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AN INTEGRATED PROCESS CONTROL PROCEDURE WITH REPEATED ADJUSTMENTS AND EWMA MONITORING UNDER AN IMA(1,1) DISTURBANCE WITH A STEP SHIFT†

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ABSTRACT

Statistical process control (SPC) and engineering process control (EPC) are based on different strategies for process quality improvement. SPC reduces process variability by detecting and eliminating special causes of process variation, while EPC reduces process variability by adjusting compensatory variables to keep the quality variable close to target. Recently there has been need for an integrated process control (IPC) procedure which combines the two strategies. This paper considers a scheme that simultaneously applies SPC and EPC techniques to reduce the variation of a process. The process model under consideration is an IMA(1,1) model with a step shift. The EPC part of the scheme adjusts the process, while the SPC part of the scheme detects the occurrence of a special cause. For adjusting the process repeated adjustment is applied according to the predicted deviation from target. For detecting special causes the exponentially weighted moving average control chart is applied to the observed deviations. It was assumed that the adjustment under the presence of a special cause may increase the process variability or change the system gain. Reasonable choices of parameters for the IPC procedure are considered in the context of the mean squared deviation as well as the average run length.

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Keywords. SPC, EPC, IMA(1,1), EWMA, observed deviation, predicted deviation, system gain, mean squared deviation, average run length.

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1. INTRODUCTION

In many processes, such as in the chemical and process industries, observations are correlated when they are closely spaced in time. The levels of such processes can usually be modelled as wandering about according to a certain stochastic rule such as a nonstationary time series model. A process whose level wanders about may be manipulated by some compensators to move the process level back to target. The wandering behavior of the process, which comes from its inherent disturbances, is often described by an IMA(1,1) model (see Box and Kramer, 1992; Vander Wiel, 1996; Montgomery, 1999) and the adjustment is usually employed by feedback control (see Box and Luceño, 1994 and 1997). This adjustment activity is called engineering process control (EPC) or automatic process control (APC).

When there is a sudden shift in the process level due to the occurrence of a special cause, it is important to detect it quickly. If the cause was known the process can be improved by removing the source of variability. This activity is called statistical process control (SPC).

When a process level has a wandering behavior as well as a level shift, an integrated process control (IPC) procedure which combines SPC and EPC can be employed in order to improve the process. Recently integration of SPC and EPC has generated a great deal of interest since the two procedures are not divided clearly in modern manufacturing industries. Examples of papers on this topic include MacGregor (1987), Vander Wiel *et al.* (1992), Montgomery *et al.* (1994), Box and Luceño (1997), Box *et al.* (1997), Janakiram and Keats (1998), Nembhard and Mastrangelo (1998), Mastrangelo and Brown (2000) and Ruhhal *et al.* (2000).

We consider a process disturbance which follows an IMA(1,1) model in the absence of a special cause and an IMA(1,1) model with a step shift in its level under the presence of a special cause. We apply an adjustment rule and a monitoring rule at the same time. Since the process level represents the combined effect of wandering behavior and a step shift, we first adjust the process by forecasting and apply a monitoring rule to the forecast errors, which are isolated from the adjustment. As is indicated by Vander Wiel (1996), a step change is more difficult to detect when buried in an IMA model than when buried in a Shewhart *iid* model.

The purpose of this paper is to develop an IPC procedure which can adjust the process level close to target and detect the occurrence of special causes.

Adjustment is made by forecast of the process and monitoring is performed by an exponentially weighted moving average (EWMA) of forecast errors.

2. PROCESS DISTURBANCE MODEL

Suppose that the process level is measured at equispaced time intervals, say, $\ldots, t-1, t, t+1, \ldots$ The process disturbance tends to wander off from target when no control action is attempted to adjust it. Moreover the disturbance, if left to itself without adjustment, would permanently drift away from target. This nonstationary wandering behavior of disturbance is often well represented by an IMA(1,1) model. A pure IMA(1,1) process is represented as

$$N_t = N_{t-1} + a_t - \theta a_{t-1}, \tag{2.1}$$

where random shock $\{a_t\}$ is a sequence of white noise with variance σ_a^2 and θ is a smoothing constant with $0 \leq \theta < 1$. In addition to the wandering behavior of the process disturbance we also consider a special cause which deploys a step shift to the level of the disturbance. Let U be the time that a special cause occurs and δ be the amount of the step shift in units of σ_a, then an IMA(1,1) process with a step shift can be modelled as

$$Z_t = Z_{t-1} + a_t - \theta a_{t-1} + \delta\sigma_a I_{\{U\}}(t), \tag{2.2}$$

where $I_{\{U\}}(t) = 1,\ 0$ if $t = U,\ t \neq U$, respectively. If we assume that $Z_0 = 0$ and $a_0 = 0$, then a random shock form equivalent to equation (2.2) is

$$Z_t = a_t + \lambda \sum_{j=1}^{t-1} a_j + \delta\sigma_a I_{\{x|x \geq U\}}(t) \tag{2.3}$$

where $\lambda\ (= 1-\theta)$ denotes the nonstationary parameter and $I_{\{x|x\geq U\}}(t) = 1,\ 0$ if $t \geq U,\ t < U$, respectively.

The model in equation (2.2) is an example of a common cause plus special cause model. In the model the common cause model is a pure IMA(1,1) process and the special cause model is a Shewhart model where the constant mean jumps to the amount of $\delta\sigma_a$ when a special cause occurs at time $t = U$.

We assume that the effect of a special cause remains in the process until it is detected and removed. We also assume that the process goes back to the common cause model and its level goes back to target after repair of the process. The interval of time from the start to the occurrence of a special cause is termed the before-special cause (BS) period and the interval of time from the occurrence of a special cause to its detection is termed the after-special cause (AS) period. A cycle of a process is defined as a series of a BS period and an AS period.

Special causes occur at random times during the operation of the process. The random occurrence of a special cause is often described by an exponential distribution for continuous time or a geometric distribution for discrete time. A geometric distribution is a discrete analog of an exponential distribution.

It is assumed here that U follows a geometric distribution with parameter p. The probability function of U is defined as

$$P(U=t)=(1-p)^{t-1}p, \qquad t=1,2,\ldots \tag{2.4}$$

The event $\{U=t\}$ in discrete time is equivalent to the event that a special cause occurs in the interval $(t-1,t]$ in continuous time.

3. IPC PROCEDURE

In some systems a residual disturbance remains to the system in spite of all the efforts of bringing the process level back to target, since the source of variations can not be removed or is even unknown by some reasons such as technical or economical problems. In such systems some type of process adjustment, *e.g.* feedback adjustment, is employed to control the disturbances.

In feedback regulation it is assumed that some compensatory variable exists which can be manipulated to adjust the process level. A system where all of the effect of a change in the compensatory variable will be realized in the output in one time interval is called a responsive system. The factor measuring the change in the output produced by one unit of change on the compensatory variable is called the system gain.

The activity of the IPC procedure is a series of adjustments and monitoring, where the adjustment action rule is to adjust the process level close to target throughout a cycle, while the monitoring action rule is to quickly detect the occurrence of a special cause during AS period. Adjustments before and after the occurrence of a special cause are called BS adjustment and AS adjustment, respectively. Since the two action rules are designed separately there are chances that both action rules are met at the same time. Since AS adjustment is not usually as efficient as BS adjustment, in such cases, we apply the monitoring action, rather than the adjustment action, in order to search for and eliminate the special cause. If AS adjustment has an effect on the level, but not on the variance or the system gain, then it can eliminate the effect of the special cause by itself. In such cases the monitoring action is not needed. We consider cases, in this paper, where AS adjustment has an effect on the level as well as the variance and the system gain.

Adjustments are usually employed under the assumption that they will change the level of the process without changing the process model or process parameters. In case of AS adjustment, however, its inefficiency may appear in several ways, such as increments of adjustment cost and/or process variance, change in system gain, delay in realization of adjustment effects, *etc.*

As an example, suppose that a house is controlled to maintain the indoor temperature at target $T = 70^o$F. The indoor temperature depends on the ambient temperature whose changes are disturbances which can not be removed. Suppose that the indoor temperature is measured every hour and a furnace is operated manually to compensate for the disturbance in temperature. If a window is open accidently and unnoticed (*i.e.* a special cause is occurred but not detected), the operation of a furnace to adjust the indoor temperature corresponds to an AS adjustment. Then we must operate the furnace for larger period of time, which will bring increments of adjustment cost, changes in system gain and/or delays in realization of adjustment effect. Also the indoor temperature depends more than before on the ambient temperature which will result in an increase in process variance.

3.1. Adjustment procedure

Denote, in a responsive system, a compensatory variable adjusted at time t by X_t and the system gain by g. Then the total output compensation at time $t+1$, Y_{t+1}, produced by the total input compensation, X_t, is

$$Y_{t+1} = gX_t.$$

The total input compensation X_t is usually determined by the forecast of the process disturbance. When there is no cost for adjustment we use repeated adjustment where the process is adjusted at every monitoring time.

The discrete adjustment equation employed here is

$$gx_t = -GO_t, \tag{3.1}$$

where $x_t = X_t - X_{t-1}$ is the one-step input compensation from the previous level X_{t-1}, G is the damping factor and O_t is the observed deviation after adjustment at time t. If the damping factor is equal to 1, the adjustment will be a full adjustment. Usually G is selected between 0 and 1 to avoid overcompensation. In a repeated adjustment scheme by forecasting, the forecast error is identical to the observed deviation.

If a one step ahead forecast of Z_{t+1}, $\widehat{Z}_{t+1}$, is available, then the adjustment is to set the total input compensation so that it cancels out the predicted value $\widehat{Z}_{t+1}$. Thus we set

$$gX_t = -\widehat{Z}_{t+1},$$

or equivalently

$$gx_t = -(\widehat{Z}_{t+1} - \widehat{Z}_t). \tag{3.2}$$

Since $O_t = Z_t - \widehat{Z}_t$, we have, from equations (3.1) and (3.2),

$$\widehat{Z}_{t+1} - \widehat{Z}_t = G(Z_t - \widehat{Z}_t).$$

Thus the forecast is calculated as

$$\widehat{Z}_{t+1} = GZ_t + (1 - G)\widehat{Z}_t \tag{3.3}$$

and solution of equation (3.3) is an EWMA of past levels of the unadjusted process,

$$\widehat{Z}_{t+1} = G\{Z_t + (1 - G)Z_{t-1} + (1 - G)^2 Z_{t-2} + \cdots\}.$$

Since adjustment action is continually taken to cancel out the forecast, we do not either see the unadjusted process level or set the total input compensation, but only see the observed deviation and set the one-step input compensation at each time. Thus it is more convenient if the adjusted process level is expressed exactly as what we see in terms of the one-step input and output compensations.

Let $y_t = Y_t - Y_{t-1}$, which is the one-step output compensation from the previous level Y_{t-1}. Then we have the discrete adjustment equation,

$$y_{t+1} = gx_t = -GO_t.$$

When AS adjustment does not change the process variance or the system gain, we have, by using $O_{t-1} = Z_{t-1} - \widehat{Z}_{t-1}$,

$$O_t - (1 - G)O_{t-1} = Z_t - Z_{t-1}$$

and thus the difference equation form of the observed deviation is derived as, by equation (2.2),

$$O_t - (1 - G)O_{t-1} = a_t - \theta a_{t-1} + \delta\sigma_a I_{\{U\}}(t). \tag{3.4}$$

Suppose that the AS adjustment increases the variance of the random shocks, then the increment of variance will appear from $(U+1)^{th}$ time. Thus the random shock a_t can be replaced by b_t defined as

$$b_t = \begin{cases} a_t, & \text{if } t \leq U, \\ v \cdot a_t, & \text{if } t \geq U+1 \end{cases}$$

for a variance inflation factor v (> 1). Then the observed deviation is expressed as

$$O_t - (1-G)O_{t-1} = b_t - \theta b_{t-1} + \delta\sigma_a I_{\{U\}}(t). \tag{3.5}$$

Since equation (3.5) can be rewritten as

$$O_t - O_{t-1} = b_t - \theta b_{t-1} + \delta\sigma_a I_{\{U\}}(t) - GO_{t-1},$$

we see that the last term on the right hand side of the equation, $-GO_{t-1}$, corresponds to the one-step output compensation, y_t. Thus it is seen that the observed deviation at time t, when adjusted up to time $t-1$ but not adjusted yet at time t, is

$$O_t = O_{t-1} + b_t - \theta b_{t-1} + \delta\sigma_a I_{\{U\}}(t).$$

If the system gain changes from g to g', the one-step output compensation will be $y_t = -G(g'/g)O_{t-1}$, and thus the observed deviation can be defined as

$$O_t - (1-Gg_t^*)O_{t-1} = b_t - \theta b_{t-1} + \delta\sigma_a I_{\{U\}}(t), \tag{3.6}$$

where $g_t^* = 1, g'/g$ if $t \leq U$, $t \geq U+1$, respectively. From this equation we see that the repeated adjustment changes a nonstationary IMA(1,1) process to a stationary ARMA(1,1) process, in which the special cause produces a step shift of size $\delta\sigma_a$ and AS adjustment changes the random shock from a_t to b_t as well as changes the autoregressive parameter from $1-G$ to $1-Gg_t^*$. Note that if $G = \lambda$ the observed deviations in a BS period are white noises. If the change in system gain is unobservable its effect will remain in the process until the special cause is removed. We assume that the delay in realization of the AS adjustment effect, although it exists, is not long enough to violate the responsive system.

Figure 1 shows an example of an unadjusted process and three corresponding adjusted processes, each of which shows one hundred values of deviations from target generated by an IMA(1,1) with a step shift, where the parameters in equations (2.2) and (3.6) are set as

$$\theta = 0.7,\ \sigma_a = 1,\ \delta = 3,\ U = 21,\ G = 0.3,\ v = 2,\ g^* = 0.5.$$

Figure 1(a) shows the unadjusted process Z_t. Figure 1(b) shows the adjusted process when AS adjustment does not change the variance of the random shock or the system gain. Figure 1(c) shows the adjusted process when AS adjustment increases the variance of the random shock ($v = 2$) and Figure 1(d) when AS adjustment increases the variance of the random shock ($v = 2$) as well as changes the system gain ($g' = 0.5g$). Since $G = \lambda$ in each graph of (b), (c) and (d), the observed deviations up to $t = 20$ constitute a series of white noise and from $t = 21$, a series of ARMA(1,1) processes defined by (3.4), (3.5) and (3.6), respectively. We see that the increment of the random shock variance from $\sigma_a = 1$ to $\sigma_b = 2$ results in AS deviations in (c) that wander about more variably than in (b). The AS deviations in (d) due to the reduction in the system gain, when compared to (c), show an additional small variation for the first 20 periods after the special cause, but resemble that in (c) afterwards.

3.2. Monitoring procedure

We use the observed deviations from target to detect the occurrence of a special cause. Observed deviations from target have been used by Vander Wiel (1996) in monitoring level shifts in IMA(1,1) process.

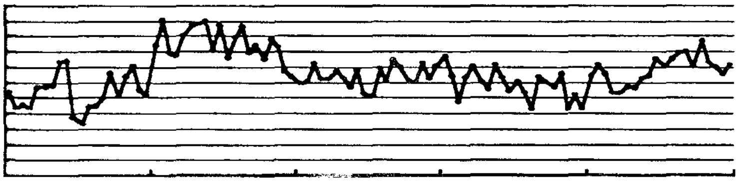
For monitoring level shifts with observed deviations, the EWMA chart statistic is defined as

$$E_t = rO_t + (1-r)E_{t-1} \tag{3.7}$$

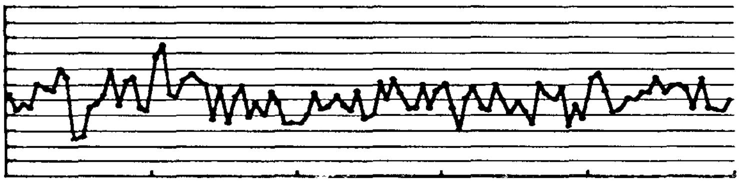
for a weight r ($0 < r \leq 1$). The monitoring procedure is to observe the EWMA statistic at every monitoring time and to signal if $|E_t| \geq c$ for a control limit c. We start the EWMA statistic at $E_0 = 0$. If a signal is found to be a false alarm, we restart the EWMA procedure with $E_t = 0$.

4. SELECTION OF CONTROL PARAMETERS

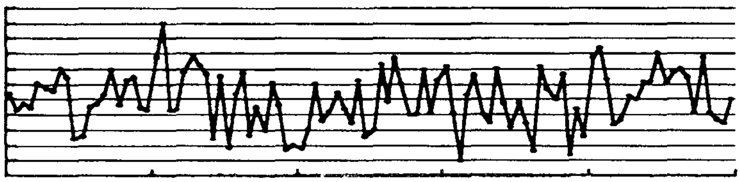
In the repeated adjustment scheme the costs to be considered are mainly those due to being off-target and due to false alarms. Thus the minimum cost policy would be to choose a scheme which minimize the expected cost per interval due to being off-target and false alarms. Often the false alarm cost, which does play an important role in determining control parameters, is not easy to estimate precisely. Instead of considering the cost of false alarms, thus, we try to minimize the mean squared deviations (MSD) subject to the constraint that the false alarm rate (FAR), say α, is bounded by a fixed probability, say f_0.



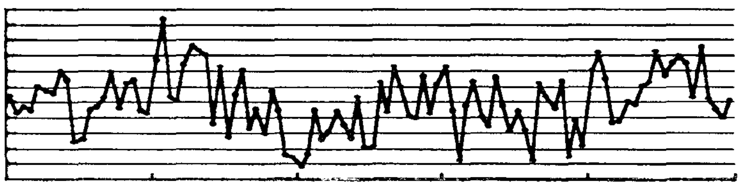
(a) $Z_t = Z_{t-1} + a_t - 0.7a_{t-1} + 3I_{\{21\}}(t)$



(b) $O_t = 0.7O_{t-1} + a_t - 0.7a_{t-1} + 3I_{\{21\}}(t)$



(c) $O_t = 0.7O_{t-1} + b_t - 0.7b_{t-1} + 3I_{\{21\}}(t)$



(d) $O_t = 0.85O_{t-1} + b_t - 0.7b_{t-1} + 3I_{\{21\}}(t)$

FIGURE 1 *Series of disturbance* Z_t *and adjusted disturbance* O_t *generated by an IMA(0,1,1) with a step shift:* $\theta=0.7$, $\sigma_a=1$, $\delta=3$, $U=21$, $G=0.3$, $v=2$ *and* $g^*=0.5$

If the process level follows a pure IMA(1,1) process, that is an IMA(1,1) process without a step shift, then it is obvious that the minimum MSD policy is to choose $G = \lambda$, which may not be exactly true in an IMA(1,1) process with a step shift because of the step shift. In Section 5, it will be shown that the MSD curve tends to be flat in the neighborhood of the minimum so that moderate departures from the theoretically best value of G are unlikely to greatly increase the MSD. Thus we suggest to use $G = \lambda$ for practical purposes. Advantages of selecting $G = \lambda$ are that the observed deviations in a BS period are a series of white noise and observed deviations in a AS period are a series of independent random variates unless the system gain is changed.

Let S and F be the total sum of squared deviations and the total number of false alarms, respectively, in a cycle and let L be the cycle length in an IPC procedure. Then the MSD and FAR in the IPC procedure are defined as

$$\text{MSD} = \frac{E(S)}{E(L)}, \tag{4.1}$$

$$\text{FAR} = \frac{E(F)}{E(U)}. \tag{4.2}$$

In evaluating the properties of the IPC procedure for given process parameters, an extensive simulation is used by repeating 200,000 cycles of the process. For the numerical results in this study, combinations of the following sets of parameters are considered: $p \in \{0.001, 0.002, 0.005, 0.01, 0.02, 0.05\}$, $\theta \in \{0, 0.3, 0.5, 0.7\}$, $\sigma_b \in \{1, 1.5, 2, 3, 5\}$, $\delta \in \{0.5, 1, 3, 5\}$, $g' \in \{0.5, 1\}$, $\alpha \in \{0.0002, 0.0005, 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1\}$.

The constrained optimization problem is to find G, r and c which minimize MSD subject to $\alpha \leq f_0$. The constrained minimization of the MSD is always achieved at the boundary $\alpha = f_0$ for each given value of r. This is quite straightforward because the MSD is an increasing function of the control limit c while FAR is a decreasing function of it. The control limit was estimated as a third order polynomial of r in Table 4.1. All the regression functions are fitted almost perfectly with $R^2 > 0.999$ for ranges of α from 0.0002 to 0.1 and r from 0.05 to 1. The control limit was quite insensitive to the geometric distribution parameter p, which represents the occurrence of the special cause.

In Figure 2 four pairs of control limit curves, which satisfy the required FAR, are drawn as a function of r. For each pair the dotted line denotes the value evaluated by simulation and the solid one denotes the line estimated by the polynomial function in Table 4.1. The four pairs of curves, from top to bottom, correspond to $\alpha = 0.001, 0.002, 0.005, 0.01$, respectively.

TABLE 4.1 *Control limit c of the EWMA chart estimated as a polynomial of r for given* α

α	$c = f(r),\ 0.05 \leq r \leq 1$
0.001	$0.254 + 4.523 \cdot r - 3.405 \cdot r^2 + 1.934 \cdot r^3$
0.002	$0.222 + 4.279 \cdot r - 3.217 \cdot r^2 + 1.823 \cdot r^3$
0.005	$0.173 + 3.958 \cdot r - 3.009 \cdot r^2 + 1.696 \cdot r^3$
0.01	$0.133 + 3.663 \cdot r - 2.750 \cdot r^2 + 1.536 \cdot r^3$

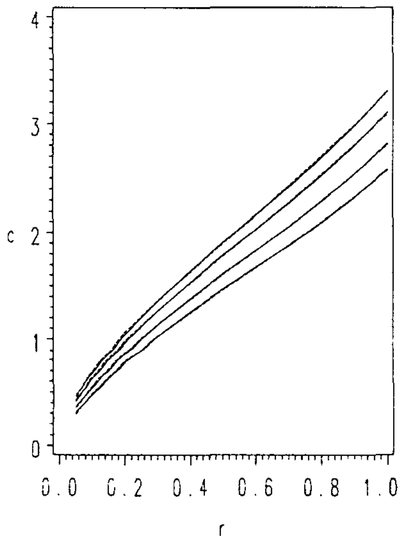


FIGURE 2 *Control limit curves of the EWMA chart determined by polynomial regression (solid curve) and simulation (dotted curve) for given r when* α *is given. Four pairs of curves correspond to* α*=0.001, 0.002, 0.005 and 0.01 from top to bottom.*

Finding the exact optimal value of r for each of the given process parameters requires complicated and tedious computations. Figure 3 shows an example of graphs that can be used as an aid in choosing a value of r for a particular application. The panels in Figure 3 show MSD's when $\sigma_b = 1$ for various values of δ, θ and r, with c calculated by Table 4.1 to provide the given α. The panels are arranged so that rows represent values of δ from 0.5 to 5 and columns represent values of θ from 0 to 0.7. In each panel the left and right graphs are for $g' = 0.5$ and 1, respectively. Each graph has four curves which correspond, from top to bottom, to IPC procedures designed to have α values of 0.001, 0.002, 0.005 and 0.01, respectively. Each curve shows MSD's for values of r for 0.05, 0.1, 0.2, ..., 0.9, 1. Similar figures have been drawn for cases when $\sigma_b = 1.5, 2, 3$ and 5, but these figures are not shown here. Each figure shows that the MSD is quite insen-

TABLE 4.2 *Values of r which provide near minimum MSD's for given σ_b and δ*

		σ_b				
		1	1.5	2	3	5
δ	0.5	0.1	0.2	0.7	0.8	0.9
	1	0.2	0.3	0.7	0.8	0.9
	3	0.7	0.7	0.8	0.8	0.9
	5	0.8	0.9	0.9	0.9	0.9

sitive to values of θ, g' and α in choosing appropriate values of r. From Figure 3, for example, the most reasonable choice of r, which minimizes MSD, seems to be 0.1, 0.2, 0.7, 0.8 for $\delta = 0.5$, 1, 3, 5, respectively, regardless of

The most reasonable values of r chosen by considering the figures are listed in Table 4.2 for various values of σ_b and δ. The reasonable choice of r tends to be larger as the amount of shift δ increases, which is the same trend as in the EWMA chart designed for minimizing out-of-control average run length (ARL) given a fixed in-control ARL. The reasonable choice of r also tends to be larger as the standard deviation of the random shock, σ_b, increases because a larger variance makes the process level deviate more variably from target, which produces an effect similar to a level shift. When $\sigma_b > 5$ and/or $\delta > 5$ the reasonable choice would be to use $r = 1$, which corresponds to the individual X-chart.

In addition to MSD graphs the ARL's during an AS period (ASARL) are also considered. Since the ASARL describes the average time delay to a true signal, considering the ASARL with c calculated by Table 4.1 corresponds to measuring sensitivity of the EWMA chart to the level shift subject to the constrained FAR. Figure 4 shows an example of ASARL graphs which correspond to the case when $\sigma_b = 1$ for various values of δ, θ and r, with c calculated by Table 4.1 to provide the specified α. The structure of Figure 4 is exactly the same as Figure 3. Similar figures also have been drawn for cases when σ_b=1.5, 2, 3 and 5, but these figures are not shown here. It can be easily noticed that reasonable choices of r according to ASARL are almost always the same as those according to MSD. This result was fortunate, although it was somewhat expected, for those who design the IPC procedure, otherwise using MSD in choosing r conflicts with using ASARL.

5. ROBUSTNESS OF G TO DEPARTURE FROM THE OPTIMAL VALUE

In the previous section the damping factor was chosen as equal to λ for practical purposes, although λ is not the exactly optimal choice for G. Another sit-

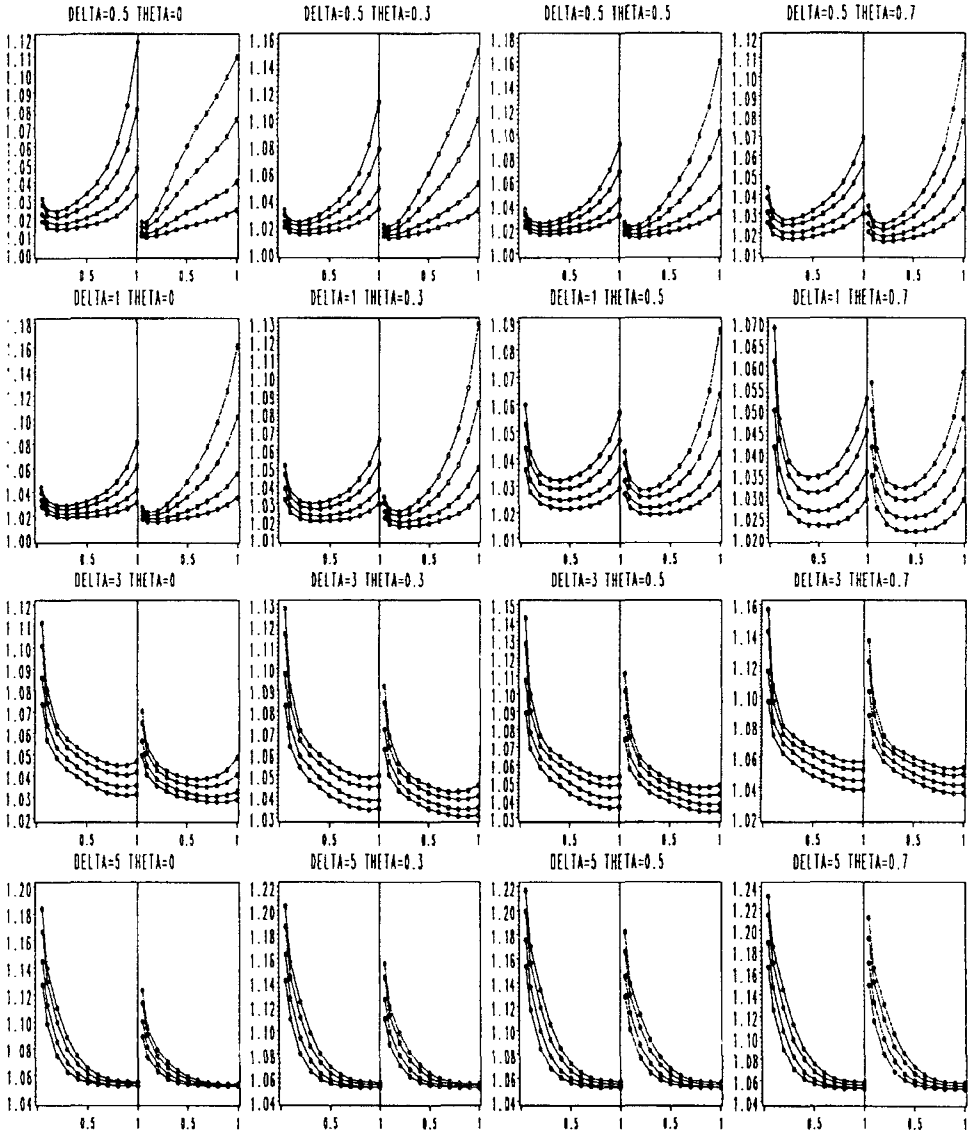


FIGURE 3 *The MSD's of the IPC procedure for r=0.05, 0.1, ..., 0.9, 1 when σ_b=1; Each row and column of panels represent δ and θ, respectively. The left and right graphs of each panel are for g'=0.5 and 1, respectively. Four curves of each graph correspond to α=0.001, 0.002, 0.005 and 0.01 from top to bottom.*

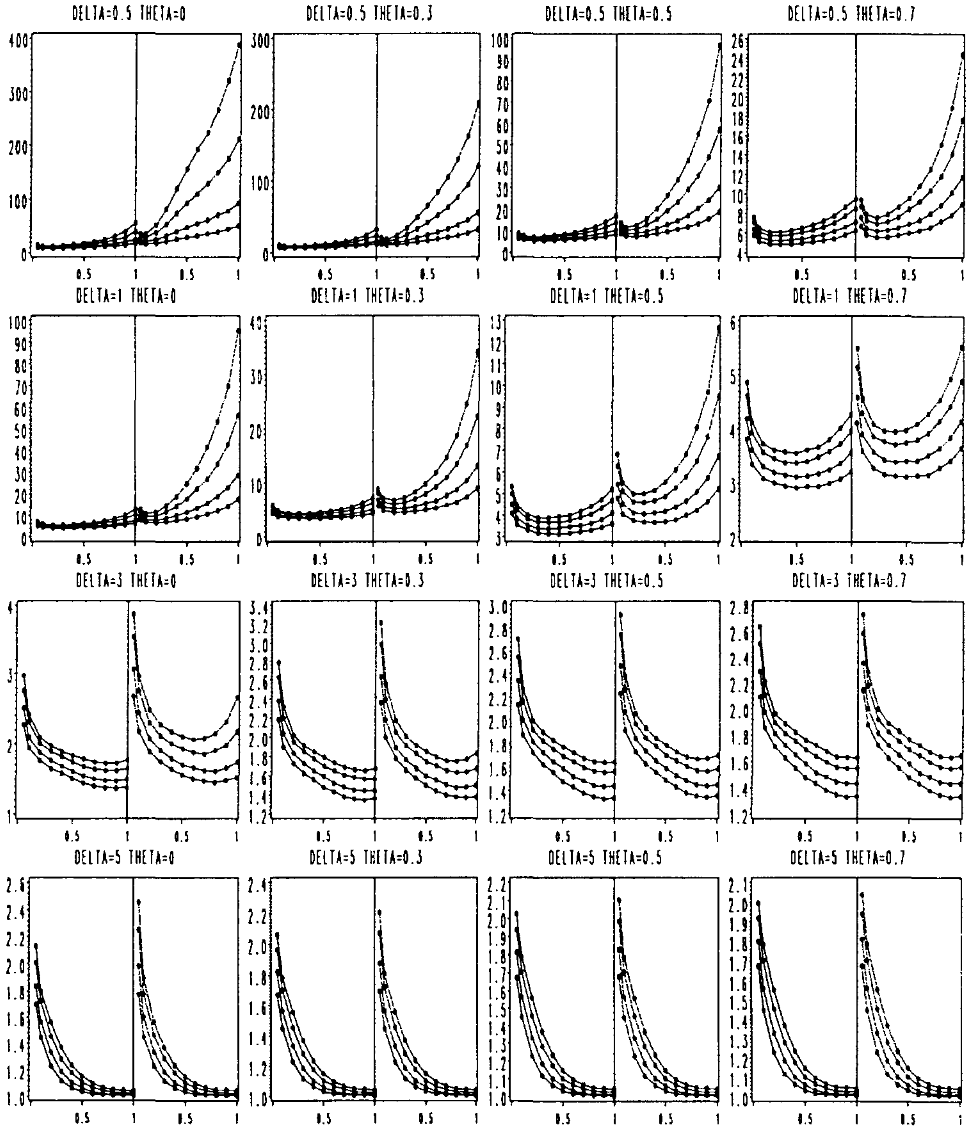


FIGURE 4 *The ASARL's of the IPC procedure for r=0.05, 0.1, ..., 0.9, 1 when σ_b=1; Each row and column of panels represent δ and θ, respectively. The left and right graphs of each panel are for g'=0.5 and 1, respectively. Four curves of each graph correspond to α=0.001, 0.002, 0.005 and 0.01 from top to bottom.*

uation expected in practice is that the estimate of λ, when is of unknown, will not be equal to the true value. Thus it is desirable to study the effect of using G not equal to the optimal value or the true λ, due to practical purposes and/or misestimation.

Figure 5 shows panels of MSD curves for values of G, where r is chosen by Table 4.2. In each panel the horizontal axis denotes G and the dotted reference line indicates the true λ. The panels are arranged so that the rows represent values of σ_b from 1 to 5 and the columns represent values of θ from 0 to 0.7. In each panel MSD curves for combinations of α=0.001, 0.002, 0.005, 0.01 and δ=0.5, 1, 2, 3, 5 are plotted for values of G. It is noticeable that the twenty MSD curves for combinations of α and δ values are almost the same when θ is given. Especially when $\theta = 0$ and 0.3 the MSD curves almost exactly coincide like a single curve. Moreover the curves are almost the same for different values of σ_b.

We see that the value of G which gives the minimum MSD is almost identical to the true λ, which implies that the use of $G = \lambda$ is a good choice.

6. AN EXAMPLE

In a chemical vapor deposition (CVD) reactor system a deposition process takes place to synthesize a thin solid film from the gaseous phase by a chemical reaction. The thickness of the film is the key quality characteristic of the CVD process and very sensitive to the thermal reactor conditions. A hot wall reactor is a kind of a reactor which uses a heater to raise the wall temperature to the desired level. A disadvantage of hot wall reactors is that the heater has a short life time. Thus it is necessary to change the heater periodically.

Since the reactor can not be fully emptied between batches, serial correlation among batches is expected. Also the thickness of the film is affected by many other physical and chemical process conditions, which can not be controlled precisely. We assume that the departure from the target thickness follows an IMA(1,1) process, where the time interval is treated as the time needed for one deposition process.

The gas flow rate is used to adjust the thickness of the film as a compensatory variable. A one unit increase in the gas flow rate produces a 0.01 unit increase in the film thickness. Since the effect of changes in the gas flow rate is completely realized in one time interval, the CVD reactor system is a responsive system. If the heater is not able to establish the thermal equilibrium of the wall, system engineers consider that the lifetime of the heater is over and the film thickness

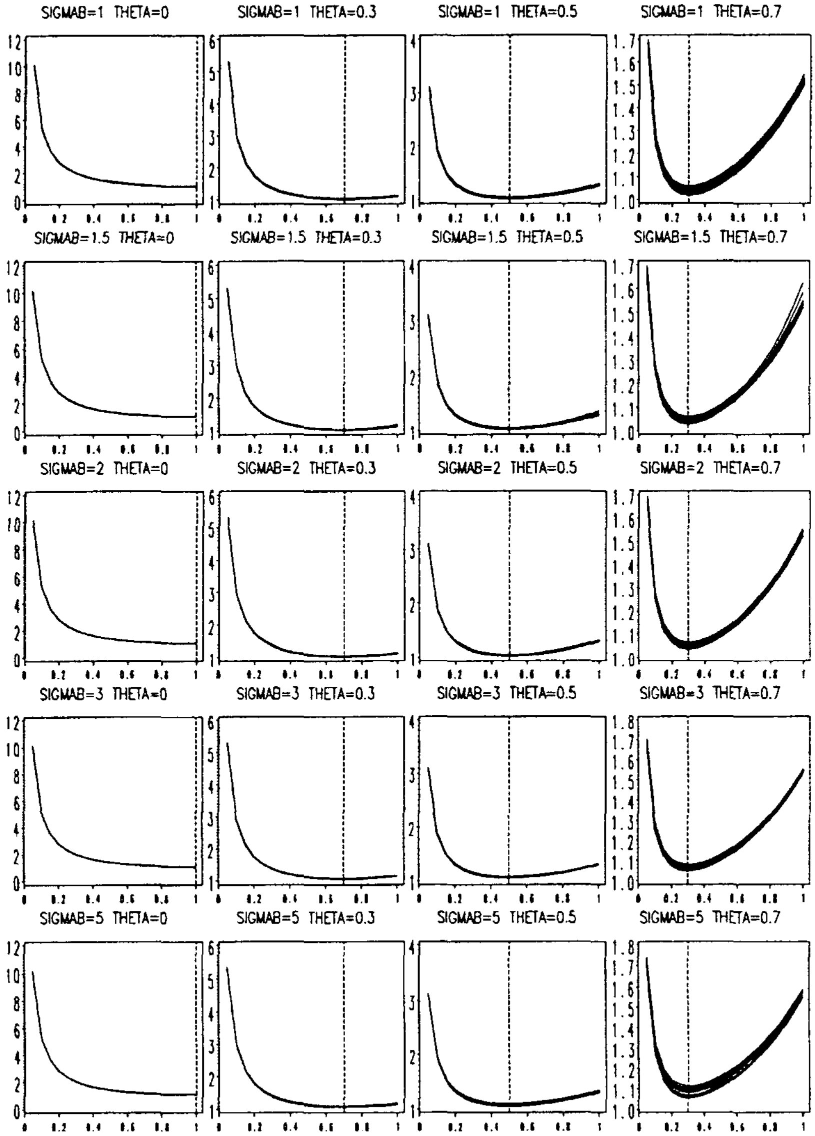


FIGURE 5 *The MSD's of the IPC procedure for values of G when r is chosen by Table 2; Each row and column of panels represent σ_b and θ, respectively. In each panel 20 MSD curves for combinations of α=0.001, 0.002, 0.005, 0.01 and δ=0.5, 1, 2, 3, 5 are plotted. The dotted reference line indicates $\lambda(=1-\theta)$.*

is expected to drop to one unit on the average of the standard deviation of the random shock ($\delta = 1$). The average life time of the heater is known to be 100 time intervals.

Suppose that the current practice of this CVD process is to use the EPC procedure, which adjusts the film thickness by compensating the gas flow rate and to replace the heater at the mean life time of the heater, *i.e.* after every 100 time intervals. System engineers are interested in improving the CVD process and decided to use the IPC procedure for controlling the process. The IPC procedure is to adjust the thickness as well as to monitor for the failure of thermal equilibrium of the heater by use of the EWMA chart.

For the purpose of comparison between EPC and IPC we assume that the parameters in the disturbance model are known as $\theta = 0.7$ (thus $G = 0.3$ is used) and $\sigma_a = 1$. If we increase the gas flow rate when the heater loses the thermal equilibrium it is expected that the standard deviation of the disturbance random shock is doubled and the system gain by the deposition rate is changed to 0.007 from 0.01 (thus $g' = 0.7$) for a one unit increment of the gas flow rate. We also assume that the time until failure of the heater follows a geometric distribution with mean of 100 time intervals. When an out-of-control signal is triggered by the EWMA chart we replace the heater if the signal is true.

For a fair comparison a cycle of both procedures is defined as the period from the start to the change of the heater and the MSD is calculated by simulation, where 200,000 cycles were repeated. For the EPC design, the MSD per cycle is calculated as $\text{MSD}_\text{E} = 2.196$. For the IPC design, we chose the FAR as 0.01 and the weight of the EWMA chart as $r = 0.2$ which corresponds to $\delta = 1$ and $\sigma_b = 2$ in Table 4.2. Thus the control limit is calculated as $c = 0.768$ by Table 4.1. The MSD and FAR of the IPC design was calculated as $\text{MSD}_\text{I} = 1.483$ and $\text{FAR} = 0.009$ with the average cycle length equal to 103.0 intervals.

Comparing the IPC procedure to the current practice, we have a significant reduction in MSD with very low FAR, *i.e.* 32.5% reduction in MSD with less than 1% FAR. When FAR is decreased we have the following results.

FAR	MSD_I	*Average cycle length*
0.0046	1.544	104.0
0.0017	1.628	105.0
0.0008	1.689	106.3

As the FAR is decreased the MSD is increasing gradually, but is still significantly less than that of the EPC procedures.

7. Concluding Remarks

The IPC procedure proposed here combines the repeated adjustment procedure and the EWMA charting procedure. The disturbance model considered is the IMA(1,1) model with a step shift when no adjustment is made. It was assumed that AS adjustment will increase the process variability and change the system gain. A modified IMA(1,1) model with a step shift designed for the AS adjustment was also proposed. The effectiveness of the procedure was evaluated by the MSD of the process cycle subject to a constraint on the FAR.

For practical reasons the damping factor used in the difference equation was set as the nonstationary parameter, which was shown to be near to the optimal value. The control limit of EWMA chart was expressed as a polynomial of the weight of the chart to satisfy the specified FAR. Reasonable choices for the weight in the EWMA chart were obtained by considering the MSD and ASARL values which are mainly dependent on the size of the step shift and the variability of the random shock under the presence of a special cause. An application of the IPC procedure to a CVD reactor system has resulted in a significant reduction in MSD, compared to the EPC procedure, with a very low FAR.

When the cost for adjustment is substantial, a different kind of feedback adjustment should be employed which can take such cost into account. The bounded adjustment scheme is one good candidate for further research of the IPC procedure when there is a fixed cost associated with an adjustment.

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References

Box, G. E. P., Coleman, D. E. and Baxley, R. V. Jr. (1997). "A comparison of statistical process control and engineering process control", *Journal of Quality Technology*, **29**, 128–130.

Box, G. E. P. and Kramer, T. (1992). "Statistical process monitoring and feedback adjustment-A discussion", *Technometrics*, **34**, 251–267.

Box, G. E. P. and Luceño, A. (1994). "Selection of sampling interval and action limit for discrete feedback adjustment", *Technometrics*, **36**, 369–378.

BOX, G. E. P. AND LUCEÑO, A. (1997). *Statistical Control by Monitoring and Feedback Adjustment*, John Wiley & Sons, New York.

JANAKIRAM, M. AND KEATS, J. B. (1998). "Combining SPC and EPC in a hybrid industry", *Journal of Quality Technology*, **30**, 189–200.

MACGREGOR, J. F. (1987). "Interfaces between process control and on-line statistical process control", *Computing and Systems Technology Division Communications*, **10**, 9–20.

MASTRANGELO, C. M. AND BROWN, E. C. (2000). "Shift detection properties of moving centerline control chart schemes", *Journal of Quality Technology*, **32**, 67–74.

MONTGOMERY, D. C. (1999). "A perspective on models and the quality sciences : Some challenges and future directions", *ASQ Statistics Division Newsletter*, **18**, 8–13.

MONTGOMERY, D. C., KEATS, J. B., RUNGER, G. C. AND MESSINA, W. S. (1994). "Integrating statistical process control and engineering process control", *Journal of Quality Technology*, **26**, 79–87.

NEMBHARD, H. B. AND MASTRANGELO, C. M. (1998). "Integrated process control for startup operations", *Journal of Quality Technology*, **30**, 201–211.

RUHHAL, N. H., RUNGER, G. C. AND DUMITRESCU, M. (2000). "Control charts and feedback adjustments for a jump disturbance model", *Journal of Quality Technology*, **32**, 379–394.

VANDER WIEL, S. A. (1996). "Monitoring processes that wander using integrated moving average models", *Technometrics*, **38**, 139–151.

VANDER WIEL, S. A., TUCKER, W. T., FALTIN, F. W. AND DOGANAKSOY, N. (1992). "Algorithmic statistical process control : Concepts and an application", *Technometrics*, **34**, 286–297.