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FATIGUE DAMAGE PARAMETER OF SPOT WELDED JOINTS UNDER PROPORTIONAL LOADING

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ABSTRACT−In this paper, the author proposes a fatigue damage parameter of spot welded joints under proportional loading. The proposed fatigue damage parameter is developed based on von Mises' equivalent stress and local structural stress at the edge of spot weld nugget. The structural stress at the edges of the weld nugget in each sheet is calculated using the forces and moments that are determined by finite element analysis. A structural equivalent stress is then calculated by von Mises' equivalent stress equation. The structural equivalent stresses are correlated to experimental fatigue life of the spot welded joints. The proposed parameter is evaluated with fatigue test data of spot welds subjected to multiaxial and tensile-shear loads. Sheppard's parameter and Rupp and co-workers' parameter are also evaluated with the same test data to compare with the author's parameter. This proposed parameter presents a better correlation with experimental fatigue data than those of Sheppard's and Rupp and co-workers' parameter. The proposed parameter should be very effective for durability calculations during the early design phase since coarsely meshed finite element models can be employed.

KEY WORDS : Fatigue life, Spot welded joint, Multiaxial load, Tensile-shear load, Structural stress, Equivalent stress

1. INTRODUCTION

Electrical resistance spot welding is a common method of joining sheet steel in the automotive industry. A typical automobile may contain more than 3000 spot welds that join various body and structural components (Rupp *et al.*, 1995). Basic fatigue life and joint characterization tests are commonly conducted using tensile-shear specimens of single spot welds. In these tests, the weld is primarily subjected to shearing forces, and the fatigue cracks are typically initiated at the nugget edge at the sheet interface.

Numerous researchers proposed fatigue damage parameters for spot welded joints. These parameters were typically formulated using either fracture mechanics based approaches (Swellam *et al.*, 1991; Swellam and Lawrence, 1991; Radaj *et al.*, 1991a, 1991b, 1992) or structural stress based approaches (Sheppard, 1993, 1996; Rupp *et al.*, 1995; Heyes and Fermer, 1996). Those parameters were then correlated with fatigue test results to predict fatigue life of spot welds. A numerical approach was also applied to estimate fatigue life of spot welds (Kang and Barkey, 1999).

Very detailed finite element models of a spot welded joint can be constructed to calculate the stress states near the joint. However, such an approach was typically used when analyzing the characteristics of a single joint (Kan, 1976), and not employed during the development phase of the ground vehicle design process. Instead, forces and moments acting on each joint were usually determined by the linear elastic finite element method and these forces and moments were used to calculate a fatigue damage parameter for the joint (Sheppard, 1993, 1996; Rupp *et al.*, 1995; Heyes and Fermer, 1996).

Generally, most empirical models correlated the fatigue life of spot welded joints to geometric factors and load conditions, so the constants in the models were valid only for the specific test data. When specimen types and geometries were different from those of the empirical model, the model had to be modified for the new test data. Therefore, Rupp and co-workers (Rupp *et al.*, 1995) and Sheppard (Sheppard, 1996) proposed fatigue damage models for spot welded joints that were independent from the geometric factors and specimen types. However, these models still experienced some limitations (Kang, 1999).

In this study a fatigue damage parameter based on a structural equivalent stress is proposed to estimate fatigue life of spot welds subjected to proportional loading. The damage parameter is correlated to the multiaxial fatigue test results (Kang, 1999) and the tensile-shear test data (Swellam *et al.*, 1991). The evaluation results of the

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structural equivalent stress approach are compared to those of other damage parameters based on a structural stress (Rupp *et al.*, 1995; Sheppard, 1996).

2. STRUCTURAL EQUIVALENT STRESS APPROACH

It is important to consider large local deformation for calculation of stresses at the edge of spot welded joints. The large local deformations generally occur at the very first cycle of fatigue loading and result in a plastic zone around the weld nugget (Rupp *et al.*, 1995). The plastic deformations make it difficult to obtain exact stresses around the weld nugget using linear elastic finite element analysis even if refined models are used. Such analyses also require extensive modeling expertise and long calculation time. Furthermore, these models can be difficult to apply to real structural components. Therefore, structural stress approaches that use coarse meshed models to obtain forces and moments at a spot weld are more useful for engineering purposes.

It is also important to determine the failure modes for applying the proper damage parameter to calculate fatigue life numerically. Based on the observations of the previous researchers (Barkey *et al.*, 2001; Sheppard, 1996; Rupp *et al.*, 1995; Swellam *et al.*, 1991), two types of failure modes can be classified. These are a crack in the top or bottom sheet metal and the crack through the weld nugget. Plate theory was applied to calculate structural stresses (Sheppard, 1996; Rupp *et al.*, 1995) in case of cracking in the sheets. Beam theory was used for cracking through the weld nugget (Rupp *et al.*, 1995).

For calculation of the forces and moments at the spot welded connection, linear elastic finite element models were used. These models contained linear elastic shell elements for the sheet metals, a stiff beam element for the spot weld, and rigid body elements for loading fixtures. The length of the beam element was equal to the thickness of the sheet.

In the current study, structural equivalent stress was calculated at the edges of the weld nugget in each sheet using the forces and moments that were determined by a finite element analysis. The von Mises' equivalent stress equation is presented as below:

$$\sigma_q^i = \frac{1}{\sqrt{2}}[(\sigma_x^i - \sigma_y^i)^2 + (\sigma_y^i - \sigma_z^i)^2 + (\sigma_z^i - \sigma_x^i)^2 + 6((\tau_{xy}^i)^2 + (\tau_{yz}^i)^2 + (\tau_{zx}^i)^2)]^{1/2}, \quad (1)$$

where σ_q^i is structural equivalent stress based on von Mises' equation. Here, i is an index representing the top sheet (i=1) or bottom sheet (i=2), and q is an index representing equivalent stress. σ_x^i, σ_y^i, and σ_z^i represent normal stresses in x, y, and z direction at the top or bottom sheet, respectively. τ_{xy}^i, τ_{yz}^i, and τ_{zx}^i represent shear stresses in xy, yz, and zx planes at the top or bottom sheet, respectively.

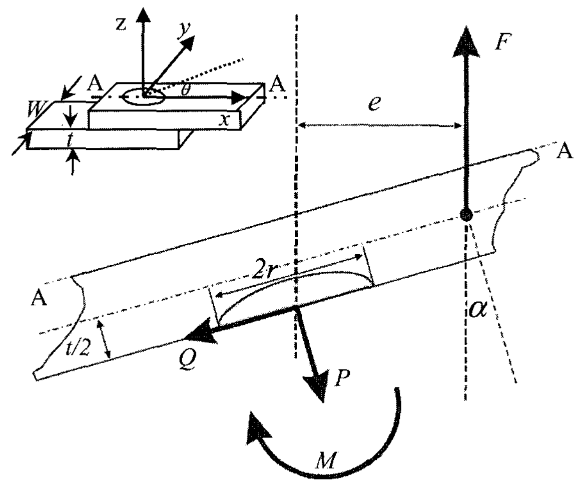

Figure 1. Resolved components P, Q and M at the nugget for a general applied load F.

The free body diagram of the weld nugget is presented in Figure 1. F represents an applied load, P represents an axial load, Q represents a membrane load, and M (M=F×e) represents a moment. The angle between the loading direction and z-axis is defined as α that is employed to resolve the applied load (F) into Q and P. The stresses due to the applied load at the edge of the weld nugget were assumed to be three normal stresses and one shear stress, and were calculated using following equations (Young, 1989):

$$\sigma_x^i = \frac{\Delta Q_x^i}{2\pi r t} + \frac{6\Delta M_y^i}{Wt^2}, \quad (2)$$

$$\sigma_y^i = \frac{\Delta Q_y^i}{2\pi r t} + \frac{6\Delta M_x^i}{Wt^2}, \quad (3)$$

$$\sigma_z^i = \frac{\Delta P_z^i}{\pi r^2}, \quad (4)$$

$$\tau_{zx}^i = \frac{\Delta P_z^i}{2\pi r t}, \quad (5)$$

where σ_z^i is a stress range in z-direction, $\frac{\Delta Q_x^i}{2\pi r t}$ and $\frac{\Delta Q_y^i}{2\pi r t}$ are membrane stress ranges, $\frac{6\Delta M_x^i}{Wt^2}$ and $\frac{6\Delta M_y^i}{Wt^2}$ are stress ranges due to bending moments, and τ_{zx}^i is a shearing stress range due to load range in z-direction (ΔP_z^i). The specimen thickness, width, and nugget radius are t, W, and r, respectively. Here, i is an index representing the top sheet (i=1) or bottom sheet (i=2).

The applied load ranges can be easily resolved into axial direction and membrane direction using knowledge of statics. The resolved load ranges are inputs for calculations of stress ranges in equation (2) through (5). The structural equivalent stress (σ_q^i) is to be then calculated at the top and bottom sheet using the equation (1) for the applied load ranges on the spot welds. Then, maximum structural equivalent stress (σ_q^{max}) is to be determined from σ_q^i and to be correlated with test results.

3. OTHER STRUCTURAL STRESS APPROACH

Sheppard (1993, 1996) calculated the structural stresses based on a nominal stress determined by bending moments, membrane forces, and axial forces by

$$\Delta S = \Delta Q/(t\omega) + 6\Delta M^*/(t^2 W) + (\Delta P)/t^2, \tag{6}$$

where $\Delta Q/t\omega$ is a membrane stress term, $6\Delta M^*/t^2W$ is a bending stress term, and $\Delta P/t^2$ is the stress at the edge of the weld nugget in longitudinal direction due to an axial load. In these relations, the effective width in shear $\omega=\pi d/3$, t is the thickness of the sheet, d is the nugget diameter, and W is the width of the piece. The structural stress range is fairly sensitive to the variation of the sheet thickness due to the third term of the Equation (6).

Then, a curve fitting equation is derived from a plot of maximum structural stress range (ΔS_{max}) versus measured fatigue life ($N_f/(1-R)$) on a log-log scale as below:

$$\frac{N_f}{(1-R)} = C(\Delta S_{max})^{-b}. \tag{7}$$

N_f is defined as the total life spent propagating the crack through the sheet thickness t. The coefficient C and exponent b are from a power law relation of maximum structural stress range versus measured fatigue life for crack propagation.

Rupp and co-workers (Rupp *et al.*, 1995) calculated the local structural stresses based on the cross-sectional forces and moments using beam, sheet, and plate theory. The equivalent stresses for the damage parameter were calculated by combination and superposition of the local structural stress. The equivalent stresses were calculated as a function of angle θ around the circumference of the spot weld. Here, θ is the angle measured from a reference axis as shown in Figure 1. The equivalent stresses for cracking in the sheet were calculated using superposition of formulae for the plate subjected to central loading as below:

$$\sigma_{eql}(\theta) = -\sigma_{max}(F_x)\cos\theta - \sigma_{max}(F_y)\sin\theta + \sigma(F_z), \\ + \sigma_{max}(M_x)\sin\theta - \sigma_{max}(M_y)\cos\theta \tag{8}$$

$$\text{where: } \sigma_{max}(F_x) = \frac{F_x}{\pi dt}, \tag{9}$$

$$\sigma_{max}(F_y) = \frac{F_y}{\pi dt}, \tag{10}$$

$$\sigma(F_z) = \kappa\left(\frac{1.744F_z}{t^2}\right) \text{ for } F_z > 0, \tag{11}$$

$$\sigma(F_z)=0 \text{ for } F_z \le 0, \tag{12}$$

$$\sigma_{max}(M_x) = \kappa\left(\frac{1.872M_x}{dt^2}\right), \tag{13}$$

$$\sigma_{max}(M_y) = \kappa\left(\frac{1.872M_y}{dt^2}\right), \tag{14}$$

$$\kappa = 0.6\sqrt{t}.$$

The parameter κ is a material dependent geometry factor applied to the stress terms calculated from the bending moment. It effectively reduces the sensitivity of these stress terms to the sheet thickness.

Structural stresses for cracking through nuggets were calculated based on the elastic formulae of a beam subjected to tension, bending and shear. The resolved tensile stress on the critical plane were taken as the damage parameter, and were calculated from the state of combined tension and shear:

$$\tau(\theta) = \tau_{max}(F_x)\cos\theta + \tau_{max}(F_y)\cos\theta, \tag{16}$$

$$\sigma(\theta) = \sigma(F_z) + \sigma_{max}(M_x)\sin\theta - \sigma_{max}(M_y)\cos\theta, \tag{17}$$

$$\text{where: } \tau_{max}(F_x) = \frac{16F_x}{3\pi d^2}, \tag{18}$$

$$\tau_{max}(F_y) = \frac{16F_y}{3\pi d^2}, \tag{19}$$

$$\sigma(F_z) = \frac{4F_z}{\pi d^2} \text{ for } F_z > 0, \tag{20}$$

$$\sigma(F_z)=0 \text{ for } F_z \le 0, \tag{21}$$

$$\sigma_{max}(M_x) = \frac{32M_x}{\pi d^3}, \tag{22}$$

$$\sigma_{max}(M_y) = \frac{32M_y}{\pi d^3}. \tag{23}$$

The total fatigue life was then correlated with the calculated maximum equivalent stress amplitude.

4. EVALUATION OF THE STRUCTURAL EQUIVALENT STRESS APPROACH

For the evaluation of the structural equivalent stress approach, two sets of test data were used. One data set was obtained from the specimens subjected to multiaxial loads (Kang, 1999). The other data set was obtained from specimens subjected to tensile-shear loads (Swellam *et* *al.*, 1991). The structural equivalent stress approach was first evaluated with the combined tension and shear test results, and then with the tensile-shear test results.

In this evaluation, the amplitudes of the maximum structural equivalent stress were correlated to the fatigue test data of combined tension and shear loads. The maximum structural equivalent stress amplitudes were calculated using nodal forces and moments at the spot weld in the center plane of the sheet. It was assumed that a fatigue crack was initiated at the edge of the weld nugget as observations of the previous researchers (Barkey *et al.*, 2001; Sheppard, 1996; Rupp *et al.*, 1995; Swellam *et al.*, 1991).

It was also assumed that the maximum structural equivalent stress amplitude at the crack initiation site could be directly related to the fatigue life of the spot welded joint. The structural equivalent stress amplitude was calculated at the two sides of the nugget edge for each metal sheet in this evaluation based on the observations of the previous researchers. Then, the maximum structural equivalent stress amplitude was determined from the two structural equivalent stress amplitudes calculated using the Equation (1).

This approach is very simple compared to other structural stress approaches (Rupp *et al.*, 1995; Sheppard, 1996). The shortcomings of Rupp and co-workers' approach and Sheppard's approach were well described in other articles (Kang and Barkey, 1999; Kang, 1999).

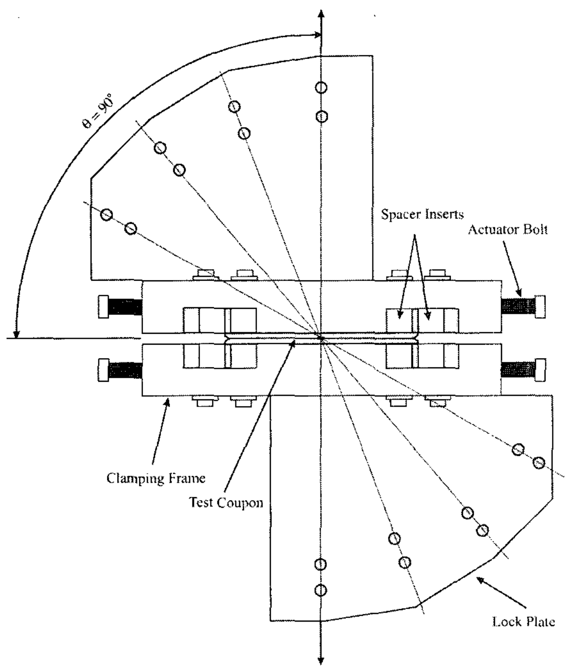

Figure 2. The test fixture to apply the combined tension and shear loads on the spot welded specimens.

The multiaxial test data includes 140 fatigue test results of high strength steel specimens subjected to combined tension and shear loads. The thickness of the specimens was 1.6 mm and the nominal diameters of the specimens were 5.4 mm and 8 mm. The three loading directions, 30°, 50°, and 90°, were employed to apply the combined tension and shear on the weld nugget as shown in Figure 2.

The maximum structural equivalent stress amplitudes were correlated to multiaxial test results as shown in Figure 3. The measured fatigue life versus the calculated fatigue life for high strength steel specimens under multiaxial loads were presented in Figure 4. All the data calculated with the approach were within the upper and

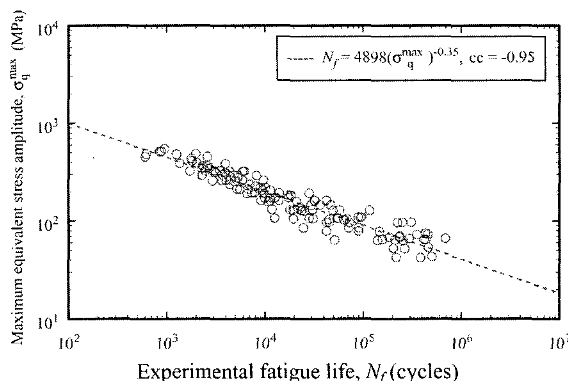

Figure 3. Experimental fatigue life versus maximum structural equivalent stress amplitude for multiaxial tests.

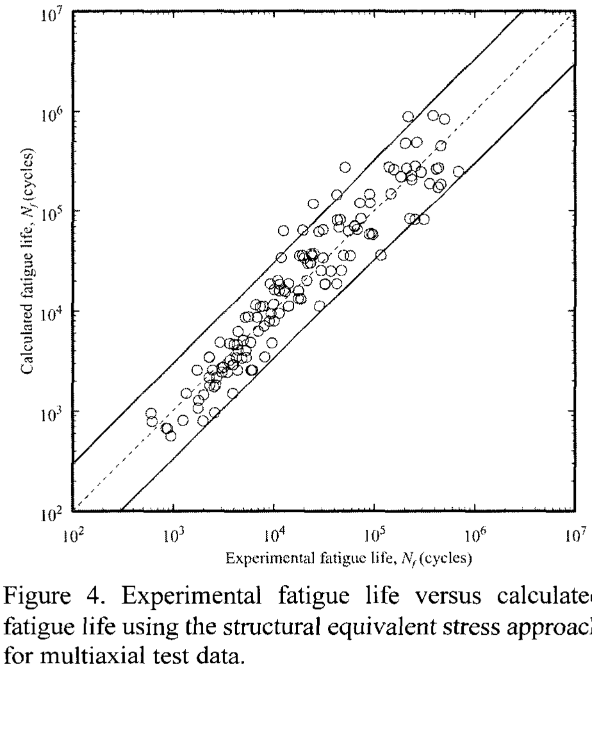

Figure 4. Experimental fatigue life versus calculated fatigue life using the structural equivalent stress approach for multiaxial test data.

lower boundaries. The dotted line in the figures represents a perfect correlation between the measurements and the calculations, and the solid lines represent a factor of three variations from a perfect correlation.

The structural equivalent stress approach was also evaluated with tensile-shear test data (Swellam *et al.*, 1991). The maximum structural equivalent stress amplitudes were correlated to tensile-shear test data as the same manner as shown in Figure 3. The experimental fatigue life versus the calculated fatigue life for high strength steel and low carbon steel specimens under tensile-shear loads was shown in Figure 5. The structural equivalent stress approach shows good correlation between experimental fatigue life and calculated fatigue life for specimens subjected to multiaxial loading and tensile-shear loading.

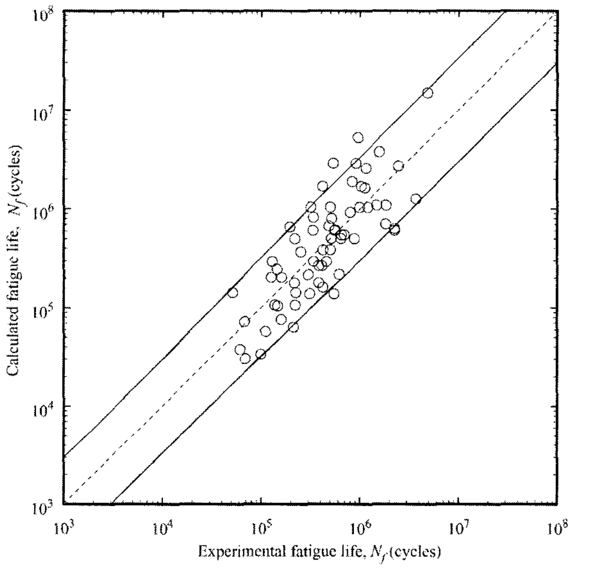

Figure 5. Experimental fatigue life versus calculated fatigue life using the structural equivalent stress approach for tensile-shear test data.

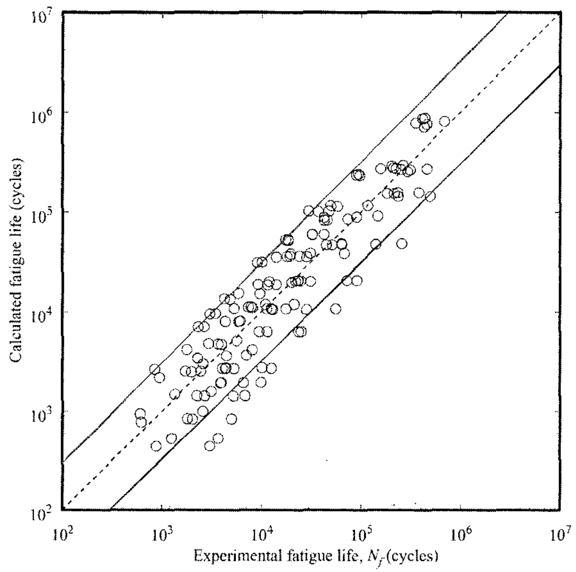

Figure 7. Experimental fatigue life versus calculated fatigue life using Rupp and co-workers' approach for multiaxial test data.

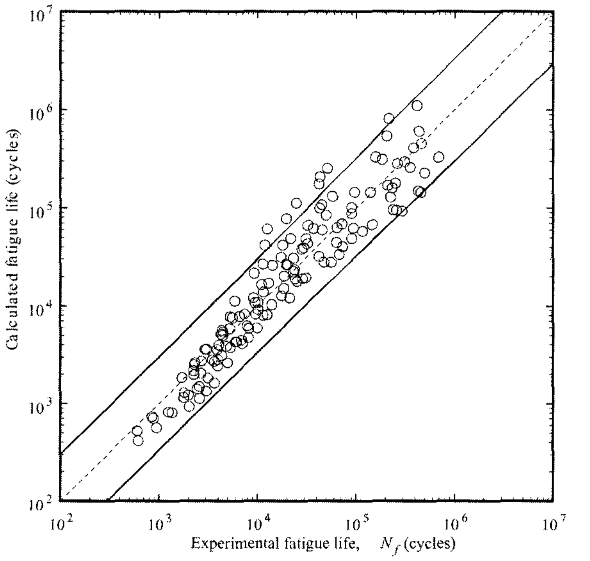

Figure 6. Experimental fatigue life versus calculated fatigue life using Sheppard's approach for multiaxial test data.

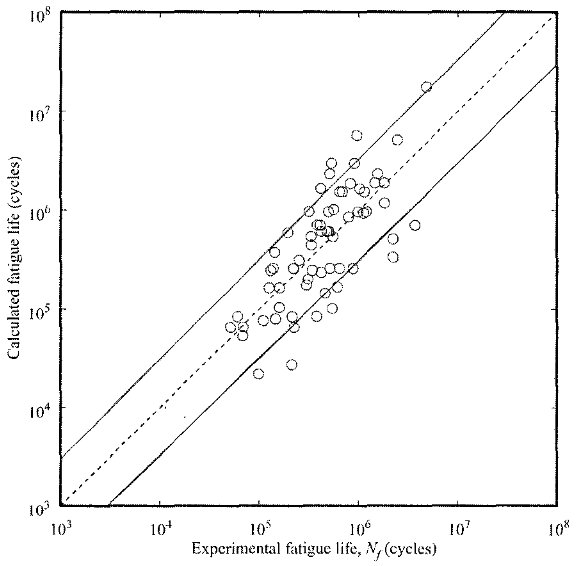

Figure 8. Experimental fatigue life versus calculated fatigue life using Rupp and co-workers' approach for tensile-shear test data.

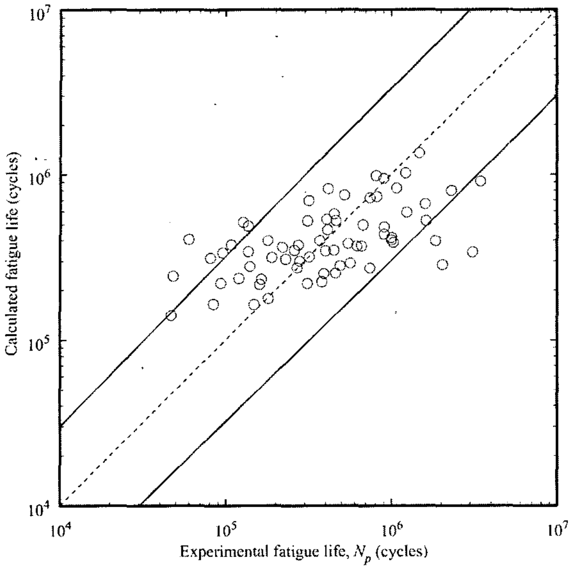

Figure 9. Experimental fatigue life versus calculated fatigue life using Sheppard's approach for tensile-shear test data.

This study has also compared the structural equivalent stress approach with other structural stress approaches, such as Sheppard's (Sheppard, 1996) and Rupp and co-workers' approach (Rupp *et al.*, 1995). Figure 6 and Figure 7 presented experimental fatigue life versus the calculated fatigue life using Sheppard's approach and Rupp and co-workers' approach for the multiaxial test data, respectively. Figure 8 and Figure 9 presented experimental fatigue life versus the calculated fatigue life using Sheppard's approach and Rupp and co-workers' approach for tensile-shear test data, respectively, to compare with those of the structural equivalent stress approach.

Sheppard's and Rupp and co-workers' approaches show good correlation between experimental fatigue life and calculated fatigue life for the specimens subjected to multiaxial loads. However, their approaches show more scatter than the structural equivalent stress parameter does for tensile-shear test data.

5. CONCLUSION

A fatigue damage parameter was proposed based on the local structural equivalent stress at the spot welded joints. The forces and moments were calculated using linear elastic finite element analysis with a coarsely meshed model. The structural equivalent stress was calculated by von Mises' equivalent stress equation using the local structural stresses.

The proposed model using structural equivalent stress amplitudes was well correlated to multiaxial and uniaxial test data. The results of this model also compared to those of Sheppard's and Rupp and co-workers' approaches. The results of the structural equivalent stress model showed better correlation than those of Sheppard's and Rupp and co-workers' damage parameters. This parameter should be very effective for durability calculations during the early design phase since coarsely meshed finite element models can be employed.

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