Multiple Target Angle-tracking Using Angular Innovations Extracted from Noise Subspace

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Abstract

Ryu et al. proposed a multiple target angle-tracking algorithm without a data association problem using angular innovations. This algorithm, however, needs the computational loads in proportion to the square number of sensors regardless of the number of targets, because it uses a nonlinear equation between the signal subspace and angular innovation. In this Paper, we proposed an efficient algorithm for the multiple target angle-tracking using angular innovations. The proposed algorithm extracts the angular innovations from noise subspace. Also, it is demonstrated by computer simulations dealing with the tracking of crossing targets. The simulation results show that the computational loads of the proposed algorithm are 80% and 60% of those of Ryu's algorithm for 3 targets and 6 targets without degrading the performance of the target tracking.

Keywords: *Target tracking, Subspace, Angular innovation*

I. Introduction

Multiple target angle-tracking problem has been studied for several decades in various fields, including sonar, radar, communications, and so on. Researches on the problem can be separated into two approaches. The one, target state model approach, establishes a dynamic model of target state and performs tracking that estimates the state vector using measurements. In this approach, tracking is performed by estimating the time delay of the target signals with respect to sensors in the array. This approaches have the data association problem in tracking multiple targets[1].

The other approach simultaneously estimates the angles of the targets and associates the data[2~8]. A signal subspace algorithm such as the MUSIC(multiple signal classification) algorithm is applied to yield the initial estimates of the number of targets, sensor noise power, target signal power, and the target angles[3]. The angular innovations of the targets during a observing time period are estimated in the least square sense using the most recent estimate of the sensor output covariance matrix. The target angles are then tracked recursively by adding the estimated angular innovations to the existing estimates of the target angles. This approach has the attractive features of simple structure and avoidance of data association problem.

Recently, Ryu et al. proposed an equation derived from the fact that the projection error is zero when the target steering vector is projected onto the signal subspace[8]. To obtain the P angular innovations from the linear array of M sensors, it deals a real matrix with the dimensions $2M\times 2M$. In this paper, we propose a linear equation for the angular innovation based on the fact that the steering vector and the noise subspace are orthogonal. This equation only requires the operation of a $2(M-P)\times 2(M-P)$ real matrix. The proposed equation can improve the computational performance of the multiple target angle-tracking algorithm. In Section II, we formulate the problem, and in Section III, we describe the proposed algorithm. In Section IV, we demonstrate the improved performance of the proposed algorithm by computer simulations dealing with the tracking of multiple targets. Section V is the conclusion.

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II. Problem Formulation

Consider the signals from P targets in the presence of noise received by M linear arrayed sensors which are separated by a distance d. The direction angles $\{\theta_i(k),\ i=1,2,\cdots,P\}$ of the target signals are tracked by using the sensor outputs. Fig. 1 illustrates the system and defines appropriate parameters. The output of the mth sensor at the sampling time k is given by

$$x_m(k)=\sum_{i=1}^{P} s_i(k)e^{j\omega(d/c)(m-1)\cos\theta_i(k)}+n_m(k) \tag{1}$$

where $s_i(k)$ is the complex signal with a carrier frequency ω emitted from the target i, which has arrived at the reference point (sensor #1), and c is the signal propagation speed. $n_m(k)$ is the sensor noise at the sensor m which is assumed to be white, zero mean with a constant power σ^2, uncorrelated between sensors, and uncorrelated with the target signals. The $M\times 1$ steering vector $\mathbf{a}_i(k)$ is defined as

$$\mathbf{a}_i(k)=\left[1\ \ \gamma_i(k)\ \cdots\ \gamma_i^{M-1}(k)\right]^T,\ \ i=1,2,\cdots,P \tag{2}$$

where $\gamma_i(k)=e^{j\omega(d/c)\cos\theta_i(k)}$ and the superscript T denotes a transpose.

III. Proposed Algorithm

It is known that the steering vector $\mathbf{a}_i(k)$ and the noise subspace $\mathbf{W}(k)=[\ \mathbf{w}_{P+1}(k)\ \ \mathbf{w}_{P+2}(k)\ \cdots\ \mathbf{w}_M(k)]$ are orthogonal[9]. Accordingly, we have

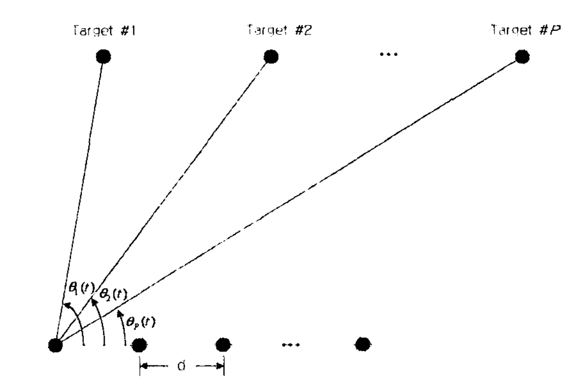

Fig. 1. Sensor array geometry for tracking.

$$\mathbf{w}_m^H(k)\ \mathbf{a}_i(k)=0,\ P+1\le m\le M,\ 1\le i\le P \tag{3}$$

where the superscript H denotes a conjugate transpose.

If the predicted angle $\bar{\theta}_i(k|k-1)$ of the ith target is obtained by a simple Kalman filter, then $\theta_i(k)=\bar{\theta}_i(k|k-1)+\delta\theta_i(k)$ where $\delta\theta_i(k)$ is the angular innovation of the ith target. Therefore, the mth element of the steering vector $\mathbf{a}_i(k)$ in (2) can be represented by

$$\begin{aligned}\gamma_i^{m-1}(k)&=e^{j\omega(d/c)(m-1)\cos\theta_i(k)}\\&=e^{j\omega(d/c)(m-1)\cos\left(\bar{\theta}_i(k|k-1)+\delta\theta_i(k)\right)}.\end{aligned} \tag{4}$$

It is assumed that the observing time period is sufficiently small so that $\delta\theta_i(k)$ is small. Since $\delta\theta_i(k)$ is small, equation (4) can be expanded into a Taylor series with the second- and higher-order terms dropped. This gives

$$\begin{aligned}\gamma_i^{m-1}(k)\simeq\ &\hat{\gamma}_i^{m-1}(k|k-1)-j\omega(d/c)(m-1)\\&\sin\bar{\theta}_i(k|k-1)\,\hat{\gamma}_i^{m-1}(k|k-1)\,\delta\theta_i(k)\end{aligned} \tag{5}$$

where $\hat{\gamma}_i^{m-1}(k|k-1)=e^{j\omega(d/c)(m-1)\cos\bar{\theta}_i(k|k-1)}$. Substituting (5) into $\mathbf{a}_i(k)$ and by straight manipulation, the following is obtained:

$$\mathbf{a}_i(k)=\ \hat{\mathbf{a}}_i(k|k-1)+\ \bar{\mathbf{b}}_i(k|k-1)\,\delta\theta_i(k),\ \ i=1,2,\cdots,P \tag{6}$$

where

$$\begin{aligned}\hat{\mathbf{a}}_i(k|k-1)&=\ [1,\hat{\gamma}_i(k|k-1),\cdots,\hat{\gamma}_i^{M-1}(k|k-1)]^T\\ \bar{\mathbf{b}}_i(k|k-1)&=\ \frac{j\omega d}{c}\begin{bmatrix}0\\-\sin(\bar{\theta}_i(k|k-1))\,\hat{\gamma}_i(k|k-1)\\ \vdots\\-(M-1)\sin(\bar{\theta}_i(k|k-1))\,\hat{\gamma}_i^{M-1}(k|k-1)\end{bmatrix}.\end{aligned}$$

Now substituting (6) into (3), a linear matrix equation is given by

$$\mathbf{Y}_i(k)=\ \mathbf{B}_i(k)\,\delta\theta_i(k) \tag{7}$$

where

$$\begin{aligned}\mathbf{B}_i(k)&=\ \mathbf{W}^H(k)\ \bar{\mathbf{b}}_i(k|k-1)\\&=\ \mathbf{B}_{iR}(k)+j\ \mathbf{B}_{iI}(k)\\ \mathbf{Y}_i(k)&=\ -\ \mathbf{W}^H(k)\ \hat{\mathbf{a}}_i(k|k-1)\\&=\ \mathbf{Y}_{iR}(k)+j\ \mathbf{Y}_{iI}(k)\end{aligned}$$

where $\mathbf{B}_{iR}(k)$, $\mathbf{Y}_{iR}(k)$ and $\mathbf{B}_{iI}(k)$, $\mathbf{Y}_{iI}(k)$ are the real and imaginary parts of $\mathbf{B}_i(k)$ and $\mathbf{Y}_i(k)$, respectively.
Since $\delta\theta_i(k)$ is a real number, the least square solution of (7) is

$$\delta\theta_i(k) = (\ \mathbf{B}_i^T(k)\ \mathbf{Y}_i(k)) / (\ \mathbf{B}_i^T(k)\ \mathbf{B}_i(k)),\quad i=1,2,\cdots,P \qquad (8)$$

where

$$\overline{\mathbf{B}}_i(k) = \begin{bmatrix} \mathbf{B}_{iR}(k) \\ \mathbf{B}_{iI}(k) \end{bmatrix},\quad \mathbf{Y}_i(k) = \begin{bmatrix} \mathbf{Y}_{iR}(k) \\ \mathbf{Y}_{iI}(k) \end{bmatrix}.$$

The estimated angle $\hat{\theta}_i(k|k)$ of the ith target is obtained by updating the predicted angle using the angular innovation $\delta\theta_i(k)$ as follows:

$$\hat{\theta}_i(k|k) = \hat{\theta}_i(k|k-1) + g_i(k)\,\delta\theta_i(k),\qquad i=1,2,\cdots,P \qquad (9)$$

where $g_i(k)$ is the Kalman filter gain.

IV. Simulation results

To demonstrate the performance of the proposed algorithm, simulations were performed for multiple targets whose direction angles cross each other. To track the direction angles, a uniform linear array consisting of 15 sensors with an inter-sensor distance of half a wavelength was used. Fig. 2 is a sketch illustrating the simulation geometry.

The sensor noise power σ^2, number of snapshots Q during a observing time interval, observing time period Δ were set at 1, 32, and 32 seconds, respectively. To achieve the required sensor output covariance matrix, we shall appeal to standard Fourier concepts. The sensor output covariance matrix $\mathbf{C}_x(\omega)$ is then approximately given by

$$\mathbf{C}_x(\omega) \simeq \frac{1}{Q}\sum_{q=1}^{Q} \mathbf{X}^{(q)}(\omega)\ \mathbf{X}^{(q)}(\omega)^H \qquad (10)$$

where

$$\mathbf{X}^{(q)}(\omega) = [\,X_1^{(q)}(\omega)\ X_2^{(q)}(\omega)\ \cdots\ X_M^{(q)}(\omega)\,]^T,\quad 1\le q\le Q$$

where $X_m^{(q)}(\omega)$ is Fourier coefficient corresponding to the qth subsequence associated with the mth sensor output.

Initially, we apply the MUSIC algorithm, which is a noise subspace algorithm based on the eigenstructure of the sensor output covariance matrix, to estimate the number of targets, and their direction angles. At Δ seconds later, we apply again the MUSIC algorithm. With two initial angle estimates, $\hat{\theta}_i(-1)$ and $\hat{\theta}_i(0)$, the initial estimates of the state vector and the corresponding covariance matrix are given by the two-point interpolation method such as

Table 1. Performance comparison of two algorithms.

Algorithm	MSE(deg2) N=3		Computational load for obtaining angular innovation (Flops)	
	0 dB	5 dB	N=3	N=6
Ryu's	0.4786	0.4110	11,163	22,326
Proposed	0.4343	0.4104	8,937	13,398

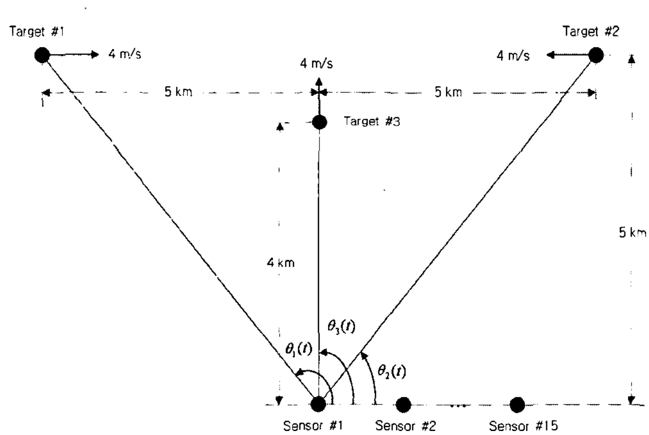

Fig. 2. Three crossing target simulation geometry.

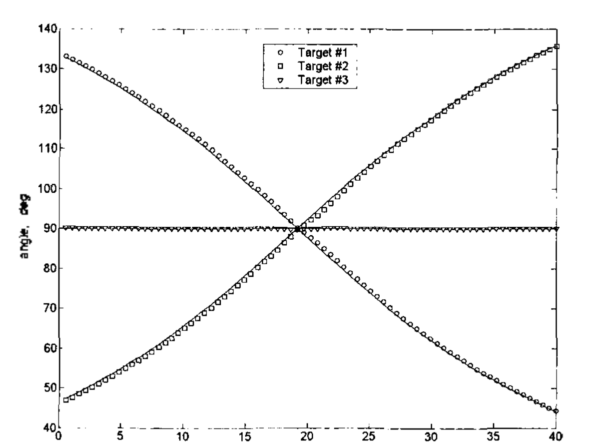

Fig. 3. Typical direction angle tracking result of proposed algorithm.

$$
\begin{aligned}
\vartheta_i(0|0) &= \vartheta_i(0) \\
\dot{\vartheta}_i(0|0) &= \frac{\vartheta_i(0) - \vartheta_i(-1)}{\Delta} \\
\mathbf{P}_i(0|0) &= \begin{bmatrix} \sigma^2_{z_i} & \sigma^2_{z_i}/\Delta \\ \sigma^2_{z_i}/\Delta & 2\sigma^2_{z_i}/\Delta \end{bmatrix}, \qquad i = 1, \cdots, P
\end{aligned}
\tag{11}
$$

where $\sigma^2_{z_i}$ is the variance of the measurement noise. σ_{z_i} is assumed to be constant with 5°.

Figure 3 is the tracking result of a typical sample run when SNR is 0 dB for all targets. Note that the tracks were followed successfully.

To compare the performance of the proposed algorithm and Ryu's, Monte Carlo simulations with 100 runs were carried out for SNR=0dB and SNR=5dB. The simulations were executed in Matlab and the computation loads were measured using the flops function built in Matlab. Table 1 shows that the computational loads of the proposed algorithm were 80% and 60% of those of Ryu's algorithm for N=3 and N=6 without degrading the performance of the target tracking.

V. Conclusion

In this Paper, we proposed an efficient algorithm for the multiple target angle−tracking using angular innovations. The proposed algorithm is based on the fact that the steering vector and the noise subspace are orthogonal. The performance of our algorithm is demonstrated by a computer simulation dealing with the tracking of crossing targets. The simulation result shows that the proposed algorithm retains comparable tracking performances and can be computationally more efficient than Ryu's. In particular, the efficiency improves as the number of targets increases.

Referance

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