minimized values with individually varying weights.
b) Let us prove the second assertion (i.e., $\Omega_{GPSD} \subseteq \Omega_{GPWD}$). Suppose that a GPSD relation exists between alternative x and y, that is, $\zeta_{\min}(x, y) > 0$ and rewrite it as
$\zeta_{\min}(x, y) = \zeta_{\min} - (y, x)$, where $\zeta_{\min} - (y, x) = \min\{\Sigma_k g^k \Sigma_i - w_i^k [v_i^k(y) - v_i^k(x)]\}$. It holds that $\zeta_{\min} - (y, x) = -\zeta_{\max}(y, x) > 0$ and in turn, $\zeta_{\min}(y, x) \le \zeta_{\max}(y, x) < 0$, which implies that

$\zeta_{\min}(y,x)$ is always negative if $\zeta_{\min}(x,y)$ is positive and thus $\zeta_{\min}(x,y) > \zeta_{\min}(y,x)$.

Corollary1 : Let us define the k-th group member's individual optimal results for alternative $x \in A$ as

$$\zeta^k_{\min}(x) = \min\{\Sigma_i w^k_i v^k_i(x) \mid w^k_i \in W^k, v^k_i(x) \in v^k_i\}$$

and

$$\zeta^k_{\max}(x) = \max\{\Sigma_i w^k_i v^k_i(x) \mid w^k_i \in W^k, v^k_i(x) \in v^k_i\}.$$

If it holds that $\zeta^k_{\min}(x) > \zeta^k_{\max}(y)$ for every group member, then alternative x is strictly pairwise dominant over alternative y from the viewpoint of group, that is, $\zeta_{\min}(x,y) > 0$.

Proof : Let us suppose $\zeta^k_{\min}(x) > \zeta^k_{\max}(y)$, $x, y \in A, k = 1, \cdots, K$. This expression becomes $\min\{\Sigma_i w^k_i v^k_i(x)\} - \max\{\Sigma_i w^k_i v^k_i(y)\} > 0, \forall k$ and thus it holds that $\zeta^k_{\min}(x,y) > 0$ in which $\zeta^k_{\min}(x,y) = \min\{\Sigma_i w^k_i [v^k_i(x) - v^k_i(y)]\}, \forall k$ according to the fact shown in part a) of the proof in Theorem 1. A GPSD value between x and y, $\zeta_{\min}(x,y)$, which is a value that results from the minimization of a convex combination of K positive numbers (i.e., $\zeta^k_{\min}(x,y) > 0, k = 1, \cdots, K$)) constrained with $\Sigma_k g^k = 1$ and $g^k \geq 0, \forall k$ is positive as well.

According to the Corollary 1, the GPSD relations can, on occasions, be identified among alternatives without solving linear programs for identifying GPSD relations though there are no GSD relations between alternatives since the intervals from the GSD checks overlap each other.

Corollary2 : The GSD and GPSD rules are asymmetric, irreflexive, and transitive. The GPWD rule is asymmetric and irreflexive, but it is not always transitive.

Proof : Corollary 2 can be easily proved by using the facts in Definition (1)~(3) and Theorem 1.

To discern group's most preferred alternative, it is required to solve *2M(KN+K+1)* linear programs by the GSD rule and *M(M-1)(KN+K+1)* by the GPSD rule, in which M stands for the number of finite alternatives. The number of linear programs to be solved with the GPSD rule increase more rapidly than those in the GSD rule, but we can identify more dominance relations among alternatives by adopting the GPSD rule as was proven in Theorem 1.

If the group's most preferred alternative is identified with the GSD or more possibly GPSD rule, the decision process ends with success. Otherwise, ensuing interactive question and response are posed possibly to change group member's preferences or to impose more restrictive preference judgments. Though such interactive approaches make sense and provide us with a useful decision principle, there exist, on the other hand, somewhat complicated problems in which we have to address how to assess more information from group members. We should further consider a situation that some of group members are not willing to provide more restrictive information on the decision parameters but a more specific recommendation for the decision making is required. Taking into account some possible situations that can happen in imprecise group decision making, we can describe the following three scenarios for reaching a group's final decision :

- A group consensus is made within a few interactivities. If the group members show strongly agreed preference tendency toward not predominant but outstanding some of alternatives, the possibility of selecting group's most preferred alternative increases with possibly a few interactive preference modifications or additions of new preferences constraints.

- Individual decision maker shows consistent decision results in her/his own perspective but the results for the group, as a whole, show much of disagreement between alternatives. In this case, focus on individual decision maker's problem and try to find out individual decision maker's rank orders of alternatives. Once individual ordinal rankings are obtained, one can use, for example, a distance measure for a group consensus [2]. They suggest the problem of determining a compromise or a consensus ranking that agrees best with all the decision-makers' rankings by using an assignment problem.
- In a case that ensuing interactive questions and responses may not sometimes guarantee to provide a best alternative to implement or individual decision approach fails to provide a rank order among alternatives, one can utilize the aggregated measure of weak dominance values, which are consistent with the sense of outranking dominance relationship, for further investigation of dominance relations [10].

3. A numerical example

To explain a solution procedure, this section illustrates an invented example developed by Salo [18], in which the marketing and production departments of a medium-sized company are to select one of three competing product suggestions for subsequent development. For brevity, the departments are assumed to consider only two attributes : 1) the design of the product and 2) aggregate costs related to production and marketing. The alternative product suggestions are labeled as x, y and z. The marketing department is less concerned with costs and believes that the weight of the cost attribute should be only about 50~70% of the weight of the design attribute. Conversely, the production department emphasizes costs by stating that the design attribute should have 60~70% of the weigh attached to the cost attribute. The normalized single-attribute value scores are shown in <Table 1>.

〈Table 1〉 Normalized Single-attribute Scores

	Marketing Department (DM 1)		Production Department (DM 2)	
	Attributes		Attributes	
Alternatives	Design	Cost	Design	Cost
x	[0.6, 0.7]	[0.3, 0.4]	1	[0.4, 0.5]
y	1	0	[0.3, 0.4]	0
z	0	1	0	1

In accordance with the notations introduced in Section 2, the imprecise information on value scores and attribute weights can be denoted as follows :

$V_1^1=\{0.6\le v_1^1(x)\le 0.7, v_1^1(y)=1, v_1^1(z)=0\}$, (7)

$V_2^1=\{0.3\le v_2^1(x)\le 0.4, v_2^1(y)=0, v_2^1(z)=1\}$, (8)

$V_1^2=\{v_1^2(x)=1, 0.3\le v_1^2(y)\le 0.4, v_1^2(z)=0\}$, (9)

$V_2^2=\{0.4\le v_2^2(x)\le 0.5, v_2^2(y)=0, v_2^2(z)=1\}$, (10)

$W^1=\{0.5w_1^1\le w_2^1\le 0.7w_1^1, w_1^1+w_2^1=1, w_1^1, w_2^1\ge 0\}$, (11)

$W^2=\{0.6w_2^2\le w_1^2\le 0.7w_2^2, w_1^2+w_2^2=1, w_1^2, w_2^2\ge 0\}$. (12)

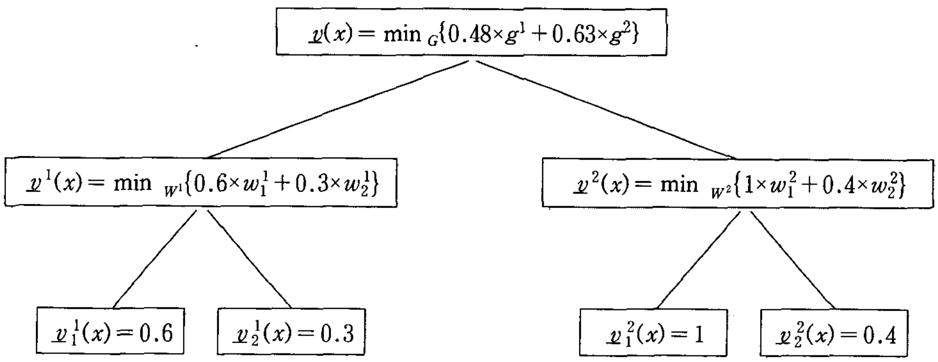
The two attributes are referred to by the subscript 1 for design and 2 for cost respectively, whereas superscript 1 denotes marketing department and 2 production department. Let us first explain how to compute lower and upper bounds that each alternative can take under given imprecise information. These bound results are then used for dominance checks with the GSD rule. The lower bound that alternative x can take, for example, is formulated as in Problem 1.

Problem 1 : a mathematical program for computing a lower bound of alternative x

$$\min\{g^1[w_1^1v_1^1(x)+w_2^1v_2^1(x)] + g^2[w_1^2v_1^2(x)+w_2^2v_2^2(x)]\} \quad (13)$$

subject to (7), (8), (9), (10), (11), and (12).

Let us select the first term in the objective function of problem 1, that is, $g^1\{w_1^1v_1^1(x)+w_2^1v_2^1(x)\}$ and explain how to obtain a minimized value in detail. The minimum value can be attained when each of sub-terms attains a minimum value and thus we can rewrite it as min $g^1\cdot\{\min\{w_1^1v_1^1(x)\}+\min\{w_2^1v_2^1(x)\}\}$. The weighted value of the marketing department on a design attribute (i.e., $\min\{w_1^1v_1^1(x)\}$) is further separable into $\min\{w_1^1\}\times\min\{v_1^1(x)\}$. If we apply the same decomposition principle to the second term, the entire process for computing a minimum value that alternative x can take can be schematically depicted as in [Figure 2].



[Figure 2] A Process of Applying a GSD Check

The values in twig levels of [Figure 2] are minimum values at which each variable (i.e., value score) can attain. For instance, a minimum value score of alternative x on attribute design in the perspective of marketing department can be computed such as $\underline{v}_1^1(x)=\min\{v_1^1(x)\mid 0.6\le v_1^1(x)\le 0.7\}=0.6$ The intermediate value $\underline{v}^1(x)$ can be computed as $\underline{v}^1(x)=\min\{0.6w_1^1+0.3w_2^1 \mid 0.5w_1^1\le w_2^1\le 0.7w_1^1, w_1^1+w_2^1=1, w_1^1, w_2^1\ge 0\}$. The elements in the topmost node are fed directly from the intermediate computation results. Similarly, a maximum value $\overline{v}(x)$, at which alternative x attains, can be computed by substituting minimization with maximization in the objective function in (13). Before assigning group members' importance weights to each of group members, *individual* group member's preference strengths on each of alternatives are shown in <Table 2>. In other words, the topmost values of each alternative appear to take the following value intervals :

$$0.48\times g^1+0.63\times g^2\le V(x)\le 0.6\times g^1+0.71\times g^2$$
$$0.59\times g^1+0.11\times g^2\le V(y)\le 0.67\times g^1+0.17\times g^2$$
$$0.33\times g^1+0.59\times g^2\le V(z)\le 0.41\times g^1+0.63\times g^2.$$

〈Table 2〉 Individual Decision Maker's Preferences on Each of Alternatives

	x	y	z
Marketing Department	[0.48, 0.6]	[0.59, 0.67]	[0.33, 0.41]
Production Department	[0.63, 0.71]	[0.11, 0.17]	[0.59, 0.63]

The value intervals signify that the final dominance relations are heavily dependent on the group members' importance weights. By convention, we include a normalized term, $g^1+g^2=1$ and we find that value intervals become $V(x)=$ [0.48, 0.71], $V(y)=$ [0.11, 0.67], and $V(z)=$ [0.33 0.63], which are the same results as Salo's approach. Further we can find that the aggregated value intervals coincide with the union (not intersection) of individual group member's value intervals indicated in <Table 2>, which implies that GSD rule has less discriminative power than GPSD rule in identifying dominant alternatives.

Before directly going to the GPSD check, the individual optimization results between alternative x and z in <Table 2> have some common features stated in Corollary 1. In the individual preference strength values, the minimum value of alternative x is greater than the maximum value of alternative z in the marketing department, and that the minimum value of alternative x is greater than or equal to the maximum value of alternative z in the production department. Thus we can conclude that alternative x is at least as pairwise dominant as alternative z in group's perspective. In fact, the value of ζ_{min} (x, z) is zero when we perform a GPSD check between alternative x and z. All of pairwise dominance results between the other alternatives (i.e., alternative x and y, y and z) are negative. For further identification of group's preferred alternatives, we assume that the departments acknowledge that in the aggregate value representation neither one of them should get more than 50% more weight than the other department. This statement can be expressed as follows :

$$G=\{g^1\le 1.5\times g^2,\ g^2\le 1.5\times g^1,\ g^1, g^2\ge 0\}. \quad (14)$$

With the inclusion of this expression, we formulate a mathematical program for checking the GPSD, for example, between alternative x and y as follows :

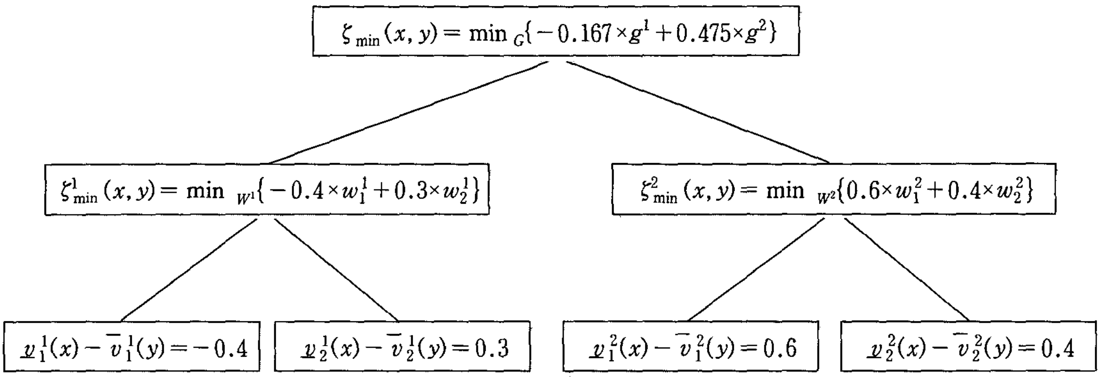
Problem2 : a mathematical program for a GPSD check between alternative x and y

$$\min\{g^1\{w_1^1[v_1^1(x)-v_1^1(y)]+w_2^1[v_2^1(x)-v_2^1(y)]\}$$
$$+g^2\{w_1^2[v_1^2(x)-v_1^2(y)]+w_2^2[v_2^2(x)-v_2^2(y)]\}\}$$

subject to (7), (8), (9), (10), (11), (12) and (14).

A process of solving a mathematical program in Problem 2 is depicted in [Figure 3]. The final

GPSD results show that $\zeta_{\min}(x,y) = 0.09$, $\zeta_{\min}(y,z) = -0.24$, $\zeta_{\min}(z,y) = -0.03$ and $\zeta_{\min}(x,z) = 0.03$. From the results, alternative x is dominant over both alternatives y and z and thus it can be concluded that alternative x is group's most preferred alternative.



[Figure 3] A Process of Applying a GPSD Check between Alternative x and y

4. Conclusions

Nowadays, the complexity of decision problems requires multiple decision makers who have their own expertise in their areas to consider the decision problem. So it is needed to expand a single decision making context into a group decision making context which has a broad range of real-world applications.

This paper presents an interactive group decision making method for identifying cooperative group's preferred alternative when imprecise preference information is specified. It is said that the reasons the decision makers provide imprecise information are 1) a decision should be made under time pressure and lack of data, 2) many of attributes are intangible or non-monetary because they reflect social and environmental impacts, and 3) decision makers have limited attention and information processing capabilities and the like [7]. In addition to the imprecise data in attribute weights and value scores, we further assume that group member's importance weights exerted in forming a group consensus can be described in imprecise ways. To aggregate imprecise judgments with an imprecise additive group value function requires some treatment for evading a nonlinearity in the imprecise group value function. To this end, we present a direct and intuitive method to circumvent the nonlinearity problem, which is a different approach from the former research works and has merits in a couple of points. First, it is possible to view individual group member's inclinations toward competing alternatives and the degree of discrepancies among group members. Second, an interactive feature in multi-criteria decision making is usual because decisions are not made at a single step especially in presence of partial preference information and we can observe how much individual decision maker is affected during interaction. Finally, the individual group member's decision results can be utilized for further investigation of dominance relations among alternatives in a case that interactive questions and responses fail to give a convergent group consensus.

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