

## **Dynamics of Industry-wide versus Firm-specific Benefits when Firms Collaborate on Building an Industry Infrastructure**

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(Received Sep. 2004 ; Revised Apr. 2005 ; Accepted May 2005)

### ABSTRACT

Firms often collaborate on building an infrastructure, which benefits all the firms in the industry, although in unequal magnitudes. Then a difficult and tricky issue is concerned with 'free riding.' Should there be only 'common, i.e., industry-wide' benefits in such collaboration, the literature indicated that the free rider problem is unavoidable. In this paper, however, we suggest that while collaborating, the firm also learns firm-specific knowledge, experience, and know-how, which can be directly utilized for its own internal improvement. That is, the collaboration between firms provides them with *not only* 'industry-wide,' *but also* 'firm-specific' benefits. Our analysis shows that if there indeed exist two types of benefits simultaneously, depending on the balance between the two, the free rider problem can be mitigated or even eliminated.

Keywords: Manufacturing Strategy, Inter-firm Collaboration, Industry Infrastructure

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## 1. INTRODUCTION

Industry infrastructure is necessary for firms to effectively conduct their economic or productive activities. For instance, in order for manufacturing companies to effectively conduct their e-SCM (e-supply chain management), there must be an efficiently functioning e-Business infrastructure in place in the industry. In a sense, such an infrastructure is a public good, which is of use to all the firms in the industry. Then, a critical question arises *'Who pays for the infrastructure?'* Companies, sometimes, fierce competitors in the same industry, collaborate with one another on building the necessary business infrastructure. Why are the competitors collaborating? Each firm probably understands that without such an infrastructure they can't effectively conduct their own business, and in most cases building such an infrastructure requires so much investment that one company can't afford to do it alone.

There exists a tricky issue: when the investment is involved with sort of public goods characteristics, there is usually an issue of opportunism, i.e., some companies enjoy the infrastructure's benefits without paying appropriate dues (Amaldoss, et al. 2000). Economically speaking, the total investment level is lower than that under an optimal condition with no free rider problem.

Our research started with a question whether the industry-wide (i.e., common) benefits are the only gains the participating firms get from such collaboration. Suppose a company in the automobile industry is collaborating with its competitors on streamlining the part codes to improve communication with suppliers, i.e., creating a kind of industry-wide standard. While collaborating with other companies on building an industry-level infrastructure, the company will learn technical knowledge as well as know-how that can be employed to improve its own internal manufacturing process and enhance its own productivity in-house without much extra cost. That is, the firm gets not only the industry-level 'public/common' benefits, but also its own 'firm-specific' benefits while collaborating with its competitors.

In our model, we take into account the firm-specific benefits, which can occur simultaneously with the common benefits, from the industry infrastructure building efforts. Then we look into whether the opportunistic behavior becomes more or less prevalent in the new setting by comparing the steady state total investment levels.

This paper is structured as follows. In the next section, we discuss our research motivation and relevant literature in detail. Section 3 develops a differen-

tial game model to formulate the problem in point. In Section 4, we solve the differential game model and suggest key observations, which help us to develop managerial implications via numerically displaying the dynamics of state/control variables and objective function. Section 5 presents managerial implications, and discusses limitations of our research and possible future directions for this line of research. Finally, appendices contain additional results of our numerical analysis and proofs of the theorems.

## 2. RESEARCH MOTIVATION AND LITERATURE REVIEW

Cooperation between firms is perceived as an important issue in management. Firms not only compete with each other, but also cooperate under certain circumstances (Brandeburger and Nalebuff 1996). That is, a firm need sometimes cooperate even with its direct competitors in order to become more competitive (Phillips and Meeker 2000). For instance, Intel and AMD, being direct competitors in the industry, collaborated on designing a new product. GM and Ford once collaborated on developing an e-marketplace in which they can communicate and transact with their suppliers.

In the economics literature, there has been an issue of creating public goods such as social and/or economic infrastructure. Economists approached the issue, focusing on the 'free rider' problem. Fershtman and Nitzan (1991) analyzed an infinite duration dynamic game where individuals' contributions to public goods are accumulated over time, and showed that the free riding problem is aggravated when players' contributions are conditional on the observable collective contributions: see also Wirl (1996), Cadsby and Maynes (1999), and Chari and Jones (2000).

In a similar vein, Baumol (2001) delved into the inter-firm coordination on innovation, postulating that coordination on innovation and on the supply of proprietary technical information to competitors can be highly beneficial to the economy, enhancing both its static efficiency and its growth. Bureth, et al. (1997) emphasized the learning aspect of inter-firm coordination, Buchholz and Kontad (1995) studied strategic transfers between coordinating firms, and Dickinson (1998) underlined the uncertainty about the ultimate size of the group (i.e., public) payoff as an important determinant in the voluntary provision of public goods (Depken, et al. 2002).

From a managerial perspective, firms' building a common industry infra-

structure shares much in common with creating public goods from the economics viewpoint. Researchers in the management literature examined mechanisms to make the inter-firm collaboration stable and sustainable. Nault and Tyagi (2001) investigated implementable mechanisms to coordinate horizontal alliances (Jap 1999 and 2000), while Madhok and Tallman (1998) offered a theoretical explanation for why inter-firm collaborations form yet fail and suggested how firms might manage them for a more positive outcome. From a more comprehensive perspective, Amaldoss, et al. (2000) investigated the issue of ‘collaborating to compete’ by looking into three key ‘structural’ variables, i.e., the profit-sharing arrangement (between the participants in the collaboration), the type of alliance as modeled by the rule for combining partners’ inputs, and the size of the market reward for winning the inter-alliance competition.

In this paper, we want to understand whether it is desirable for firms to collaborate to create a common infrastructure for the industry as a whole. As others already alluded, if there are only the public/common benefits in doing so, then the collaborating firms are not investing as much as they should do from an optimal economy-wide perspective, implying that the situation is socially less desirable. But, from a managerial viewpoint, we put forth that by participating in building a common infrastructure, the firms will also acquire valuable knowledge and expertise, which can be directly contributing to the firms’ internal improvement and productivity increase. Our main thrust is that should the firms understand this, i.e., the existence of ‘firm-specific’ benefits in addition to the ‘industry-wide’ benefits, they will be willing to make an investment as much as a ‘socially desirable’ level.

### 3. A DIFFERENTIAL GAME MODEL AND ITS SOLUTION

In our model, we consider ‘firm-specific’ benefits as well as common industry-wide benefits, in the process of creating an industry infrastructure by potentially competing firms in the same industry (Fershtman and Nitzan 1991). For analytical tractability, we assume a differential game setting, where two firms are collaborating on building the infrastructure.

#### 3.1 A Markov perfect Nash equilibrium case

Fershtman and Nitzan (1991) analyzed a dynamic version of the symmetric static

game of voluntary contributions to the supply of collective goods, by developing a game-theoretic model consisting of  $n$  players:

$$\underset{x_1(t), \dots, x_n(t)}{\text{Max}} \quad \sum_{i=1}^n \int_0^{\infty} e^{-rt} \{ \alpha f(K(t)) - C(x_i(t)) \} dt \quad (1a)$$

$$\text{Subject to} \quad \dot{K}(t) = \sum_{i=1}^n x_i(t) - \delta K(t), \quad K(0) = K_0. \quad (2a)$$

$K(t)$  is the stock of total contributions in terms of some physical input at time  $t$ ,  $x(t)$  the contribution rate of player  $i$ , and  $\delta$  a constant decay factor. In addition,  $f(\cdot)$  is a continuous monotonically increasing function and  $\alpha > 0$  a given constant, and  $C(x(t))$  the cost of contribution for player  $i$ .

Table 1. Definitions of Variables and Parameters

- $x_i(t)$ : Firm  $i$ 's cumulative investment in the common infrastructure by time  $t$  (a state variable)
- $\delta \geq 0$ : The decaying rate, i.e., the real value of cumulative investment is decayed by  $\delta x$  at  $t$
- $u_i(t)$ : Firm  $i$ 's current investment in the common infrastructure at  $t$  (a control variable)
- $\alpha_i > 0$ : A coefficient associated with the industry-wide benefits
- $b_i > 0$ : A coefficient associated with the firm-specific benefits
- $\rho \geq 0$ : A coefficient associated with the cost of investment
- $s_i$ : A dummy variable;  $s = 1$  when the firm-specific benefits exist,  $s = 0$  otherwise
- $r \geq 0$ : The discounting factor

Dockner, et al. (2000) further simplified the model by considering only two players and using specific functional forms. Using the same notation as that in our model (Table 1), their simplified model can be expressed as follows:

$$\text{Maximize} \quad J^i = \int_0^{\infty} e^{-rt} \left\{ x [\alpha_i - x] - \rho u_i - \frac{1}{2} u_i^2 \right\} dt \quad (1b)$$

$$\text{Subject to} \quad \dot{x} = u + u - \delta x, \text{ where } i = 1, 2 \text{ and } x = x_1 + x_2. \quad (2b)$$

Our improvement on the model is twofold. First, we introduce the term for the firm-specific benefits. Second, we separate  $x$  into  $x_1$  and  $x_2$  so that each individual player's dynamic decisions can be observed and analyzed independently. Our model is structured as follows:

$$\text{Maximize } J^i = \int_0^{\infty} e^{-rt} \left\{ (x_1 + x_2)[a_i - (x_1 + x_2)] + s_i x_i [b_i - x_i] - \rho u_i - \frac{1}{2} u_i^2 \right\} dt \quad (1)$$

$$\text{Subject to } \dot{x}_1 = u_1 - \delta x_1 \quad (2)$$

$$\dot{x}_2 = u_2 - \delta x_2 \quad (3)$$

Variables and parameters are defined in Table 1. Using the definitions, we explain each term in the constraints as well as the objective functional.

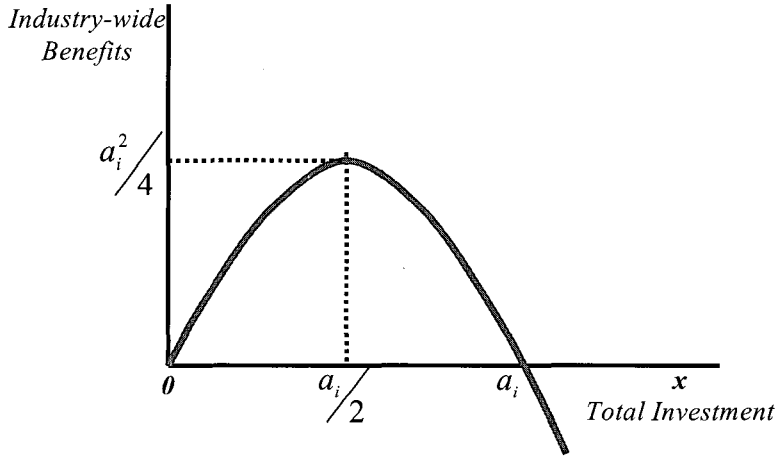


Figure 1. Industry-wide Benefit and Total Investment in Industry Infrastructure

#### A. The industry-wide benefits: $(x_1 + x_2)[a_i - (x_1 + x_2)]$

The first element in (1),  $(x_1 + x_2)[a_i - (x_1 + x_2)]$ , represents the industry-wide benefit enjoyed by Firm  $i$ , where  $x = x_1 + x_2$  stands for the industry-wide, i.e., Firm 1's plus Firm 2's investment in the common infrastructure. That is, the amount of benefit Firm 1 can derive from the industry infrastructure is determined by *not only* its own, *but also* its competitor Firm 2's investment in building the common infrastructure, and *vice versa*. As further explained below, formulating the benefits/revenues in a quadratic form like in (1) is considered as robust to depict the realistic situation: both Dockner, et al. (2000) and Fershtman and Nitzan (1991) adopted the formulation.

Another important observation is that the potential of the industry-wide benefit Firm  $i$  can enjoy is also determined by its own capability, e.g., efficiency, as represented by  $a_i$ . In effect,  $a_i$  embodies Firm  $i$ 's ability to transform the industry infrastructure to its industry-wide benefit vis-à-vis the competitor's. For

example, suppose  $a_1 > a_2$ . Then, it implies that given the same industry infrastructure, Firm 1 can enjoy more benefit than Firm 2 does.

Now  $(x_1 + x_2)[a_i - (x_1 + x_2)] = x(a_i - x) = -\left(x - \frac{a_i}{2}\right)^2 + \frac{a_i^2}{4}$ , which implies that

the maximum industry-wide benefit Firm  $i$  can obtain is  $\frac{a_i^2}{4}$ ; while  $0 \leq x \leq \frac{a_i}{2}$ , Firm  $i$ 's industry-wide benefit increases as the firms' total investment increases; but, while  $\frac{a_i}{2} \leq x \leq a_i$ , Firm  $i$ 's industry-wide benefit decreases as the firms' total investment increases; finally, once  $a_i \leq x$ , the benefit becomes negative. Therefore, under a normal situation (i.e., the objective function is to maximize the total profit), the Firm  $i$  try to invest as long as  $0 \leq x \leq \frac{a_i}{2}$ .

### B. The firm-specific benefits: $s_i x_i [b_i - x_i]$

It makes our model primarily different from those in Dockner, et al. (2000) and Fershtman and Nitzan (1991): in effect, it embodies our contribution to the literature. That is,  $s_i x_i [b_i - x_i]$  captures the firm-specific benefits an individual firm gets from its own effort to build the common industry infrastructure. The basic behavior of these firm-specific benefits looks similar with that of the industry-wide benefits in Figure 1. The most significant difference is that unlike the industry-wide benefit, a firm's firm-specific benefit is determined by its own effort only, not affected by the other's. Now the firm has an additional incentive to participate in building the industry infrastructure, i.e., the enhanced ability to improve its own/individual operations. We use  $s_i$  to give more flexibility to the model:  $s_i = 1$  means the model contains firm-specific benefits, whereas  $s_i = 0$  implies no firm-specific benefits. Since we focus on the case where both firm-specific and common benefits are present, we use  $s_1 = s_2 = 1$  for the ensuing formulation.

### C. The cost of investment: $C(u_i) = \rho u_i + \frac{1}{2} u_i^2$

This is a convex cost function, which is also widely accepted in the literature as consistent with the real-world context. In effect,  $C(u_i) = \rho u_i + \frac{1}{2} u_i^2 = \frac{1}{2}(u_i + \rho) - \frac{1}{2}\rho^2$ , which implies that as the investment increases, the cost increases in a convex manner, i.e., in an accelerating pattern. In addition, when there is no in-

vestment, the cost is zero, i.e.,  $C(0) = 0$ ; since  $u_i \geq 0$  is always assumed,  $C(0) \geq 0$  holds throughout the decision time horizon.

**D. The dynamics of investment efforts:**  $\dot{x}_i = u_i - \delta x_i$

Firm  $i$ 's cumulative investment in the common infrastructure accumulates following a dynamic,  $\dot{x}_i = u_i - \delta x_i$ , where  $\delta \geq 0$  is a constant decaying rate. It implies that if the firms don't make any effort to sustain the industry infrastructure, the real value/utility of the infrastructure actually deteriorates at the instantaneous rate of  $\delta x_i$  at  $t$ . Since  $\dot{x}_i = -\delta x_i$  if  $u_i = 0$ , the value/utility (i.e., the effective level of investment) of the entire industry infrastructure actually declines proportionally to the current level of investment, i.e.,  $\dot{x} = -\delta(x_1 + x_2)$ , if both firms make no effort at the same time, i.e.,  $u_1 = u_2 = 0$ . As in (2) and (3), including a decay effect into the state variable's dynamics is an important feature in optimal control theory that tries to depict the real-world systems behavior.

Now consider the case for  $i = 1$ , using HJB equation for an infinite time horizon case.

$$rJ = \max_{u_1} \left\{ (x_1 + x_2)[a_i - (x_1 + x_2)] + x_i[b_i - x_i] - \rho u_i - \frac{1}{2} u_i^2 + J_{x_1}(u_1 - \delta x_1) + J_{x_2}(u_2 - \delta x_2) \right\} \quad (4)$$

By taking partial differentiation to (4), we obtain  $u_1 = J_{x_1} - \rho$ . Similarly, we can obtain  $u_2 = J_{x_2} - \rho$  for  $i = 2$ . In order to solve the dynamic HJB equation, we suggest the following value functions:

$$J^1 = \frac{\alpha}{2}(x_1 + x_2)^2 + \beta_1(x_1 + x_2) + \frac{\theta}{2}x_1^2 + \phi_1 x_1 + \gamma_1 \quad (5)$$

for  $i = 1$ , and

$$J^2 = \frac{\alpha}{2}(x_1 + x_2)^2 + \beta_2(x_1 + x_2) + \frac{\theta}{2}x_2^2 + \phi_2 x_2 + \gamma_2 \quad (6)$$

for  $i = 2$ , where  $\alpha$ ,  $\beta_i$ ,  $\theta$ ,  $\phi_i$ , and  $\gamma_i$  are constants to be determined. Using a value function to solve the HJB equations in differential games is a well-established, widely used methodology (Kamien and Schwartz, 1991).



**Theorem 1.** Using the value function suggested above and the standard dynamic optimization method, we obtain a solution to the Markov perfect Nash equilibrium case as follows:

$$x_1 = c_1 e^{(2\alpha + \theta - \delta)t} + c_2 e^{(\theta - \delta)t} + \frac{\alpha(\beta_2 + \phi_2 - \rho) - (\alpha + \theta - \delta)(\beta_1 + \phi_1 - \rho)}{(\alpha + \theta - \delta)^2 - \alpha^2} \quad (7)$$

$$x_2 = c_1 e^{(2\alpha + \theta - \delta)t} - c_2 e^{(\theta - \delta)t} + \frac{\alpha(\beta_1 + \phi_1 - \rho) - (\alpha + \theta - \delta)(\beta_2 + \phi_2 - \rho)}{(\alpha + \theta - \delta)^2 - \alpha^2} \quad (8)$$

For a steady state solution, we require that  $2\alpha + \theta - \delta < 0$  and  $\theta - \delta < 0$ , where

$$\theta = \frac{(r + 2\delta) - \sqrt{(r + 2\delta)^2 + 8}}{2}, \quad (9)$$

$$\alpha = \frac{(r + 2\delta - 2\theta) - \sqrt{(r + 2\delta - 2\theta)^2 + 24}}{6}, \quad (10)$$

$$\phi_1 = \frac{b_1 - \theta\rho}{r + \delta - \theta}, \quad (11)$$

$$\phi_2 = \frac{b_2 - \theta\rho}{r + \delta - \theta}, \quad (12)$$

$$W_i = \frac{\alpha(2r\rho + 2\delta\rho - b_1 - b_2)}{r + \delta - \theta} - a_i, \quad (13)$$

$$\beta_1 = \frac{2\alpha + \theta - r - \delta}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_1 + \frac{-\alpha}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_2, \quad (14)$$

$$\beta_2 = \frac{-\alpha}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_1 + \frac{2\alpha + \theta - r - \delta}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_2. \quad (15)$$

Once initial values of the state variables ( $x_1(0) = x_{10}$  and  $x_2(0) = x_{20}$ ) are given, we can determine  $c_1$  and  $c_2$ . Since we are concerned with the steady state ‘investment level,’ however, we do not intend to specify such values.

**Proof.** See Appendix 2.

For future reference, we define  $\lim_{t \rightarrow \infty} x_i(t) \equiv x_{is}^N$  as the firm  $i$ 's steady state cumulative investment. Therefore,

$$\lim_{t \rightarrow \infty} x_1(t) \equiv x_{1s}^N = \frac{\alpha(\beta_2 + \phi_2 - \rho) - (\alpha + \theta - \delta)(\beta_1 + \phi_1 - \rho)}{(\alpha + \theta - \delta)^2 - \alpha^2}, \quad (16)$$

$$\lim_{t \rightarrow \infty} x_2(t) \equiv x_{2s}^N = \frac{\alpha(\beta_1 + \phi_1 - \rho) - (\alpha + \theta - \delta)(\beta_2 + \phi_2 - \rho)}{(\alpha + \theta - \delta)^2 - \alpha^2}, \quad (17)$$

where  $N$  stands for ‘Nash equilibrium’ and  $s$  for ‘steady state.’

### 3.2 A perfect coordination (collusive) case

Now let’s consider a perfect coordination case, where the two firms make a decision in harmony, i.e., it is almost like the two firms are under one decision-making authority. Note that the two firms are still separate and independent entities.

The optimal control theory problem for this situation is formulated as follows:

*Maximize*

$$J = \int_0^{\infty} e^{-rt} \left\{ (x_1 + x_2)[a_1 + a_2 - 2(x_1 + x_2)] + x_1[b_1 - x_1] + x_2[b_2 - x_2] - \rho(u_1 + u_2) - \frac{1}{2}(u_1^2 + u_2^2) \right\} dt \quad (18)$$

$$\text{Subject to} \quad \dot{x}_1 = u_1 - \delta x_1 \quad (19)$$

$$\dot{x}_2 = u_2 - \delta x_2 \quad (20)$$

Applying the optimal control theory, we use the current value Hamiltonian to solve the problem.

$$H = (x_1 + x_2)[a_1 + a_2 - 2(x_1 + x_2)] + x_1[b_1 - x_1] + x_2[b_2 - x_2] - \rho(u_1 + u_2) - \frac{1}{2}(u_1^2 + u_2^2) + \lambda_1(u_1 - \delta x_1) + \lambda_2(u_2 - \delta x_2) \quad (21)$$

Necessary conditions are:

$$\frac{\partial H}{\partial u_1} = -\rho - u_1 + \lambda_1 = 0, \quad u_1 = \lambda_1 - \rho \quad (22)$$

$$\frac{\partial H}{\partial u_2} = -\rho - u_2 + \lambda_2 = 0, \quad u_2 = \lambda_2 - \rho \quad (23)$$

$$\dot{\lambda}_1 = r\lambda_1 - \frac{\partial H}{\partial x_1} = (r + \delta)\lambda_1 + 6x_1 + 4x_2 - (a_1 + a_2 + b_1) \quad (24)$$

$$\text{Likewise, } \dot{\lambda}_2 = r\lambda_2 - \frac{\partial H}{\partial x_2} = (r + \delta)\lambda_2 + 4x_1 + 6x_2 - (a_1 + a_2 + b_2) \quad (25)$$

**Theorem 2.** The steady state solution of the state variables for the perfect coordination case, i.e.,  $x_1$  and  $x_2$ , is as follows:

$$\lim_{t \rightarrow \infty} x_1(t) \equiv x_{1s}^C = \frac{4[\rho(r + \delta) - (a_1 + a_2 + b_2)] - [\rho(r + \delta) - (a_1 + a_2 + b_1)][\delta(r + \delta) + 6]}{[\delta(r + \delta) + 6]^2 - 16}, \quad (26)$$

$$\lim_{t \rightarrow \infty} x_2(t) \equiv x_{2s}^C = \frac{4[\rho(r + \delta) - (a_1 + a_2 + b_1)] - [\rho(r + \delta) - (a_1 + a_2 + b_2)][\delta(r + \delta) + 6]}{[\delta(r + \delta) + 6]^2 - 16}. \quad (27)$$

As in the Nash equilibrium case,  $s$  stands for ‘steady state’ and  $C$  for ‘coordination.’

**Proof.** See Appendix 2.

**Corollary 1.** Firm 1’s steady state cumulative investment under the collusive decision-making scheme, i.e.,  $\lim_{t \rightarrow \infty} x_1(t) \equiv x_{1s}^C$ , is larger than Firm 2’s if and only if  $(b_1 - b_2)[\delta(r + \delta) + 10] > 0$ , i.e.,  $b_1 > b_2$ . That is,  $x_{1s}^C > x_{2s}^C$  iff  $b_1 > b_2$ . It implies that the steady state investment level is determined by the magnitudes of  $b_i$ .

It is also interesting to note that if there are no firm-specific benefits, i.e.,  $s_1 = s_2 = 0$ , then the two firms will invest the same amount in the long run, i.e.,  $x_{1s}^C = x_{2s}^C$ : see Appendix 2. It holds regardless of the magnitudes of  $a_i$ .

#### 4. ANALYSIS OF THE MODEL

Theorems 1 and 2 provide us with very important implications. In this section, we more thoroughly analyze them, to explore our main research question related with the issue of free riding in the investment. Dockner, et al. (2000) showed the solutions for the differential game problem with public benefits only and at the integrated level, i.e., the total investment level, not differentiated between two firms. Table A1 in Appendix 1 summarizes those results.

Our approach to solving the problem is different from that in Dockner, et al. (2000) in two folds. First, we have calculated the individual firm’s investment level, not just the summed one. Thus, it becomes possible for us to see how the difference between the firms’ investment levels is caused as shown in the corol-

lary. Second, we have considered two cases separately, i.e., the case with common benefits only and that with firm-specific benefits in addition to the common ones. Table A2 in Appendix 1 summarizes our results.

In order to answer our primary research question, we would like to compare the steady state investment levels in different situations: see Table 2.

Table 2. Notations of Steady State Total Investment Levels

	A differential game situation (A Markov perfect Nash equilibrium)	A collusive situation (A perfect coordination)
Common (industry-wide) benefits only	$X_{CN}$	$X_{CC}$
Common and firm-specific benefits	$X_{FN}$	$X_{FC}$

$X_{CN}$  is the steady state total investment (including both firms') in building the infrastructure, when there exist common benefits only and the two firms are competing against each other (i.e., Nash equilibrium).  $X_{CC}$  is the same as  $X_{CN}$  except that the two firms coordinate their decision-making, i.e., a collusive situation.

When there are two different types of benefits (i.e., firm-specific and common) coexist, we use  $X_{FN}$  and  $X_{FC}$  to represent the steady state total investment summing the two firms' together:  $X_{FN}$  is for the Nash equilibrium, whereas  $X_{FC}$  for the collusive case.

Using Table A2 in Appendix 1, we derive the exact values of  $X_{CN}$  and  $X_{CC}$ :

$$X_{CN} = \frac{2\rho(r + \delta - \alpha) - (a_1 + a_2)}{(r + \delta - 3\alpha)(2\alpha - \delta)}, \quad (28)$$

$$X_{CC} = \frac{2[(a_1 + a_2) - \rho(r + \delta)]}{\delta(r + \delta) + 8}. \quad (29)$$

Employing the notation in the previous section (see (7), (8), (26), and (27)), we rewrite  $X_{FN}$  and  $X_{FC}$  as follows:

$$X_{FN} \equiv x_{1s}^N + x_{2s}^N = \frac{(a_1 + a_2)(r + \delta - \theta) - (2r\rho + 2\delta\rho - b_1 - b_2)(r + \delta - \alpha - \theta)}{(r + \delta - \theta)(3\alpha + \theta - r - \delta)(2\alpha + \theta - \delta)}, \quad (30)$$

$$X_{FC} \equiv x_{1s}^C + x_{2s}^C = \frac{2[(a_1 + a_2) - \rho(r + \delta)] + (b_1 + b_2)}{\delta(r + \delta) + 10}. \quad (31)$$

**Observation 1.**  $X_{CN} \leq X_{CC}$ . It implies that the coordinated investment decision brings about a steady state investment level higher than when the decision is made under a Markov perfect Nash equilibrium. This observation's key managerial implication is that when there are industry-wide benefits only, it is impossible to avoid the opportunistic behavior on the part of the firms that invest in building the industry infrastructure. Therefore, the total investment in the infrastructure is far less than the optimal level, which is achievable when there is no free-rider problem. The proof will be available upon request.

**Observation 2.** A Necessary Condition for  $X_{CC} \leq X_{FC}$ .

It is possible to determine parameter values so that

$$X_{2N} - X_{1C} = [\delta(r + \delta) + 8] \cdot [(a_1 + a_2)(r + \delta - \theta) - (2\rho(r + \delta) - b_1 - b_2)(r + \delta - \alpha - \theta)] \\ - 2[(a_1 + a_2) - \rho(r + \delta)] \cdot (r + \delta - \theta)(3\alpha + \theta - r - \delta)(2\alpha + \theta - \delta) \geq 0 \quad (32)$$

which implies  $X_{CC} \leq X_{FC}$ . This is the key theorem of our research. It shows that under certain circumstances (that largely depend on the magnitudes of  $a_i$  and  $b_i$ ), the steady state total investment of a Markov perfect Nash equilibrium can be higher than that in a perfect coordination case, i.e., the opportunistic behavior can be lessened. Its managerial implication is that when there exist both firm-specific and industry-wide benefits in building the industry infrastructure, the free-riding problem can be avoided or at least mitigated. This was not possible when firm-specific benefits did not exist.

**Observation 3.** A Necessary Condition for  $X_{CC} \leq X_{FC}$ .

It holds if  $b_1 + b_2 \geq \frac{4[(a_1 + a_2) - \rho(r + \delta)]}{\delta(r + \delta) + 8}$ , which is equivalent to  $\frac{b_1 + b_2}{2} \geq X_{CC}$ . (33)

We can derive a managerial implication that if the potential of firm-specific benefits is sufficiently large (i.e.,  $\frac{b_1 + b_2}{2} \geq X_{CC}$ ), having both industry-wide and firm-specific benefits is better than having just the industry-wide benefit even when the firms cooperate fully. It indicates that the existence of firm-specific benefits can get rid of any possible opportunistic behavior even when the firms are coordinating their operations completely.

## 5. NUMERICAL ANALYSIS AND ITS IMPLICATIONS

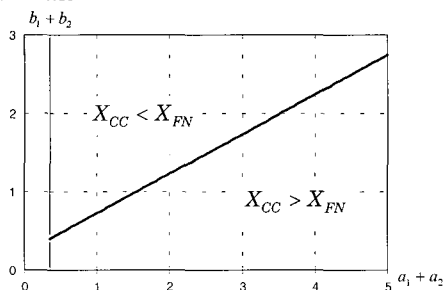
In order to more clearly see the implications of our analysis, we conduct numerical analysis using the parameter values, which are used to depict a realistic situation in Wirl (1996).

Table 3. Base Values of Parameters in the Numerical Analysis

$r$	$\delta$	$\rho$	$s_1$	$s_2$	$a_1$	$a_2$	$b_1$	$b_2$
0.1	0.2	1	1	1	1.1	1	0.5	0.5

From Figure 2, note that Observation 2 is valid only when  $b = b_1 + b_2$  is not so (relatively) small relative to  $a = a_1 + a_2$ . It indicates that only when the firm-specific benefits are significant in relation to the common benefits, it is possible to avoid the free-riding problem. Figure 3 shows the cases where  $X_{CN} \leq X_{FN}$  and  $X_{FN} \leq X_{FC}$ .

Observation 2



Observation 3

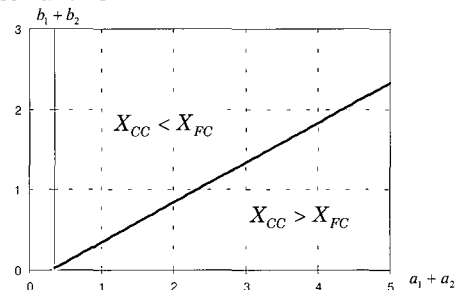
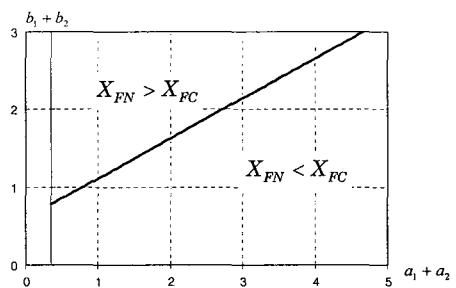
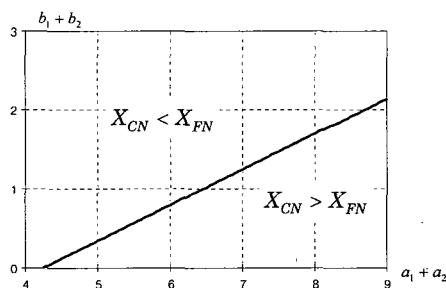


Figure 2. Observation 2 and 3

Figure 3. Cases where  $X_{CN} \leq X_{FN}$  and  $X_{FC} \leq X_{FN}$

From the first and the second observation, we are able to suggest that under an appropriate condition, it is possible to have  $X_{CN} \leq X_{CC} \leq X_{FN}$ . Its key implication is that if there are firm-specific benefits when firms work together to build a business infrastructure for common benefits, the coordination can be more desirable from the whole economy's perspective: that is, the total investment to the infrastructure building can be greater than that in a complete coordination case where there may be no competition between the firms. Therefore, the free-rider problem can be mitigated.

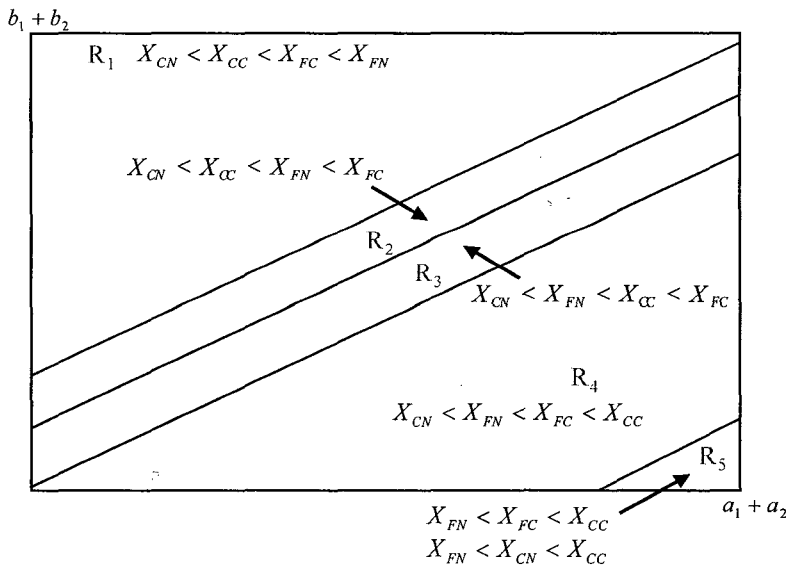


Figure 4. Combined Observations

Figure 4 recapitulates the key observations of our paper in an integrated way. Based on the numerical analysis results, we divide the  $a - b$  space into 5 regions  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$ , and  $R_5$ . There is an extreme case,  $R_5$ , where the common benefit is very large, but the firm-specific benefit is very small. Thus, in  $R_5$  it is difficult to avoid the free-riding problem, implying that the collusive case offers the largest investment in creating the public goods. Also in  $R_3$  and  $R_4$ , the firm-specific benefit is not big enough for the competitive setting to avoid the free riding problem: the main difference between  $R_3$  and  $R_4$ , though, is that in  $R_3$  the firm-specific benefit is larger than that in  $R_4$  so that the existence of firm-specific benefits is better for the economy as a whole than when there are only the industry-wide benefits, even under the collusive situation.

Finally, in  $R_1$  and  $R_2$ , the firm-specific benefit becomes large enough to mitigate the free riding problem, i.e., the total investment level becomes larger than that in the collusive case. The important difference between  $R_1$  and  $R_2$  is that in  $R_1$  the firm-specific benefit becomes sufficiently large so that the cumulative investment level of the competitive setting is larger than that in any other situation including both ‘common benefit only’ collusive case and ‘common plus firm-specific’ collusive case.

Figure 4 again underlines our primary observation: if the firm-specific benefit is sufficiently large compared with the common benefit, it will be possible to avoid the free riding problem even when there is no collusive arrangement between the firms. From an individual firm’s perspective, it is also an important conclusion. If the company can figure out how to utilize knowledge derived from the ‘common effort’ to improve its own internal operations, it will be more willing to get involved in developing the common industry infrastructure.

## 6. MANAGERIAL IMPLICATIONS AND DISCUSSION

We started with a question “Why do firms collaborate on building a common industry-level infrastructure?” It is also related with other managerial questions focused on the issues of coordination or alliances between (sometimes competing) firms. The industry-level infrastructure acts like public goods in that it offers benefits to most of the firms in the industry, regardless of whether the beneficiaries have contributed to building the infrastructure in the first place. Thus, an issue of ‘free riding’ is a serious one. Fershtman and Nitzan (1991) showed that when providing a public good, a collusive case generates more investment than a (Markov perfect) competitive case, implying that the free rider problem hampers achieving an optimal investment from the entire economy’s perspective.

From a more managerial viewpoint, we suggested that the firms get not only ‘common industry-wide,’ but also some ‘firm-specific’ benefits by participating in developing the public goods, i.e., industry infrastructure. For instance, by participating in creating an industry-wide e-Business infrastructure, the firm will be able to *not only* do business with other partners in the industry more efficiently, *but also* improve its internal manufacturing process so as to enhance its own productivity. Should the firm understand this mechanism, it will be more willing to collaborate with other firms on providing public goods (such as common infrastructure or industry-wide standardization), which will benefit the economy as a



whole in the end.

Including both industry-wide and firm-specific benefits simultaneously in the model, we were able to prove that under certain circumstances, it is indeed possible to avoid the free riding problem: such circumstances, in large part, depend on the balance of difference between sizes of industry-wide and firm-specific benefits: if the firm-specific benefit is sufficiently large compared with the common benefit, it will be possible to avoid the free riding problem even when there is no collusive arrangement between the firms. It has a profound implication to an individual firm. If the company is capable of finding out how to utilize knowledge derived from the 'common effort' to enhance its own internal productivity, it can afford to get *more* involved in developing the common infrastructure than otherwise. It will have a positive repercussion in the economy as a whole.

Following the same reasoning in reverse, one can state that unless there are firm-specific benefits in the inter-firm collaboration, any such endeavor is unsustainable, implying that it can be affected by some form of opportunism, i.e., 'free riding.' That is, only when external collaboration also enables the firm to enhance its internal learning as 'firm-specific' benefits, it becomes meaningful for the firm to participate in such collaboration.

Despite the well-focused implication of our research, we are mindful of potential shortcomings. First, although it is not unusual in the literature, our model was just able to depict a two-person differential game setting. But, we envisage the key implications of our analysis will be applicable to a more generalized situation as well. Second, although we believe it would not pose a serious threat to the validity of our analysis, the linearity assumption in our model could be a vulnerable one.

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## APPENDIX 1

Table A1. Industry Level Investment – Common Benefits Only

A Markov perfect Nash equilibrium	A perfect coordination
$\lim_{t \rightarrow \infty} x(t) = -\frac{\beta_1 + \beta_2 - 2\rho}{2\alpha - \delta}$ $\beta_i = \frac{1}{\Delta_m} [2\alpha^2\rho + (r + \delta)(a_i - 2\alpha\rho) - \alpha(2a_i - a_j)]$ $\Delta_m = (r + \delta - 2\alpha)^2 - \alpha^2$ $\alpha = \frac{r + 2\delta}{6} - \sqrt{\left(\frac{r + 2\delta}{6}\right)^2 + \frac{2}{3}}$	$\lim_{t \rightarrow \infty} x(t) = \frac{2}{\Delta_c} [\rho(r + \delta) - (a_1 + a_2)]$ $\Delta_c = -\delta(r + \delta) - 8$

Table A2. Individual Firm Investment – Common and Firm-specific Benefits

	Markov perfect Nash equilibrium	Full coordination
<b>Common Benefits Only</b>	$\lim_{t \rightarrow \infty} x_1(t) = \frac{\alpha(\beta_2 - \rho) - (\alpha - \delta)(\beta_1 - \rho)}{(\alpha - \delta)^2 - \alpha^2}$ $\lim_{t \rightarrow \infty} x_2(t) = \frac{\alpha(\beta_1 - \rho) - (\alpha - \delta)(\beta_2 - \rho)}{(\alpha - \delta)^2 - \alpha^2}$	$\lim_{t \rightarrow \infty} x_i(t) = \frac{1}{\Delta_c} [\rho(r + \delta) - (a_1 + a_2)]$ $\Delta_c = -\delta(r + \delta) - 8$ <p>That is, <math>\lim_{t \rightarrow \infty} x_1(t) = \lim_{t \rightarrow \infty} x_2(t)</math></p>
<b>Common &amp; Firm-specific Benefits</b>	$\lim_{t \rightarrow \infty} x_1(t) = \frac{\alpha(\beta_2 + \phi_2 - \rho) - (\alpha + \theta - \delta)(\beta_1 + \phi_1 - \rho)}{(\alpha + \theta - \delta)^2 - \alpha^2}$ $\lim_{t \rightarrow \infty} x_2(t) = \frac{\alpha(\beta_1 + \phi_1 - \rho) - (\alpha + \theta - \delta)(\beta_2 + \phi_2 - \rho)}{(\alpha + \theta - \delta)^2 - \alpha^2}$ $\beta_i = \frac{2\alpha + \theta - r - \delta}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_i + \frac{-\alpha}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_j$ $\theta = \frac{(r + 2\delta) \pm \sqrt{(r + 2\delta)^2 + 8s_1}}{2}$ $\alpha = \frac{(r + 2\delta - 2\theta) \pm \sqrt{(r + 2\delta - 2\theta)^2 + 24}}{6}$ $W_i = \frac{\alpha(2r\rho + 2\delta\rho - s_1b_1 - s_2b_2)}{r + \delta - \theta} - a_i$ $\phi_1 = \frac{s_1b_1 - \theta\rho}{r + \delta - \theta}, \quad \phi_2 = \frac{s_2b_2 - \theta\rho}{r + \delta - \theta}$	$\lim_{t \rightarrow \infty} x_1(t) = \frac{4[\rho(r + \delta) - (a_1 + a_2 + s_2b_2)] - [\rho(r + \delta) - (a_1 + a_2 + s_1b_1)][\delta(r + \delta) + 2s_2 + 4]}{[\delta(r + \delta) + 2s_1 + 4][\delta(r + \delta) + 2s_2 + 4] - 16}$ $\lim_{t \rightarrow \infty} x_2(t) = \frac{4[\rho(r + \delta) - (a_1 + a_2 + s_1b_1)] - [\rho(r + \delta) - (a_1 + a_2 + s_2b_2)][\delta(r + \delta) + 2s_1 + 4]}{[\delta(r + \delta) + 2s_1 + 4][\delta(r + \delta) + 2s_2 + 4] - 16}$

## APPENDIX 2 – PROOFS OF THEOREM 1 AND 2

**Theorem 1.**

Consider for  $i = 1$ , using HJB equation for an infinity case:

$$rJ = \max_{u_i} \left\{ (x_1 + x_2)[a_i - (x_1 + x_2)] + s_i x_i [b_i - x_i] - \rho u_i - \frac{1}{2} u_i^2 + J_{x_1}(u_1 - \delta x_1) + J_{x_2}(u_2 - \delta x_2) \right\} \quad (\text{A1})$$

Thus, we obtain:  $u_1 = J_{x_1} - \rho$ ,  $u_2 = J_{x_2}^2 - \rho$ .

We suggest the following value functions:

$$J = \frac{\alpha}{2}(x_1 + x_2)^2 + \beta_1(x_1 + x_2) + \frac{\theta}{2}x_1^2 + \phi_1 x_1 + \gamma_1 \quad \text{for } i = 1, \text{ and}$$

$$J^2 = \frac{\alpha}{2}(x_1 + x_2)^2 + \beta_2(x_1 + x_2) + \frac{\theta}{2}x_2^2 + \phi_2 x_2 + \gamma_2 \quad \text{for } i = 2.$$

Take appropriate partial differentiations and plug them into (A1):

$$J_{x_1} = \alpha(x_1 + x_2) + \beta_1 + \theta x_1 + \phi_1$$

$$J_{x_2} = \alpha(x_1 + x_2) + \beta_1$$

$$J_{x_2}^2 = \alpha(x_1 + x_2) + \beta_2 + \theta x_2 + \phi_2$$

$$\begin{aligned} \therefore rJ &= (x_1 + x_2)[a_1 - (x_1 + x_2)] + s_1 x_1 [b_1 - x_1] - \rho(J_{x_1} - \rho) - \frac{1}{2}(J_{x_1} - \rho)^2 \\ &\quad + J_{x_1}(J_{x_1} - \rho - \delta x_1) + J_{x_2}(J_{x_2}^2 - \rho - \delta x_2) \end{aligned}$$

$$\begin{aligned} \therefore rJ &= (x_1 + x_2)[a_1 - (x_1 + x_2)] + s_1 x_1 [b_1 - x_1] + \frac{1}{2}(J_{x_1})^2 - \rho J_{x_1} - \delta x_1 J_{x_1} + \frac{1}{2}\rho^2 \\ &\quad + J_{x_2} J_{x_2}^2 - \rho J_{x_2} - \delta x_2 J_{x_2} \end{aligned}$$

After rearranging appropriately, we have

$$\begin{aligned} \therefore &\left\{ -r \frac{\alpha}{2} - 1 + \frac{3}{2} \alpha^2 \right\} (x_1 + x_2)^2 \\ &+ \{ -r \beta_1 + a_1 + \alpha(\beta_1 + \phi_1) - \alpha \rho + \alpha(\beta_2 + \phi_2) + \alpha \beta_1 - \alpha \rho \} (x_1 + x_2) \\ &+ \left\{ -r \frac{\theta}{2} - s_1 + \frac{1}{2} \theta^2 - \theta \delta \right\} x_1^2 + \{ -r \phi_1 + s_1 b_1 + \theta(\beta_1 + \phi_1) - \theta \rho - \delta(\beta_1 + \phi_1) \} x_1 \\ &+ \{ \alpha \theta - \alpha \delta \} x_1 (x_1 + x_2) + \{ \alpha \theta - \alpha \delta \} x_2 (x_1 + x_2) + \{ \theta \beta_1 - \delta \beta_1 \} x_2 \end{aligned}$$

$$+\left\{-r\gamma_1 + \frac{1}{2}(\beta_1 + \phi_1)^2 - \rho(\beta_1 + \phi_1) + \frac{1}{2}\rho^2 + \beta_1(\beta_2 + \phi_2) - \rho\beta_1\right\} = 0$$

$$\therefore Ax_1^2 + Bx_1x_2 + Cx_2^2 + Dx_1 + Ex_2 + F = 0.$$

Therefore, we must have:  $A = B = C = D = E = F = 0$ , where

$$A = \left\{-r\frac{\alpha}{2} - 1 + \frac{3}{2}\alpha^2\right\} + \left\{-r\frac{\theta}{2} - s_1 + \frac{1}{2}\theta^2 - \theta\delta\right\} + \{\alpha\theta - \alpha\delta\}$$

$$B = 2\left\{-r\frac{\alpha}{2} - 1 + \frac{3}{2}\alpha^2\right\} + \{\alpha\theta - \alpha\delta\} + \{\alpha\theta - \alpha\delta\}$$

$$C = \left\{-r\frac{\alpha}{2} - 1 + \frac{3}{2}\alpha^2\right\} + \{\alpha\theta - \alpha\delta\}$$

$$D = \{-r\beta_1 + \alpha_1 + \alpha(2\beta_1 + \beta_2 + \phi_1 + \phi_2 - 2\rho)\} + \{-r\phi_1 + s_1b_1 + (\theta - \delta)(\beta_1 + \phi_1) - \theta\rho\}$$

$$E = \{-r\beta_1 + \alpha_1 + \alpha(2\beta_1 + \beta_2 + \phi_1 + \phi_2 - 2\rho)\} + \{\theta\beta_1 - \delta\beta_1\}$$

$$F = -r\gamma_1 + \frac{1}{2}(\beta_1 + \phi_1)^2 - \rho(\beta_1 + \phi_1) + \frac{1}{2}\rho^2 + \beta_1(\beta_2 + \phi_2) - \rho\beta_1.$$

After taking appropriate steps for both firms in order to make sure that  $A = B = C = D = E = F = 0$  holds, we reach the following.

$$\therefore \phi_1 = \frac{s_1b_1 - \theta\rho}{r + \delta - \theta}, \quad \phi_2 = \frac{s_2b_2 - \theta\rho}{r + \delta - \theta},$$

$$\beta_1 = \frac{2\alpha + \theta - r - \delta}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_1 + \frac{-\alpha}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_2,$$

$$\beta_2 = \frac{-\alpha}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_1 + \frac{2\alpha + \theta - r - \delta}{(2\alpha + \theta - r - \delta)^2 - \alpha^2} W_2,$$

$$\text{where } W_i = \frac{\alpha(2r\rho + 2\delta\rho - s_1b_1 - s_2b_2)}{r + \delta - \theta} - \alpha_i.$$

In addition,  $\gamma_1 = \frac{1}{r} \left[ \frac{1}{2}(\beta_1 + \phi_1)^2 - \rho(\beta_1 + \phi_1) + \frac{1}{2}\rho^2 + \beta_1(\beta_2 + \phi_2) - \rho\beta_1 \right]$  and

$$\gamma_2 = \frac{1}{r} \left[ \frac{1}{2}(\beta_2 + \phi_2)^2 - \rho(\beta_2 + \phi_2) + \frac{1}{2}\rho^2 + \beta_2(\beta_1 + \phi_1) - \rho\beta_2 \right].$$

Now, using  $u_1 = J_{x_1} - \rho = \alpha(x_1 + x_2) + \theta x_1 + \beta_1 + \phi_1 - \rho$  and

$$u_2 = J_{x_2} - \rho = \alpha(x_1 + x_2) + \theta x_2 + \beta_2 + \phi_2 - \rho, \text{ we eventually obtain}$$

$$x_1 = c_1 e^{(2\alpha + \theta - \delta)t} + c_2 e^{(\theta - \delta)t} + \frac{\alpha(\beta_2 + \phi_2 - \rho) - (\alpha + \theta - \delta)(\beta_1 + \phi_1 - \rho)}{(\alpha + \theta - \delta)^2 - \alpha^2},$$

$$x_2 = c_1 e^{(2\alpha + \theta - \delta)t} - c_2 e^{(\theta - \delta)t} + \frac{\alpha(\beta_1 + \phi_1 - \rho) - (\alpha + \theta - \delta)(\beta_2 + \phi_2 - \rho)}{(\alpha + \theta - \delta)^2 - \alpha^2}.$$

To determine  $c_1$  and  $c_2$ , assume  $x_1(0) = x_{10}$  and  $x_2(0) = x_{20}$ .  
(Further suppose  $x_1(0) = x_2(0) = 0$  to calculate  $c_1$  and  $c_2$ .)

### Theorem 2.

Now, consider a perfect coordination (i.e., collusive) case. The current value Hamiltonian is:

$$H = (x_1 + x_2)[a_1 + a_2 - 2(x_1 + x_2)] + s_1 x_1 [b_1 - x_1] + s_2 x_2 [b_2 - x_2]$$

$$- \rho(u_1 + u_2) - \frac{1}{2}(u_1^2 + u_2^2) + \lambda_1(u_1 - \delta x_1) + \lambda_2(u_2 - \delta x_2) \Big\} dt$$

Necessary conditions:

$$\frac{\partial H}{\partial u_1} = -\rho - u_1 + \lambda_1 = 0, \quad u_1 = \lambda_1 - \rho; \quad \frac{\partial H}{\partial u_2} = -\rho - u_2 + \lambda_2 = 0, \quad u_2 = \lambda_2 - \rho$$

$$\dot{\lambda}_1 = r\lambda_1 - \frac{\partial H}{\partial x_1} = (r + \delta)\lambda_1 + (2s_1 + 4)x_1 + 4x_2 - (a_1 + a_2 + s_1 b_1)$$

$$\text{Likewise, } \dot{\lambda}_2 = r\lambda_2 - \frac{\partial H}{\partial x_2} = (r + \delta)\lambda_2 + 4x_1 + (2s_2 + 4)x_2 - (a_1 + a_2 + s_2 b_2).$$

Solving the necessary conditions, we have the following. For instance, suppose the case when  $s_1 = s_2 = 1$ .

$$\left[ \{(\delta + z)(r + \delta - z)\} + 2 \right] \left[ \{(\delta + z)(r + \delta - z)\} + 10 \right] = 0$$

Therefore,  $(\delta + z)(r + \delta - z) = -2$  or  $(\delta + z)(r + \delta - z) = -10$

$$\text{From } (\delta + z)(r + \delta - z) = -2, \text{ we know } z = \frac{r \pm \sqrt{r^2 + 4\{\delta(r + \delta) + 2\}}}{2}.$$

$$\text{From } (\delta + z)(r + \delta - z) = -10, \text{ we know } z = \frac{r \pm \sqrt{r^2 + 4\{\delta(r + \delta) + 10\}}}{2}.$$

In order for the problem to have a solution, it must hold that  $\lim_{t \rightarrow \infty} x_i(t)$  converges.

Therefore, out of four possible values, we can have only two, i.e.,

$$z_1 = \frac{r - \sqrt{r^2 + 4\{\delta(r + \delta) + 2\}}}{2} \quad \text{and} \quad z_3 = \frac{r - \sqrt{r^2 + 4\{\delta(r + \delta) + 10\}}}{2} \quad \text{by posing}$$

$$A_2 = A_4 = B_2 = B_4 = C_2 = C_4 = D_2 = D_4 = 0.$$

Now, we have the following system.

$$x_1 = A_1 e^{z_1 t} + A_3 e^{z_3 t} + \text{Constant}$$

$$x_2 = B_1 e^{z_1 t} + B_3 e^{z_3 t} + \text{Constant}$$

$$\lambda_1 = C_1 e^{z_1 t} + C_3 e^{z_3 t} + \text{Constant}$$

$$\lambda_2 = D_1 e^{z_1 t} + D_3 e^{z_3 t} + \text{Constant}$$

In order to determine the constant terms, we can use the following set of relations.

$$0 = -\delta x_1 + \lambda_1 - \rho \tag{A2}$$

$$0 = -\delta x_2 + \lambda_2 - \rho \tag{A3}$$

$$0 = (2s_1 + 4)x_1 + 4x_2 + (r + \delta)\lambda_1 - (a_1 + a_2 + s_1 b_1)$$

$$0 = 4x_1 + (2s_2 + 4)x_2 + (r + \delta)\lambda_2 - (a_1 + a_2 + s_2 b_2)$$

After some arrangements, we have the following:

$$\text{Const}(x_1) = \frac{4[\rho(r + \delta) - (a_1 + a_2 + s_2 b_2)] - [\rho(r + \delta) - (a_1 + a_2 + s_1 b_1)][\delta(r + \delta) + 2s_2 + 4]}{[\delta(r + \delta) + 2s_1 + 4][\delta(r + \delta) + 2s_2 + 4] - 16}$$

$$\text{Const}(x_2) = \frac{4[\rho(r + \delta) - (a_1 + a_2 + s_1 b_1)] - [\rho(r + \delta) - (a_1 + a_2 + s_2 b_2)][\delta(r + \delta) + 2s_1 + 4]}{[\delta(r + \delta) + 2s_1 + 4][\delta(r + \delta) + 2s_2 + 4] - 16}$$

Now we have:

$$x_1 = A_1 e^{z_1 t} + A_3 e^{z_3 t} + \text{Const}(x_1)$$

$$x_2 = B_1 e^{z_1 t} + B_3 e^{z_3 t} + \text{Const}(x_2)$$

From (A2) and (A3), we know:

$$\text{Const}(\lambda_1) = \delta \text{Const}(x_1) + \rho \quad \text{and} \quad \text{Const}(\lambda_2) = \delta \text{Const}(x_2) + \rho.$$

$$\lambda_1 = C_1 e^{z_1 t} + C_3 e^{z_3 t} + \text{Const}(\lambda_1)$$

$$\lambda_2 = D_1 e^{z_1 t} + D_3 e^{z_3 t} + \text{Const}(\lambda_2)$$

Now, how to determine  $A_1, A_3, B_1, B_3, C_1, C_3, D_1, D_3$ ?

Since  $\dot{x}_1 = -\delta x_1 + \lambda_1 - \rho$  and  $\dot{x}_2 = -\delta x_2 + \lambda_2 - \rho$ ,

$$A_1 z_1 e^{z_1 t} + A_3 z_3 e^{z_3 t} = -\delta A_1 e^{z_1 t} - \delta A_3 e^{z_3 t} - \delta \text{Const}(x_1) + C_1 e^{z_1 t} + C_3 e^{z_3 t} + \text{Const}(\lambda_1) - \rho.$$

$$A_1(z_1 + \delta) = C_1, \quad A_3(z_3 + \delta) = C_3, \quad \delta \text{Const}(x_1) - \text{Const}(\lambda_1) + \rho = 0.$$

Likewise,  $B_1(z_1 + \delta) = D_1, \quad D_3(z_3 + \delta) = D_3$ .

Therefore, we only need to determine  $A_1, A_3, B_1, B_3$ . It will be possible to determine exact values of these parameters, if specific initial conditions for the state variables are provided. For our purpose, we only need the steady-state values, which are represented by the constants in the state variables.