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Nonlinear Representation of Two-Stage Power-Factor-Correction AC/DC Circuits

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ABSTRACT

Two-stage Power-Factor-Correction (PFC) converters are the most common circuits for drawing sinusoidal and in phase current waveforms from an ac source with a good regulated output voltage. The first stage is a boost PFC converter with average-current-mode control for achieving the near-unity power factor and the second stage is a forward converter with voltage-mode control to regulate the output voltage. Stability analysis and design methods of two-stage PFC converters have previously been discussed using linear models. Recently, new nonlinear phenomena have been detected in pre-regulator boost PFC circuits and a new nonlinear model has been proposed for pre-regulated PFC converters. Therefore, investigation of two-stage PFC converters from the nonlinear viewpoint becomes important because the second stage DC/DC converter adds more complexity to the circuit. So, this paper introduces a study of the stability of two-stage PFC converters. A novel nonlinear model of two-stage PFC converters is proposed. Then, a stability analysis is made based upon this nonlinear model. The high correspondence between the simulated and experimental results confirms our analysis.

Keywords: two-stage PFC converter, stability, bifurcation, nonlinear analysis

1. Introduction

Recently, power factor correction (PFC) converters have becomes an important part of many power systems. Many converter topologies can function as PFC converters. Boost converters are the most common[1]. Many topologies for controlling PFC have been introduced. Average current mode control works very well for power factor correction [2].

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A PFC converter can be implemented using either a two-stage technique or a single-stage technique. The two-stage technique shown in Fig. 1 is the most commonly used due to its high power factor. This is especially useful for higher power applications. The PFC stage is employed as the pre-regulator stage of the two-stage PFC circuit to force the line current to track the line voltage. A second DC/DC converter is used to regulate the output voltage. Many researchers have investigated and analyzed the dynamic behavior of PFCs. However, very few have attempted to properly analyze the stability of this system. Unfortunately, most prior research has been directed toward linear models that are easily analyzed. Most prior research has made assumptions to reduce nonlinear PFC systems to linear systems. The stability of the nonlinear system is then analyzed by using a smooth linearized small signal model instead of taking the non-linearity into account[3]-[6]. The input time-varying voltage is replaced by its root mean square (rms) value. The output capacitance is assumed to be very high (an unacceptable assumption in industry) to establish the small signal analysis. All these assumptions lead the linear solution to be inaccurate. They cannot derive the correct predictions for the stability problem.

Unstable phenomena are detected in regions which are predicted to be stable by these linear models. This leads us to take the non-linearity of pre-regulator boost PFC converters into account when we investigate them. Period doubling bifurcation and chaos instability are detected in PFC converter circuits [7]-[9]. PFC boost converters are analyzed using nonlinear analysis where the second stage DC/DC converter is represented by a large signal. Then, the stable and unstable regions are determined [10]-[11].

In a two-stage PFC circuit, the second stage DC/DC converter adds more complexity to the circuit analysis. In previous research, the second stage DC/DC converter has been represented as a negative resistance connected to the small-signal model of the first stage boost PFC converter[5], [12]. Then, stability analysis and circuit design were done based upon these prior linear models. However, in this paper, the two-stage PFC converter is analyzed using the new nonlinear model for the first stage (boost PFC converter) and representing the second stage DC/DC converter by a large signal. Then, the stability matter is discussed. The next section will show that the simulation and experimental results agree.

2. Two-Stage PFC Circuit

A two-stage PFC circuit consists of a pre-regulator boost PFC stage and a forward DC/DC converter. The boost PFC uses average-current-mode control with UC 3854A[13]. Conventional voltage-mode control is used for the forward DC/DC converter. The test circuit conditions are: 70-120 Vac/60Hz input, 180Vdc at pre-regulated boost PFC output, 48V/2A dc forward converter output.

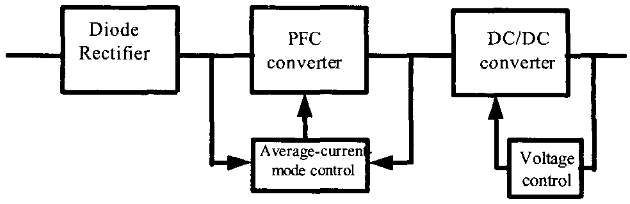

Fig. 1 Two-stage PFC converter with control circuits block diagram.

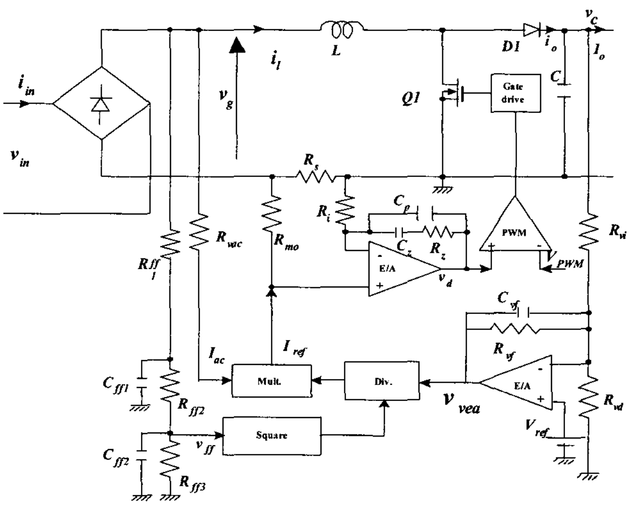

Fig. 2 Pre-regulator PFC boost converter circuit configuration.

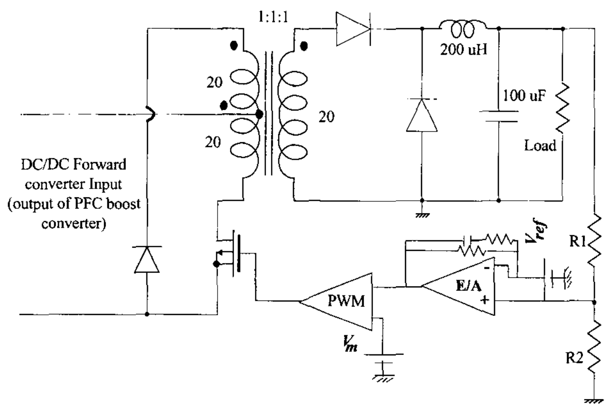

Fig. 3 Forward DC/DC converter circuit configuration.

Figure 2 shows the circuit configuration for the boost pre-regulator PFC with average current mode control. Circuit parameters for the boost PFC converter are selected according to the Unitrode Application note[13]. The circuit configuration of the second stage forward DC/DC converter is shown in Fig. 3. The forward converter is designed to provide a constant and regulated output voltage (48 volts).

As with the pre-regulator PFC converter, the main parameters affecting system stability are the output capacitor C and the feedback capacitor C_{vf} [10]-[11]. Therefore, we follow the same procedure here as we did for the two-stage PFC circuit. The output capacitor C and the feed back capacitor C_{vf} are varied from small to large values to examine their effect on the system stability. The results are summarized in Fig. 4 for the stable case, nominal periodic waveform, where the input current and voltage are sinusoidal with period at line frequency and emphasized; also the output voltage ripple of the boost PFC converter (the intermediate voltage) is sinusoidal with double line frequency. There is a good correspondence between the simulation and experimental values. (In the figure the simulation values are represented as square dots above the experimental waveforms.) Although this operation point has heavy loads and small output capacitance, the power factor is very high (0.97). Figure 5 shows one case for instability phenomena. The test parameters are the same as for the stable case except that the output capacitor has been decreased. The input current is distorted and becomes asymmetrical (PF =0.67) so that there is no symmetry of the periodic waveform. Also, the output voltage ripple of the intermediate voltage changes its periodic frequency from the double line frequency to the line frequency while increasing its amplitude and distorting the input current through the feedback cycles. This instability points out the limitations of linear models to predict the behavior of a two-stage PFC converter. (This will be explained in section 4.) It is also noteworthy that instability phenomena have been detected at heavy loads in two-stage PFC circuits instead of at light loads as is the case with pre-regulator PFCs connected to a resistive load [10]-[11].

This reveals new instability phenomena that must be corrected or at least have its parameter's defined so that it can be avoided. In [7]-[11], these unstable phenomena were only discovered in pre-regulator PFC converters with a resistive load when the proposed nonlinear model was applied. In this paper,

these unstable phenomena will be explained and studied so that two-stage PFC converters can be used as power

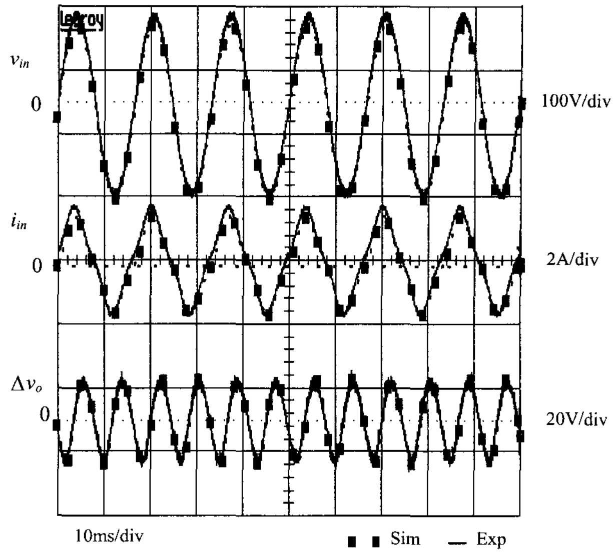

Fig. 4 Response of input voltage, input current, and output voltage ripple in the stable case. (PF=0.97) At C=100μF, C_{vf}=47nF & full load two-stage PFC.

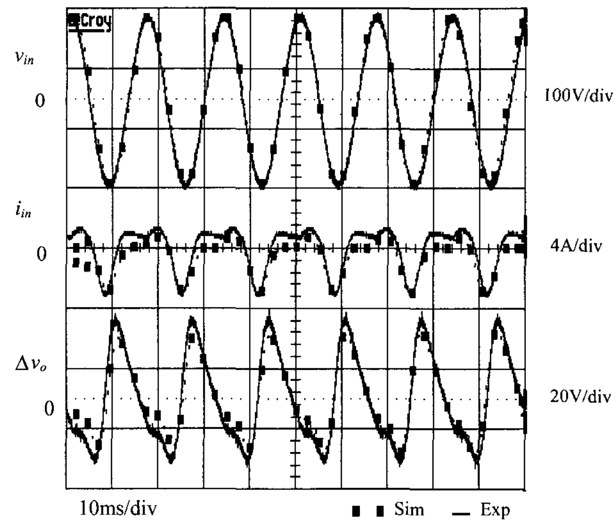

Fig. 5 Response of input voltage, input current, and output voltage ripple the unstable case (period doubling bifurcation). (PF=0.67) At C=60μF, C_{vf}=47nF & full load two-stage PFC.

supplies in practical applications.

3. DC/DC Converter Representation

First, to perform a simple analysis of a two-stage PFC converter, a representation for the second stage is needed.

As shown in Fig. 2, any regulated DC/DC converter can be represented by a constant power source[12]. Figure 6 shows the relation between the input voltage v_{in} and the input current for the regulated DC/DC converter, assuming ideal operation where the output power, P is equal to the input power. By prior analysis we can calculate the input impedance by small-signal analysis. The power P can be calculated as:

$$v_{in} i_{in} = P = K. \tag{1}$$

Applying small-signal analysis to the input voltage and current, then:

$$\begin{aligned} v_{in} i_{in} &= K = (V + \Delta v_{in})(I + \Delta i_{in}) \\ &= VI + I\Delta v_{in} + V\Delta i_{in} + \Delta v_{in}\Delta i_{in} \end{aligned} \tag{2}$$

where $\Delta v_{in}, \Delta i_{in}$ are the small-signal values of the input voltage and the input current, respectively.

Removing the DC part and the nonlinear term, then:

$$0 = I\Delta v_{in} + V\Delta i_{in} \tag{3}$$

Therefore, the small-signal impedance $Z_{in,ss}$ can be calculated as:

$$Z_{in,ss} = \frac{\Delta v_{in}}{\Delta i_{in}} = -\frac{V}{I} \tag{4}$$

From equation (4), it is clear that the small-signal input impedance of any regulated DC/DC converter appears as a negative resistance. This is clear from the negative slope in Fig. 7.

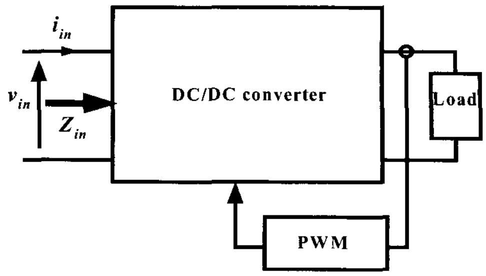

Fig. 6 Input impedance of any regulated DC/DC converter.

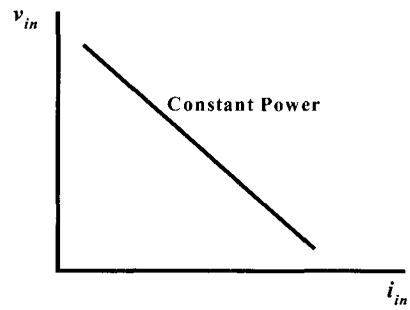

Fig. 7 Relation between the input voltage and current of any regulated DC/DC converter appears as a constant power.

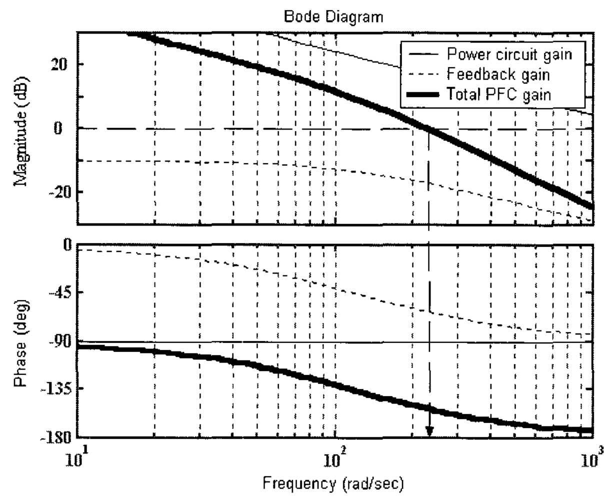

Fig. 8 Bode diagram for the linear analysis of the two-stage PFC converter. It appears as a stable system, however, in practice, it is nonlinear.

4. Prior Linear Design

Prior research depends on a simple linear model to calculate the stability regions. It assumes a huge capacitance at the output terminal and replaces the time-varying input voltage with its rms value. The nonlinear terms are ignored and a linear model is derived[5]. The transfer function is calculated for the power and feedback stages using small-signal analysis and using the definition of small-signal input impedance of the second stage DC/DC converter:

1) The converter power-stage transfer function from the output voltage $\hat{v}_c$ (small-signal) to the feedback voltage $\hat{v}_{vea}$ (small-signal) gain is:

$$G_P = \frac{\hat{v}_c}{\hat{v}_{vea}} = \frac{V}{\sqrt{KV_{vea}\frac{V_c}{I_o}}}\frac{1}{s} \tag{5}$$

2) The feedback voltage to the output capacitance voltage transfer function gain (feedback gain) is:

$$G_{FB} = \frac{\hat{v}_{vea}}{\hat{v}_c} = \frac{R_{vf} / R_{vi}}{1 + sC_{vf}R_{vf}} \tag{6}$$

Then, the total system transfer function gain is:

$$G_{total} = \frac{VR_{vf} / R_{vi}}{\sqrt{K V_{vea}\frac{V_c}{I_o}}}\frac{1}{s\left(1 + sC_{vf}R_{vf}\right)} \tag{7}$$

where K, V_c and V_{vea} are a constant, the DC value of the output voltage, and the DC value of the feedback voltage, respectively. Figure 8 shows an example of the frequency response using (7). The results shown in Fig. 8 have a phase margin of +20 degrees that assures the stability of the system. However, this test condition was actually unstable (both experimentally and in the simulation) as shown in Fig. 5. This assures the invalidity of the linear model for the two stage PFC converter as well.

5. Novel Nonlinear Model for the Two-Stage PFC Converter

Whereas the small-signal representation of a DC/DC converter is a negative resistance, the large-signal representation is a constant power source. Therefore, the two-stage PFC converter shown in Fig. 9 can be represented as in Fig. 10. The instantaneous power equation of the boost PFC converter as shown in Fig. 11 is given at the MOSFET switch *Q1* by representing the second stage (forward DC/DC converter) as a constant power source *P*:

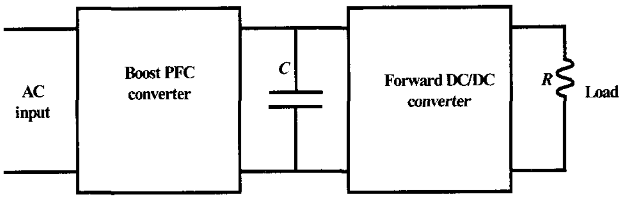

Fig. 9 Two-stage PFC converter block diagram before representation.

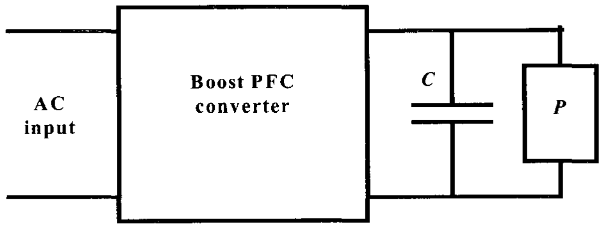

Fig. 10 Two-stage PFC converter block diagram after representating the second stage as a constant power source.

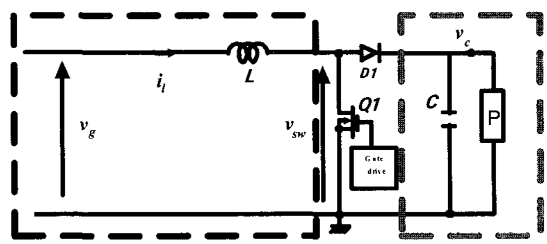

Fig. 11 The power loops around the switch Q1 for the two-stage PFC converter after the second stage DC/DC converter representation.

$$v_c(t)\times C\frac{dv_c(t)}{dt} + P = v_{sw}(t)\times i_l(t) \tag{8}$$

$$v_{sw}(t) = v_g(t) - L\frac{di_l(t)}{dt} \tag{9}$$

where, $v_{sw}(t)$ is the voltage across the MOSFET switch. The feed-forward voltage signal obtained from a second-order filter can be assumed to be constant.

Therefore, the reference current can be calculated as:

$$I_{ref}(t) = \frac{v_{vea}(t) I_{ac}(t)}{v_{ff}^2(t)} = \frac{v_{vea}(t) - 1.5 v_g(t)}{v_{ff}^2(t) R_{vac}}$$

$$= K_1 (v_{vea}(t)\ v_g(t) \qquad (10)$$

$$K_1 = \frac{1}{v_{ff}^2(t) R_{vac}} \qquad (11)$$

If we assume the current error amplifier gain to be very high, we can simplify this nonlinear model and the following equation is obtained:

$$i_l R_s = I_{ref} R_{mo} \qquad (12)$$

Substituting (10) into (12), the inductor current is expressed as:

$$i_l(t) = \frac{R_{mo}}{R_s} I_{ref} = K_2\, v_{vea}(t)\ v_g(t) \qquad (13)$$

$$K_2 = \frac{R_{mo}/R_s}{v_{ff}^2(t) R_{vac}} \qquad (14)$$

Then, substituting (9) and (13) into (8), the main equation of the simplified nonlinear model is developed. The main two-stage PFC converter system power stage equation can be calculated as:

$$v_c(t)\frac{d v_c(t)}{dt} + \frac{P}{C} = \frac{K_2 V^2}{C}(1-\cos(2\omega t)) v_{vea}(t) - \frac{\omega K_2^2 L V^2}{C}\sin(2\omega t)\ v_{vea}^2(t)$$

$$-\frac{K_2^2 L V^2}{C}(1-\cos(2\omega t)) v_{vea}(t)\frac{d v_{vea}(t)}{dt} \qquad (15)$$

The feedback voltage equation can be expressed as:

$$\frac{d v_{vea}(t)}{dt} + \frac{v_{vea}(t)}{C_{vf} R_{vf}} = \left(\frac{1}{R_{vf} C_{vf}} + \frac{R_{vd} + R_{vi}}{R_{vd} R_{vi} C_{vf}}\right) V_{ref} - \frac{v_c(t)}{R_{vi} C_{vf}} \qquad (16)$$

These two nonlinear equations ((15) and (16)) demonstrate the proposed nonlinear two-stage PFC converter model. These two equations can be solved easily using various mathematical methods. In this paper, a FORTRAN program is used to solve these equations. This new nonlinear model is analyzed for stable and unstable phenomena and the results agree well with the experimental and simulation results. Fig. 12 shows the

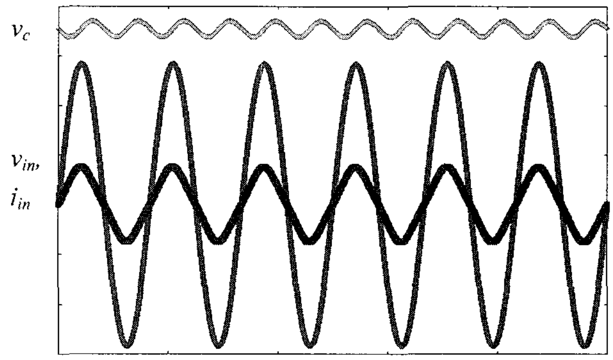

Fig. 12 The input voltage, input current and output voltage response during stable operation.

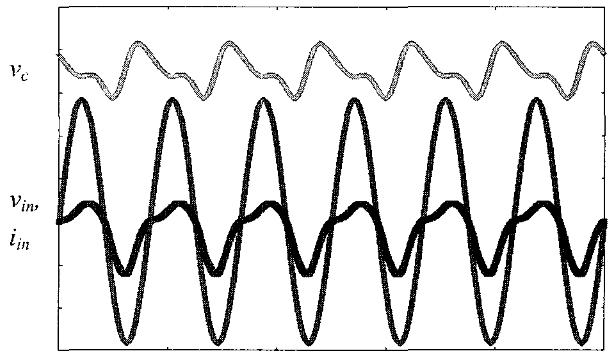

Fig. 13 The input voltage, input current and output voltage response during unstable operation.

output voltage, input voltage, and input current waveforms from the mathematical model for stable operation using the same parameters as Fig. 4. Additionally, Fig. 13 shows the same waveforms for unstable operation (period doubling bifurcation) using the same parameters as Fig. 5.

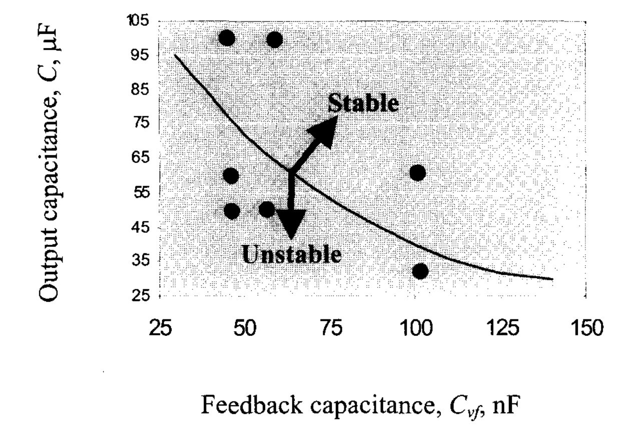

Fig. 14 Two-Stage PFC converter stability map

These figures show a high correlation between the simulation, the experiment and the proposed nonlinear model. The stable and unstable operation regions and their borderlines(critical points) are illustrated in Fig. 14. The dots above the borderline show the stable tested points (experiment) and the dots under the borderline show the unstable tested points. Therefore, safe operation can be assured by operating within the stable operation region shown in Fig. 14.

5. Conclusions

We have analyzed a two-stage PFC converter from a non-linear perspective. New unstable phenomena were discovered in a region that the prior linear model predicted would be stable. The failure of the linear model forces us to develop nonlinear models to study two-stage PFC converters. This forces us to develop new mathematical approaches. A novel nonlinear model that can detect these unstable phenomena has been proposed here. This nonlinear model is then used to investigate the operation of the two-stage PFC converter.

In conclusion, the mathematical model and its experimental confirmation presented in this paper have improved the understanding of the dynamic behavior of two-stage PFC converters.

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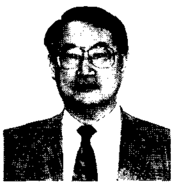

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