공정평균 이동을 탐지하기 위한 적응 합성 관리도*

임태진†
숭실대학교 산업·정보시스템공학과

An Adaptive Synthetic Control Chart for Detecting Shifts in the Process Mean

Taejin Lim
Dept. of Industrial & Information Systems Engineering, Soongsil University

Key Words : Adaptive scheme, ARL, ATS, Double sampling chart, EWMA, Synthetic control chart, Variable sampling scheme

Abstract

The synthetic control chart (SCC) proposed by Wu and Spedding (2000) is to detect shifts in the process mean. The performance was re-evaluated by Davis and Woodall (2002), and the steady-state average run length (ARL) performance was shown to be inferior to cumulative sum (CUSUM) or exponentially weighted moving average (EWMA) chart. This paper proposes a simple adaptive scheme to improve the performance of the synthetic control chart. That is, once a non-conforming (NC) sample occurs, we investigate the next L-consecutive samples with larger sample sizes and shorter sampling intervals. We employ a Markov chain model to derive the ARL and the average time to signal (ATS). We also propose a statistical design procedure for determining decision variables. Comprehensive comparative study shows that the proposed control chart is uniformly superior to the original SCC or double sampling (DS) $\overline{X}$ chart and comparable to the EWMA chart in ATS performance.

1. Introduction

The SCC proposed by Wu and Spedding (2000) is composed of sub-$\overline{X}$ chart and conforming run length (CRL) chart. The CRL is the number of samples to get a non- conforming (NC) sample after the most recent previous NC sample. A set of sample is called an NC sample whenever

† 교신저자 tjlim@ssu.ac.kr
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the sample mean falls outside of sub-$\overline{X}$ control limits. The SCC produces the 'out-of-control' signal, if we have $CRL \leq L$, where L is a specified positive integer. Wu and Spedding (2000) argued that the SCC consistently produce smaller out-of- control ARL than the Shewhart $\overline{X}$ chart and outperform EWMA chart when the shift is greater than 0.8σ. The ARL proposed by them is

$$ARL(\delta) = \frac{1}{p(\delta)} \times \frac{1}{1-[1-p(\delta)]^{L}}, \quad (1)$$

where

$p(\delta) = 1 - \Phi(k - \delta\sqrt{n}) + \Phi(-k - \delta\sqrt{n})$ denotes the probability that an NC sample occurs when the process mean shifts as much as $\delta\sigma$.

Davis and Woodall (2002) pointed out that the SCC is based on a head start (HS) feature which postulates that the current sample be NC. The HS feature makes the SCC more sensitive to a mean shift, but the out-of-control ARL can not be obtained in real processes, because there is no guaranty that the sample just before an assignable cause occurs is NC. The Markov chain model was presented by Davis and Woodall (2002) in order to calculated the steady-state ARL. They showed, however, that the SCC did not perform as well as CUSUM and EWMA charts.

In this paper, we propose a simple adaptive scheme to improve the performance of the SCC. That is, once an NC sample occurs, we investigate the next L-consecutive samples with larger sample sizes and shorter sampling intervals. If no more NC sample occurs in the L-consecutive severe samples, we go back to the normal sampling scheme. By this way, we may improve the power of the control chart while reducing the chance of false alarms. We present a Markov chain model describing the adaptive scheme, and derive formulae for the ARL and the ATS. We also propose a statistical design procedure for determining decision variables. We perform a comprehensive comparative study and show that the proposed control chart is uniformly superior to the original SCC or DS $\overline{X}$ chart and comparable to the EWMA chart in ATS performance.

2. Adaptive SCC

The SCC signals if two out of $L+1$ successive samples are NC. The regular SCC employes a fixed sample size n_0 and a sampling interval h_0. Once an NC sample occurs, we should determine whether assignable cause really occurs or not as soon as possible. Therefore, we may need a severe sampling scheme with a bigger sample size and a shorter sampling interval than usual situation. If we get no more NC sample in the next

L-consecutive samples, we may conclude that the first NC sample occurred by chance, then we go back to the normal sampling scheme. Otherwise, we may conclude that a shift in mean occurs, so the chart produces an 'out-of-control' signal.

Let n_1 and h_2 denote normal sampling size and sampling interval, respectively. Let n_2 and h_1 denote severe sampling size and sampling interval, so that $n_1 < n_2$ and $h_1 < h_2$. Let Z_i denote a standardized sample mean statistic, and $[-k,\ k]$ denote the corresponding control limit. Then we may summarize the adaptive sampling scheme as

$$[n(i), h(i)] = \begin{cases} [n_1, h_2], & \text{if } -k \le Z_{i-L}, \cdots, Z_{i-1} \le k \\ [n_2, h_1], & \text{otherwise.} \end{cases} \tag{2}$$

We assume that the process starts with in-control state($\mu = \mu_0$), and the process mean shifts to $\mu_1 = \mu_0 \pm \delta\sigma$, upon an assignable cause occurs. Given the process mean shifts as much as $\delta\sigma$ from the in-control value, let $p_1(\delta)$ and $p_2(\delta)$ denote the probability that an NC sample occurs under normal and severe sampling scheme, respectively. Then we have

$$p_i(\delta) = 1 - \Phi(k - \delta\sqrt{n_i}) + \Phi(-k - \delta\sqrt{n_i}),$$
$$(i = 1, 2). \tag{3}$$

As Davis and Woodall (2002) did, we define $(L+2)$ states as follows.

State 0 $(O, O, \cdots, O)$: the latest L samples are conforming

State 1 $(X, O, \cdots, O)$: the L-step ahead sample is NC, whereas the latest $L-1$ samples are conforming

...

State L $(O, O, \cdots, X)$: the previous sample is NC, whereas the latest $L-1$ samples before that one are conforming

State $L+1$ (Signal) : an absorbing state where the 'out-of-control' signal occurs, that is, $CRL \le L$

Then the following transition matrix would govern the Markov chain

$$P_\delta = \begin{pmatrix} Q_\delta & (I - Q_\delta)1 \\ 0^T & 1 \end{pmatrix},$$

where Q_δ denotes the transition matrix for transient states as

$$Q_\delta = \begin{pmatrix} q_1(\delta) & 0 & 0 & \cdots & 0 & p_1(\delta) \\ q_2(\delta) & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2(\delta) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & q_2(\delta) & 0 \end{pmatrix}, \tag{4}$$

$$p_1(\delta) \equiv P[|Z_i| > k \mid n_1, \delta]$$
$$= 1 - q_1(\delta),$$

$$p_2(\delta) \equiv P[|Z_i| > k \mid n_2, \delta]$$
$$= 1 - q_2(\delta).$$

Let $\pi^T = (\pi_0, \pi_1, \pi_2, \cdots, \pi_L)$ denotes the vector of steady state probabilities for transient states. Then the steady state ARL and ATS can be obtained by

$$ARL_\delta = \pi^T(I - Q_\delta)^{-1}1, \quad (5)$$

$$ATS_\delta = \pi^T(I - Q_\delta)^{-1}h, \quad (6)$$

where 1 is an $(L+1)$ unit vector, and $h^T = (h_2, h_1, h_1, \cdots, h_1)$.

In order to obtain the steady state probabilities, we should modify Q_0 so that we may impose the condition that no signal occurs. The modified transition matrix $Q_{M,0}$ is

$$Q_{M,0} = \begin{pmatrix} q_1(0) & 0 & 0 & \cdots & 0 & p_1(0) \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix}. \quad (7)$$

Then the steady state probabilities π can be obtained by solving following the equations

$$\pi^T Q_{M,0} = \pi^T, \quad \text{and}$$

$$\pi^T 1 = 1. \quad (8)$$

The steady state probabilities from Equation (8) is easily obtained by

$$\pi_0 = \frac{1}{1 + Lp(0)},$$

$$\pi_i = \frac{p(0)}{1 + Lp(0)}, \text{ for } i = 1, \cdots, L \quad (9)$$

3. Statistical Design of the Adaptive SCC

It is important for the ASCC(Adaptive Synthetic Control Chart) not to use more resources than the original Shewhart chart. We may impose restrictions for resource conservation such as

$$E[n(i) \mid Z_i \in [-k, k]; \delta = 0] \le n_0, \quad (10)$$

$$E[h(i) \mid Z_i \in [-k, k]; \delta = 0] \ge h_0 \quad (11)$$

From Equation (10), $n_1\pi_0 + n_2(\pi_1 + \pi_2 + \cdots + \pi_L) \le n_0$, and from Equation (9), we may impose restriction on n_2 as

$$n_2 \le n_0 + \frac{n_0 - n_1}{Lp(0)}. \quad (12)$$

Likewise, from Equation (9) and (11), we may impose restriction on h_2 as

$$h_2 \ge h_0 + Lp_0(h_0 - h_1). \quad (13)$$

Let $\overrightarrow{ARL_0} = (I - Q_0)^{-1}1 \equiv (a_0, \cdots, a_L)^T$. Then from Equation (4),

$$a_i = \frac{[1 - p(0)]^i}{D_1} + \frac{1}{p(0)}, \quad (14)$$

for $i = 0, \cdots, L$,

where $D_1 = p(0)[1 - (1 - p(0))^L]$, and $p(0) \equiv P[|Z_i| > k \mid \delta = 0]$ $= 2[1 - \Phi(k)]$, which does not depend on n. Note that each element a_i represents the ARL_0 starting from state

i, $(i=0,\cdots,L)$. As Davis and Woodall mentioned, the last element a_L represents the ARL of the HS SCC. From Equation (9) and (14), we can obtain the steady state ARL as

$$ARL_0 = \frac{D_2}{D_1\,[1+Lp(0)]} + \frac{1}{p(0)} \quad ,(15)$$

where

$$D_2 = 1 + [1-p(0)][1-\{1-p(0)\}^L].$$

Once the CRL limit L is given, we can determine the control limit k by solving

$$ARL_0 = \frac{D_2}{D_1\,[1+Lp(0)]} + \frac{1}{p(0)} = \frac{1}{\alpha} = \frac{1}{2\,[1-\Phi(k_0)]}, \quad (16)$$

where k_0 denotes the standardized control limit of Shewhart $\overline{X}$ chart. We first obtain $p(0)$, then we get k from $p(0)=2\,[1-\Phi(k)]$. Table 1 provides the value of k for each L, given $k_0=3$. The value of control limit k without HS feature is a little tighter than that with HS feature for each L.

Let $\overrightarrow{ATS_0} = (I-Q_0)^{-1}h \equiv (t_0, t_1, \cdots, t_L)^T$.

Then from Equation (4),

$$t_i = \frac{h_2[1-p(0)]^i}{D_1} + \frac{h_1}{p(0)}, \quad (17)$$

for $i=0,\cdots,L$.

TABLE 1. Different Sets of Values of k and L

L	k with HS	k for steady state
1	1.94347	1.93283
2	2.08481	2.07057
3	2.16404	2.14716
4	2.21877	2.19974
5	2.26040	2.23952
6	2.29388	2.27137
7	2.32183	2.29784
8	2.34576	2.32043
9	2.36667	2.34008
10	2.38520	2.35745

Also from Equation (9) and (17), we can obtain the steady state ATS as

$$ATS_0 = \frac{h_2 D_2}{D_1\,[1+Lp(0)]} + \frac{h_1}{p(0)} \quad (18)$$

Once L, h_0, and h_1 is given, we may determine h_2 by solving

$$ATS_0 = \frac{h_0}{\alpha} = \frac{h_0}{2\,[1-\Phi(k_0)]},$$ or from Equation (18),

$$h_2 = \left(\frac{h_0}{2\,[1-\Phi(k_0)]} - \frac{h_1}{p(0)}\right)\frac{D_1\,[1+Lp(0)]}{D_2}. \quad (19)$$

As a result, once L, h_0, n_0, n_1, and n_2 is given, the variable to be determined is h_1 only. Moreover, h_1 can be easily determined, because ATS_δ under restriction (19) is a linear function of h_1. That is, the optimal h_1 is either the minimum value $h_{\min}$ or h_0.

Another useful property is that the ARL_δ and ATS_δ decreases as n_2 increases for fixed L, k, and n_1, because lager n_2 would make the ASCC more sensitive. Therefore, it is best to use maximum n_2 satisfying the resource constraint in Equation (12). Employing all these properties, we propose a procedure for designing the ASCC as follows.

1. Specify δ, n_0, h_0, k_0, and ARL_0 for fixed sample size and sampling interval (FSSI) chart.
2. Initialize L as 1.
3. Determine k by solving Equation (16) numerically.
4. Initialize n_1 as 1.
5. Determine $n_2 = \left[\dfrac{n_0 D_2 - n_1}{1-[1-p(0)]^L} \right]$.
6. Minimize ATS_δ w.r.t. h_1, while h_2 satisfying Equation (19).
7. Store the ATS_δ, and the corresponding L, n_1, n_2, k, h_1.
8. Increase n_1 by 1 and go back to Step (5), while $n_1 \le n_0 - 1$.
9. Store the minimum ATS_δ for given L, and the corresponding L, n_1, n_2, k, h_1.
10. If L is less than the specified maximum value, increase L by 1 and go back to Step (3). Otherwise, go to the next step.
11. Take the minimum ATS_δ and the corresponding L, k, n_1, n_2, h_1, h_2 as the final design variables for the ASCC.

4. Numerical Experiments

4.1 Comparison with the Steady State SCC

Fixing $n_0 = 4$ and $h_0 = 1$, we compare ATS_δ of the proposed chart with those of steady state (SS) SCC and HS SCC. Figure 1 and Table 2 show the results. For each shift δ and CRL limit L, the optimal value of design parameters n_1, n_2, h_2, the ARL_δ, ATS_δ of the ASCC, the ARL_δ of the SS SCC, and the ARL_δ of the HS SCC are listed. The optimal value of h_1 is not listed, because $h_2 = 1$ implies $h_1 = 1$, and $h_2 > 1$ implies $h_1 = h_{\min}$. Recall that the HS SCC is impractical, because it is not quarantied that the sample just before the assignable cause occurs is NC. Therefore, it is more meaningful to compare the

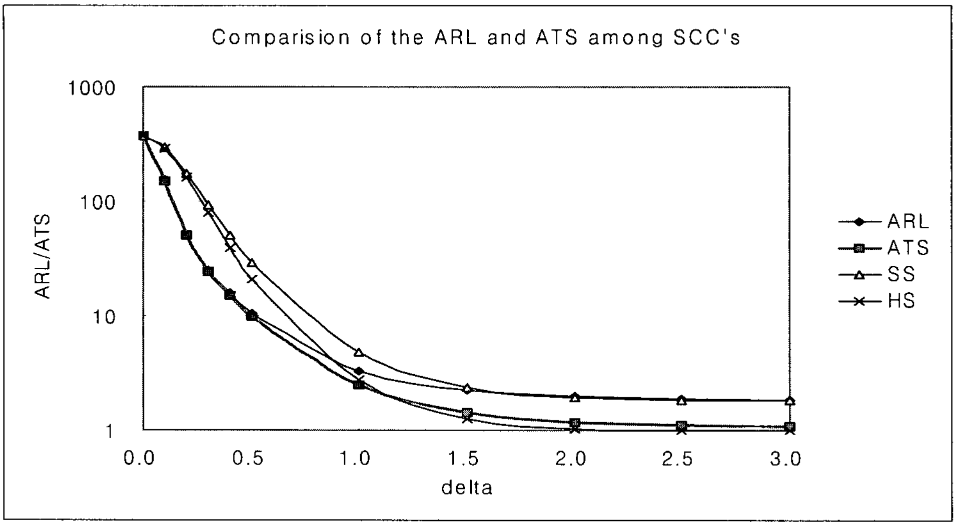

FIGURE 1. Comparison of ASCC with original SCC ($h_{\min}=0.1$)

ARL_δ, ATS_δ of the ASCC with the ARL_δ of the SS SCC. For the ASCC, the ARL_δ is the same as ATS_δ whenever $h_2=h_0=1$.

For $\delta\leq 2.0$, the optimum value of the CRL limit L is obtained to be 1. It can be interpreted that once an NC sample occur, the increased sample size n_2 makes the ASCC more powerful, so the next sample is highly likely to be NC. It can also be found that the ARL_δ of the ASCC is almost as low as the ATS_δ for small shift. However, as the shift increases, the ATS_δ gets shorter than the ARL_δ. To put it in another way, adjusting sampling size is crucial for small shifts, but adjusting sampling interval in more important to reducing the ATS_δ for large shifts in ASCC.

It is noticeable that the ARL_δ, ATS_δ of the ASCC is substantially shorter than the ARL_δ of the SS SCC or the HS SCC, for small shift ($\delta\leq 0.5$). In special for $\delta=0.3$, the reduction rate of the ARL_δ is almost 70%. For such small shift, even the HS feature is not effective, but the adaptive feature is much more useful. For large shift such as $\delta=2.0$, the ARL_δ of the ASCC is worse than that of the SS SCC. However, the ATS_δ of the ASCC is still better than that of the SS SCC. That is, the ATS_δ of the ASCC is uniformly better than that of the SS SCC, as shown in Figure 1.

4.2 Comparison with the EWMA chart

It would be more interesting to compare the performance of ASCC with other control chart such as EWMA or DS chart. Lucas and Saccucci (1990) provided a comprehensive table of ARL_δ for EWMA chart with λ ranging from 0.03 to 1.0. The table is for $n_0=1$, but the ASCC can be applied for $n_0 \geq 2$. For fair comparison, the shift in Table 3 of Lucas and Saccucci (1990) should be divided by $\sqrt{n_0}$, because $\delta\sigma$ shift in the original sample is equivalent to $\delta\sigma/\sqrt{n_0}$ shift in $\overline{X}$ with sample size n_0. Fixing ARL_δ and ATS_δ to be around 500, we compare the performance of ASCC with that of EWMA for $n_0=2,5$, and 10 as in Table 3-5, and Figure 2-4. The ASCC is extended to allow n_1 up to n_0, because it is not necessary to exclude the case $n_1=n_2=n_0$. The EWMA ARL_δ values in the Tables are the minimum value for each δ among 10 cases of weight λ. In the Tables, the ratio of ASCC ATS to that of minimum EWMA is denoted by 'ATS-R'. For each δ, ATS-R less than 1 implies that the ATS of ASCC is shorter than that of any EWMA chart. For $n_0=2$, the ATS performance of ASCC was better than any EWMA for $1.4 \leq \delta \leq 2.8$. Let's call this range to be 'superior region', because for any mean shift in the region, the ATS performance of ASCC is guarantied to be uniformly better than EWMA with any weight. For $n_0=5$, the 'superior region' shifted to $0.7 \leq \delta \leq 1.8$. For $n_0=10$, the 'superior region' shifted more toward zero such that $0.5 \leq \delta \leq 1.3$. We may say that the ASCC is uniformly better than EWMA for some range of shift, and the range can be controlled by adjusting the sample size n_0.

The ATS of the ASCC and all the 10 EWMA ARLs are shown in Figure 2, 3 and 4. They show more interesting features. For all cases of $n_0=2,5$, and 10, the ASCC locates in the lower part among 11 cases. That is, the ASCC is comparable to EWMA with arbitrary weight, for any range of mean shift.

4.3 Comparison with the DS chart

Suppose the minimum value of h_1 can be allowed to be zero. Then the adaptive scheme may be compared to the DS scheme by Daudin (1992). There are two major differences between the two schemes. First, ASCC does not employ threshold limit. Second, ASCC does not utilize the observed values of the first sample. These features may make ASCC more insensitive to mean shifts, but they make ASCC more robust to false alarms.

TABLE 2. Comparison of ASCC with original SCC ($h_{min}=0.1$) (small shift)

δ	L	1	2	3	4	5	6	7	8	9	10
	k	1.93283	2.07058	2.14718	2.19977	2.23956	2.27143	2.29791	2.32051	2.34018	2.35756
	$p(0)$	0.05326	0.0384	0.03178	0.02782	0.02512	0.02312	0.02157	0.02031	0.01927	0.0184
0.0	ARL	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
	ATS	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
	SS	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
	HS	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
0.1	n1	1	1	1	1	1	1	1	1	1	1
	n2	60	43	35	30	27	25	23	22	21	20
	E[n]	3.98	4.00	3.96	3.90	3.90	3.92	3.89	3.94	3.96	3.95
	h2	1.048	1.068	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	E[h]	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ARL	151.83	172.82	186.87	197.80	205.06	210.22	216.21	218.98	222.07	225.45
	ATS	151.65	172.74	186.87	197.80	205.06	210.22	216.21	218.98	222.07	225.45
	SS	313.30	308.01	305.22	303.40	302.09	301.09	300.30	299.66	299.13	298.68
	HS	311.45	305.29	301.83	299.46	297.68	296.27	295.10	294.11	293.27	292.53
0.2	n1	1	1	1	1	1	1	1	1	1	1
	n2	60	43	35	30	27	25	23	22	21	20
	E[n]	3.98	4.00	3.96	3.90	3.90	3.92	3.89	3.94	3.96	3.95
	h2	1.048	1.068	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	E[h]	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ARL	51.09	62.52	71.18	78.50	83.80	87.81	92.49	94.94	97.63	100.56
	ATS	50.84	62.46	71.18	78.50	83.80	87.81	92.49	94.94	97.63	100.56
	SS	203.41	192.75	187.47	184.17	181.89	180.21	178.92	177.91	177.09	176.42
	HS	198.87	186.34	179.69	175.29	172.08	169.59	167.57	165.90	164.49	163.27
0.3	n1	1	1	1	1	1	1	2	1	1	1
	n2	60	43	35	30	27	25	17	22	21	20
	E[n]	3.98	4.00	3.96	3.90	3.90	3.92	3.97	3.94	3.96	3.95
	h2	1.048	1.068	1.000	1.000	1.000	1.000	1.129	1.000	1.000	1.000
	E[h]	1.000	0.999	1.000	1.000	1.000	1.000	0.994	1.000	1.000	1.000
	ARL	24.69	30.90	35.70	39.82	42.98	45.51	48.95	50.01	51.81	53.69
	ATS	24.43	30.89	35.70	39.82	42.98	45.51	48.14	50.01	51.81	53.69
	SS	116.18	106.30	101.76	99.09	97.33	96.10	95.20	94.53	94.02	93.62
	HS	110.69	98.90	93.02	89.29	86.65	84.65	83.09	81.81	80.76	79.87
0.4	n1	2	2	2	2	2	2	2	2	2	3
	n2	41	30	24	21	19	18	17	16	15	9
	E[n]	3.97	4.00	3.91	3.90	3.90	3.95	3.97	3.96	3.92	3.93
	h2	1.048	1.068	1.084	1.097	1.108	1.119	1.129	1.138	1.147	1.155
	E[h]	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991
	ARL	15.59	18.83	21.71	23.75	25.46	26.60	27.78	29.01	30.32	33.78
	ATS	15.05	18.35	21.21	23.26	24.97	26.16	27.37	28.61	29.89	30.48
	SS	64.79	57.87	54.93	53.32	52.33	51.68	51.25	50.96	50.76	50.64
	HS	59.66	51.24	47.27	44.86	43.21	42.00	41.08	40.36	39.77	39.30
0.5	n1	2	3	3	3	3	3	3	3	3	3
	n2	41	17	14	12	11	11	10	10	9	9
	E[n]	3.97	4.00	3.96	3.90	3.89	3.97	3.92	3.98	3.89	3.93
	h2	1.048	1.068	1.084	1.097	1.108	1.119	1.129	1.138	1.147	1.155
	E[h]	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991
	ARL	10.42	13.07	14.63	16.11	17.07	17.19	18.32	18.48	19.79	19.94
	ATS	9.86	12.01	13.34	14.55	15.32	15.46	16.33	16.51	17.46	17.64
	SS	37.23	32.84	31.14	30.27	29.79	29.52	29.36	29.29	29.27	29.28
	HS	32.90	27.42	24.99	23.57	22.63	21.98	21.50	21.13	20.86	20.64

TABLE 2. Comparison of ASCC with original SCC ($h_{min}=0.1$) (large shift)

δ	L	1	2	3	4	5	6	7	8	9	10
	k	1.93283	2.07058	2.14718	2.19977	2.23956	2.27143	2.29791	2.32051	2.34018	2.35756
	$p(0)$	0.05326	0.0384	0.03178	0.02782	0.02512	0.02312	0.02157	0.02031	0.01927	0.0184
1.0	n1	3	3	3	3	3	3	3	3	3	3
	n2	22	17	14	12	11	11	10	10	9	9
	E[n]	3.96	4.00	3.96	3.90	3.89	3.97	3.92	3.98	3.89	3.93
	h2	1.048	1.068	1.084	1.097	1.108	1.119	1.129	1.138	1.147	1.155
	E[h]	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991
	ARL	3.27	3.55	3.76	3.94	4.08	4.16	4.30	4.36	4.50	4.55
	ATS	2.47	2.80	3.03	3.21	3.35	3.46	3.57	3.66	3.76	3.83
	SS	5.41	4.93	4.85	4.87	4.92	4.98	5.04	5.10	5.16	5.22
	HS	3.66	3.00	2.81	2.74	2.73	2.75	2.78	2.82	2.86	2.90
1.5	n1	3	3	3	3	3	3	3	3	3	3
	n2	22	17	14	12	11	11	10	10	9	9
	E[n]	3.96	4.00	3.96	3.90	3.89	3.97	3.92	3.98	3.89	3.93
	h2	1.048	1.068	1.084	1.097	1.108	1.119	1.129	1.138	1.147	1.155
	E[h]	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991
	ARL	2.27	2.32	2.35	2.38	2.39	2.40	2.41	2.42	2.43	2.44
	ATS	1.43	1.51	1.57	1.61	1.64	1.67	1.69	1.71	1.73	1.74
	SS	2.47	2.39	2.40	2.42	2.44	2.46	2.47	2.48	2.49	2.50
	HS	1.37	1.26	1.26	1.28	1.30	1.32	1.33	1.34	1.36	1.37
2.0	n1	3	3	3	3	3	3	3	3	3	3
	n2	22	17	14	12	11	11	10	10	9	9
	E[n]	3.96	4.00	3.96	3.90	3.89	3.97	3.92	3.98	3.89	3.93
	h2	1.048	1.068	1.084	1.097	1.108	1.119	1.129	1.138	1.147	1.155
	E[h]	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991
	ARL	2.01	2.01	2.01	2.00	2.00	1.99	1.99	1.98	1.98	1.98
	ATS	1.16	1.18	1.19	1.20	1.21	1.21	1.22	1.22	1.22	1.23
	SS	2.01	1.98	1.98	1.97	1.97	1.96	1.96	1.95	1.95	1.94
	HS	1.04	1.03	1.03	1.04	1.04	1.05	1.05	1.05	1.05	1.06
2.5	n1	3	3	3	3	3	3	3	3	3	3
	n2	22	17	14	12	11	11	10	10	9	9
	E[n]	3.96	4.00	3.96	3.90	3.89	3.97	3.92	3.98	3.89	3.93
	h2	1.048	1.068	1.084	1.097	1.108	1.119	1.129	1.138	1.147	1.155
	E[h]	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991
	ARL	1.96	1.94	1.93	1.92	1.90	1.90	1.89	1.88	1.87	1.87
	ATS	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10
	SS	1.95	1.93	1.92	1.90	1.89	1.88	1.88	1.87	1.86	1.85
	HS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3.0	n1	3	3	3	3	3	3	3	3	3	3
	n2	22	17	14	12	11	11	10	10	9	9
	E[n]	3.96	4.00	3.96	3.90	3.89	3.97	3.92	3.98	3.89	3.93
	h2	1.048	1.068	1.084	1.097	1.108	1.119	1.129	1.138	1.147	1.155
	E[h]	1.000	0.999	0.998	0.997	0.996	0.995	0.994	0.993	0.992	0.991
	ARL	1.95	1.93	1.91	1.90	1.89	1.88	1.87	1.86	1.85	1.85
	ATS	1.10	1.09	1.09	1.09	1.09	1.08	1.08	1.08	1.08	1.08
	SS	1.95	1.93	1.91	1.90	1.89	1.88	1.87	1.86	1.85	1.84
	HS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 3. Comparison of the ASCC with EWMA ($n_0=2$, $h_{\min}=0.1$)

δ	L	k	n_1	n_2	h_1	h_2	ARL	ATS	EWMA	ATS-R	E[n]
0.000							499.61	499.61	480.00	1.04	2.00
0.177	1	1.998	1	23	0.1	1.04122	167.06	166.55	74.10	2.25	1.96
0.354	1	1.998	1	23	0.1	1.04122	46.06	45.49	28.00	1.62	1.96
0.530	1	1.998	1	23	0.1	1.04122	19.25	18.72	15.50	1.21	1.96
0.707	1	1.998	1	23	0.1	1.04122	11.36	10.80	10.10	1.07	1.96
1.061	1	1.998	1	23	0.1	1.04122	6.46	5.78	5.37	1.08	1.96
1.414	5	2.301	2	2	0.1	1.09284	5.30	3.19	3.47	0.92	2.00
1.768	3	2.209	2	2	0.1	1.07163	3.25	1.90	2.47	0.77	2.00
2.121	2	2.134	2	2	0.1	1.05836	2.46	1.41	1.87	0.76	2.00
2.475	2	2.134	2	2	0.1	1.05836	2.13	1.21	1.46	0.83	2.00
2.828	10	2.418	2	2	0.1	1.13244	1.98	1.13	1.22	0.93	2.00
3.536	10	2.418	2	2	0.1	1.13244	1.87	1.08	1.03	1.05	2.00

TABLE 4. Comparison of the ASCC with EWMA ($n_0=5$, $h_{\min}=0.1$)

δ	L	k	n_1	n_2	h_1	h_2	ARL	ATS	EWMA	ATS-R	E[n]
0.000							499.61	499.61	480.00	1.04	5.00
0.112	1	1.998	1	92	0.1	1.04122	123.81	123.64	74.10	1.67	4.98
0.224	1	1.998	1	92	0.1	1.04122	35.96	35.75	28.00	1.28	4.98
0.335	1	1.998	2	70	0.1	1.04122	18.59	18.17	15.50	1.17	4.97
0.447	1	1.998	3	48	0.1	1.04122	10.98	10.34	10.10	1.02	4.97
0.671	1	1.998	4	26	0.1	1.04122	5.14	4.34	5.37	0.81	4.96
0.894	1	1.998	4	26	0.1	1.04122	3.31	2.50	3.47	0.72	4.96
1.118	1	1.998	4	26	0.1	1.04122	2.61	1.78	2.47	0.72	4.96
1.342	1	1.998	4	26	0.1	1.04122	2.27	1.42	1.87	0.76	4.96
1.565	4	2.261	5	5	0.1	1.08288	2.15	1.22	1.46	0.84	5.00
1.789	4	2.261	5	5	0.1	1.08288	2.00	1.14	1.22	0.93	5.00
2.236	10	2.418	5	5	0.1	1.13244	1.87	1.08	1.03	1.05	5.00

TABLE 5. Comparison of the ASCC with EWMA ($n_0=10$, $h_{\min}=0.1$)

δ	L	k	n_1	n_2	h_1	h_2	ARL	ATS	EWMA	ATS-R	E[n]
0.000							499.61	499.61	480.00	1.04	10.00
0.079	1	1.998	1	206	0.1	1.04122	114.85	114.76	74.10	1.55	9.96
0.158	1	1.998	1	206	0.1	1.04122	34.77	34.66	28.00	1.24	9.96
0.237	1	1.998	4	141	0.1	1.04122	18.52	18.10	15.50	1.17	9.99
0.316	1	1.998	6	97	0.1	1.04122	10.93	10.30	10.10	1.02	9.98
0.474	1	1.998	8	53	0.1	1.04122	5.12	4.31	5.37	0.80	9.97
0.632	1	1.998	9	31	0.1	1.04122	3.30	2.43	3.47	0.70	9.96
0.791	1	1.998	9	31	0.1	1.04122	2.50	1.66	2.47	0.67	9.96
0.949	1	1.998	9	31	0.1	1.04122	2.19	1.34	1.87	0.72	9.96
1.107	1	1.998	9	31	0.1	1.04122	2.05	1.20	1.46	0.82	9.96
1.265	2	2.134	10	10	0.1	1.05836	2.00	1.13	1.22	0.93	10.00
1.581	10	2.418	10	10	0.1	1.13244	1.87	1.08	1.03	1.05	10.00

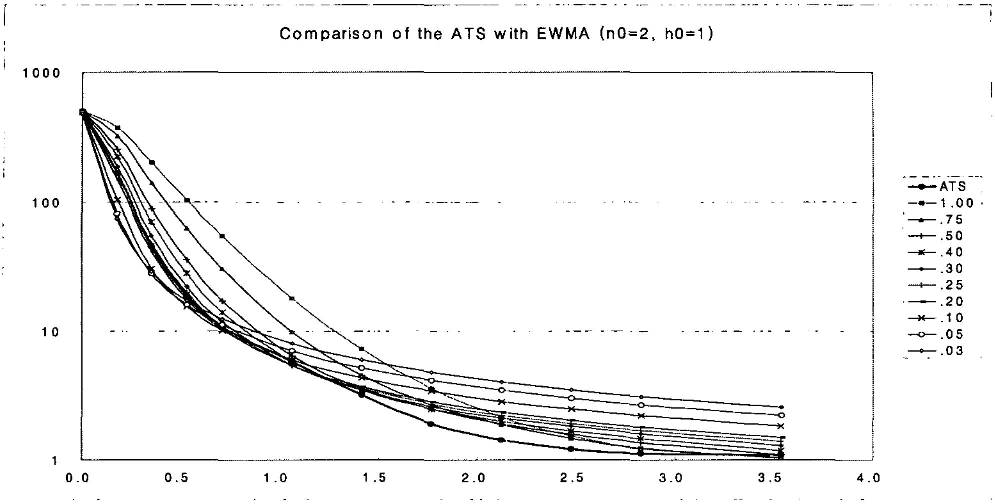

FIGURE 2. Comparison of the ASCC with EWMA ($n_0=2$, $h_{\min}=0.1$)

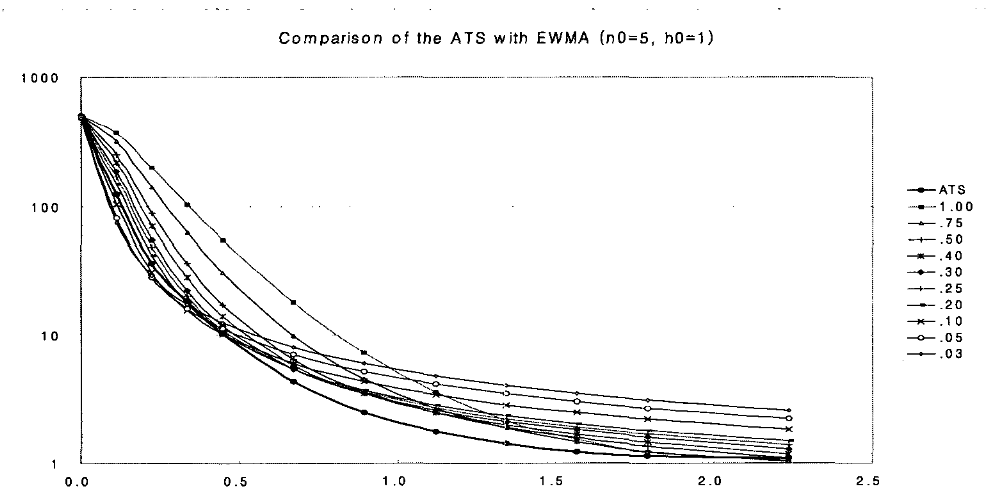

FIGURE 3. Comparison of the ASCC with EWMA ($n_0=5$, $h_{\min}=0.1$)

Therefore, it would be interesting to compare the ATS performance between ASS and DS schemes. Table 6 and Figure 5 show the result. The DS scheme employed in Table 6 is as follows [Daudin (1992)].

Chart	n_1	n_2	k_1	w	k_2
DS1	3	6	3.53	1	3
DS2	3	6	3.675	1	3
DS2	3	6	4.5	1	3.05

The DS scheme can be rephrased as follows.

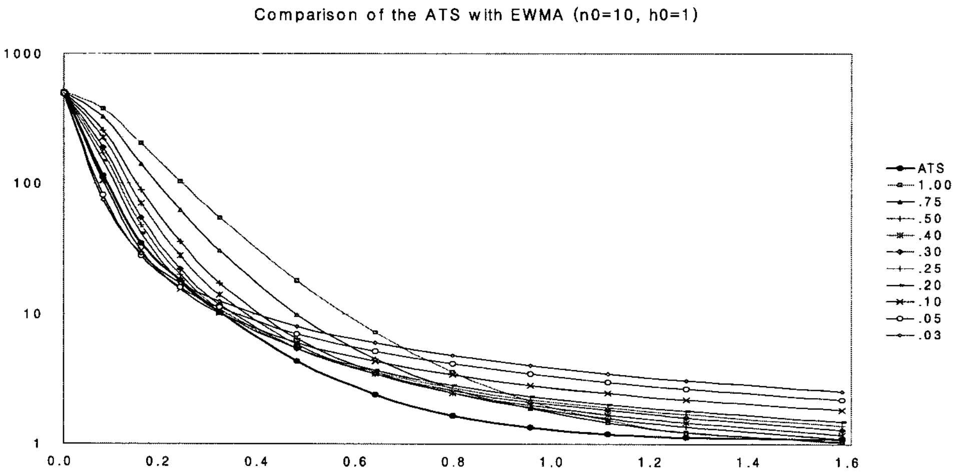

FIGURE 4. Comparison of the ASCC with EWMA ($n_0 = 10$, $h_{\min} = 0.1$)

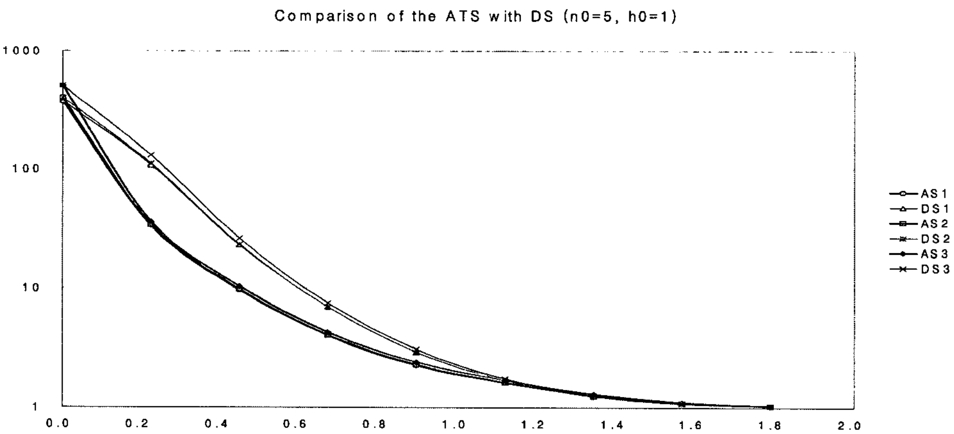

FIGURE 5. Comparison of the ASCC with DS ($n_0 = 5$, $h_{\min} = 0.0$)

(1) If $\overline{X_1}$ is not more than $w\sigma/\sqrt{n_1}$ from μ_0, conclude the process is in control.

(2) If $\overline{X_1}$ is more than $k_1\sigma/\sqrt{n_1}$ from μ_0, conclude the process is out of control.

(3) If $\overline{X_1}$ is more than $w\sigma/\sqrt{n_1}$, but nor more than $k_1\sigma/\sqrt{n_1}$ from μ_0, take a second sample with size n_2.

(4) At the second stage, if the combined sample mean $\overline{Y}$ is not more than

TABLE 6. Comparison of the ASCC with DS ($n_0=5$, $h_{min}=0.0$)

δ	L	k	n_1	n_2	h_1	h_2	ARL	ATS	DS1	ATS-R	E[n]
0.000							374.50	374.50	374.50	1.00	5.00
0.224	1	1.935	1	80	0	1.0531	33.33	33.10	107.40	0.31	4.97
0.447	1	1.935	3	42	0	1.0531	10.32	9.60	22.80	0.42	4.96
0.671	1	1.935	4	23	0	1.0531	4.94	4.03	6.90	0.58	4.96
0.894	1	1.935	4	23	0	1.0531	3.18	2.29	2.90	0.79	4.96
1.118	1	1.935	4	23	0	1.0531	2.54	1.62	1.69	0.96	4.96
1.342	3	2.150	5	5	0	1.0923	2.40	1.26	1.25	1.01	5.00
1.565	2	2.073	5	5	0	1.0752	2.10	1.09	1.082	1.01	5.00
1.789	2	2.073	5	5	0	1.0752	1.98	1.03	1.025	1.00	5.00

δ	L	k	n_1	n_2	h_1	h_2	ARL	ATS	DS2	ATS-R	E[n]
0.000							397.60	397.60	397.60	1.00	5.00
0.224	1	1.948	1	82	0	1.0515	33.98	33.75	110.52	0.31	4.96
0.447	1	1.948	3	43	0	1.0515	10.47	9.75	23.08	0.42	4.95
0.671	1	1.948	4	24	0	1.0515	4.96	4.05	6.90	0.59	4.98
0.894	1	1.948	4	24	0	1.0515	3.20	2.31	2.93	0.79	4.98
1.118	1	1.948	4	24	0	1.0515	2.55	1.63	1.69	0.97	4.98
1.342	3	2.162	5	5	0	1.0895	2.42	1.27	1.25	1.02	5.00
1.565	2	2.086	5	5	0	1.0729	2.11	1.09	1.08	1.01	5.00
1.789	2	2.086	5	5	0	1.0729	1.99	1.03	1.03	1.00	5.00

δ	L	k	n_1	n_2	h_1	h_2	ARL	ATS	DS3	ATS-R	E[n]
0.000							502.90	502.90	502.90	1.00	5.00
0.224	1	2.000	1	92	0	1.0456	36.11	35.88	130.30	0.28	4.96
0.447	1	2.000	3	48	0	1.0456	11.01	10.30	25.90	0.40	4.96
0.671	1	2.000	4	26	0	1.0456	5.15	4.26	7.50	0.57	4.96
0.894	1	2.000	4	26	0	1.0456	3.31	2.41	3.09	0.78	4.96
1.118	1	2.000	4	26	0	1.0456	2.61	1.69	1.74	0.97	4.96
1.342	3	2.211	5	5	0	1.0793	2.47	1.29	1.26	1.03	5.00
1.565	2	2.135	5	5	0	1.0646	2.13	1.10	1.09	1.02	5.00
1.789	2	2.135	5	5	0	1.0646	2.00	1.03	1.03	1.01	5.00

$k_2\sigma/\sqrt{n_1+n_2}$ from μ_0, conclude the process is in control. Otherwise, conclude the process is out of control.

Table 6 shows that ASCC may reduce the ATS as much as 70% for small shift. Even for large shifts, ASCC performs almost as well as DS as shown in Figure 5. Especially, for $ATS_0=374.5$, ASCC performance is uniformly better than DS in the whole range of shift considered. All of these ASCC have an average sample size less than or equal to 5.0 when $\mu=\mu_0$.

5. Summary and Conclusion

We have proposed an adaptive sampling scheme for the SCC. The proposed ASCC improves the ATS performance of the SCC remarkably, by concentrating resources on L-consecutive samples following the NC sample. The ATS performance of ASCC is uniformly better than EWMA for some range of mean shift, and is comparable to EWMA for any range of shift. ASCC also shows better performance than DS, in general. The ASCC is simple, because it does not employ multiple limits as in DS scheme, nor depend on weight as in EWMA. Davis and Woodall (2002) argued that SCC cannot compete with appropriately designed EWMA charts, because the use of SCC involves a loss of information about the process data. However, our work showed that this drawback may be overcome by employing a simple adaptive scheme.

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