EVOLUTION OF NUCLEAR FUEL MANAGEMENT AND REACTOR OPERATIONAL AID TOOLS

PAUL J. TURINSKY, PAUL M. KELLER and HANY S. ABDEL-KHALIK*
Electric Power Research Center
Department of Nuclear Engineering
P.O. Box 7909
North Carolina State University
Raleigh, NC USA 27695-7909
E-mail : turinsky@ncsu.edu, keller@ncsu.edu and Hany.Abdel-Khalik@framatome-anp.com
*Currently with Framatome ANP, Inc., Lynchburg, VA 24501

In this paper are reviewed the current status of nuclear fuel management and reactor operational aid tools. In addition, we indicate deficiencies in current capabilities and what future research is judged warranted. For the nuclear fuel management review the focus is on light water reactors and the utilization of stochastic optimization methods applied to the lattice, fuel bundle, core loading pattern, and for BWRs the control rod pattern/core flow design decision making problems. Significant progress in addressing separately each of these design problems on a single cycle basis is noted; however, the outstanding challenge of addressing the integrated design problem over multiple cycles under conditions of uncertainty remains to be addressed. For the reactor operational aid tools review the focus is on core simulators, used to both process core instrumentation signals and as an operator aid to predict future core behaviors under various operational strategies. After briefly reviewing the current status of capabilities, a more in depth review of adaptive core simulation capabilities, where core simulator input data are adjusted within their known uncertainties to improved agreement between prediction and measurement, is presented. This is done in support of the belief that further development of adaptive core simulation capabilities is required to further significantly advance the utility of core simulators in support of reactor operational aid tools.

1. INTRODUCTION

Nuclear fuel management and reactor operational aid tools today largely build upon the software capabilities developed to complete core design analysis. This has been made possible by both advances in computational efficiency due to methods, e.g. nodal methods, and computer hardware, e.g. higher performing CPUs. The advantages of building upon the core design analysis software capabilities are that the effort to establish the models needed to complete nuclear fuel management and for reactor operational aid tools is minimized, and once established one need not spend time reconciling differences between different models. So much of the discussion presented in the companion paper entitled "Current Research Activities on Diffusion and Transport Calculation Methodologies" authored by Professor Nam-Zin Cho serves as a basis for this paper. In this paper the focus will be on light water reactor (LWR) cores of the pressurized water reactor (PWR) and boiling water reactor (BWR) types, but much of what will be written is also applicable to other core types. We will first discuss the current status of nuclear fuel management and then follow with a discussion of reactor operator aid tools.

2. NUCLEAR FUEL MANAGEMENT

Nuclear fuel management [1] involves making the following decisions: the quantity and attributes of the fresh fuel assemblies that will be purchased, the partially burnt fuel assemblies that will be reinserted, the locations of both the fresh and partially burnt fuel assemblies within the core, i.e. core loading pattern (LP), and for a boiling water reactors the control rod program/core flow (CRP/CF) strategy. These decisions need to be made for each reload cycle. Since fuel assemblies are irradiated several core cycles, the nuclear fuel management decisions made for the current cycle will impact those made in subsequent cycles. The objective of nuclear fuel management is to minimize the nuclear fuel cycle cost while satisfying the cycle energy requirement. This must be done such that all safety and operational constraints are satisfied with sufficient margin. To get a grasp of the magnitude of the decision space, consider the following decisions that need to be made each reload cycle: fresh fuel lattice designs (pin-by-pin radial position fuel enrichment and burnable poison loading), fresh fuel bundle designs (axial span of different lattice designs), number of fresh fuel bundle designs of each type, partially burnt fuel assemblies

to be reinserted, location of fresh and partially burnt fuel assemblies within the core, and for BWRs the CRP/CF strategy as a function of cycle exposure. Constraining these decisions are various operational and safety limits as now indicated: reactivity limits (coolant/moderator density reactivity coefficient, shutdown margin, ejected or dropped rod worth, hot excess reactivity), thermal margins (total and radial power peaking factors, MFLCPR, MAPRAT, MFLPD), and mechanical/material limits (pin, assembly and region average discharge burnups, vessel fluence, excore detector count rate, and restrictions on fuel assembly placement {control cell core}). In the past and to this day, the ingenuity of the reload core design engineer has been used to make these decisions, finding feasible and near optimum core designs. To assist the reload core design engineer various computer aides have been developed, such as computer code linkage buffer codes, automated design calculational sequences (determination of moderator temperature coefficient versus moderator temperature, cycle burnup and soluble boron concentration), graphical interfaces to setup input (core loading pattern) and interpret output, and the application of mathematical optimization capabilities [2]. It is this last item, the application of mathematical optimization capabilities that this paper will focus on.

From the above discussion, we see that the nuclear fuel management decision making problem is highly constrained and has a decision space that is very large (can approach 10^{100}). Further complicating the application of mathematical optimization capabilities are that to evaluate the objective functions and core response constraints requires considerable computational effort (solution of the few-group neutron diffusion equation as a function of rodded configuration over the cycle); that the problem is nonlinear; that there is a lack of derivative information with respect to objective function and constraint values dependencies on decision variables; that the feasible space is disjointed; that both continuous and integer decision variables enter; and that multi-cycles should be evaluated, since decisions made for the current cycle impact subsequent cycle decisions, and since to evaluate fuel cycle costs, fuel must be tracked from initial fabrication to final disposition. Two prominent mathematical optimization methodologies have evolved that appear to have various degrees of applicability to the nuclear fuel management decision making problem. More classical mathematical optimization methods, such as linear, quadratic and dynamic programming, so far have met with limited success with a few exceptions [3], since they are likely not appropriate due to the attributes noted above. Stochastic methods, such as Simulated Annealing (SA) and Genetic Algorithm (GA), are well suited but carry a heavy computational burden due to the large number of potential sets of decision variable values that must be considered. Pseudo-heuristic methods, with the Tabu Search method [4-6] appearing to be the most popular for nuclear fuel management decision making, have the advantage of reduced search length but perhaps at the price of a reduced capability of coming close to finding the true optimum. In this paper we will concentrate on stochastic optimization methods for two reasons. Firstly, these methods appear to be more developed and utilized in practice. Secondly, the authors can draw upon their own experiences in developing such methods.

Stochastic optimization methods as noted above suffer from requiring a large number of combinations of decision variables, i.e. histories, to be evaluated. Constraints are addressed as either being hard, i.e. any violation results in rejection of the associated history case, or soft, implemented via penalty functions that allow constraint violations to be accepted early in the search. The advantage of utilizing soft constraints is that it facilitates moving across the infeasible decision space to locate disjointed feasible decision subspaces in search of the vicinity of the global optimum. Since the family of feasible near-optimum solutions is determined using stochastic methods, the reload core design engineer is presented with a number of possible decision variable sets that can be further examined factoring in constraints and objectives not addressed in the mathematical optimization since they may be difficult to express in a quantitative manner. To be discussed later, stochastic optimization methods are also well suited for multi-objective optimization. An example of where this would prove useful is in determining the trade-off surface of feed enrichment to satisfy the stated cycle energy requirement versus radial power peaking factor. The decision maker is now provided with quantitative information that can be used to determine the cost of introducing design margin.

Likely the most developed of the stochastic optimization methods is SA [7,8]. Casually, it appears to be a very simple method to implement; however, the reality is that to obtain both a robust, i.e. consistently locate the family of near-optimum decisions, and efficient, i.e. minimizes number of histories that must be examined to locate the family of near-optimum decisions, some thought must be given. SA is based upon the analogy of a solid slowly cooling to its lowest energy state, i.e. annealing. Given a minimization objective, let F denote the value of the objective function for the currently accepted decision variables, and F^* denote the value of the objective function after some perturbations are made to the currently accepted decision variables. Whether the perturbed decision variables are accepted, implying they now become the currently accepted decision variables, is determined by the following rules.

$$If\ F^* \le F\ or\ if\ F^* > F\ and\ random[0,1] < e^{-(F^*-F)/T}\ then$$
$$F = F^*, otherwise\ F = F \quad (1)$$

So if the perturbed decision variables produce a

lower objective function value, then they become the currently accepted decision variables. However, if the perturbed decision variable produce a higher objective function value, they only become the currently accepted decision variables if a random number between zero and one exceeds the expression noted in Eq. (1); otherwise, the currently accepted decision variables are unaltered. The conditional acceptance of inferior solutions allows the search algorithm to escape local minimums, and as we shall see when utilizing penalty functions, traverse the infeasible decision space. The parameter T plays the analog of material temperature in annealing. At high temperatures, many inferior solutions are accepted, allowing the search space to be extensively transversed in search of the vicinity of the global minimum. As temperature decreases, the probability of acceptance of inferior solutions decreases, since if the cooling schedule is done appropriately one should now be within the vicinity of the family of global optimum solutions. What the initial temperature should be and how fast cooling should occur determine the robustness and efficiency of the SA implementation. Fortunately, both of these attributes can be determined based upon the specific behavior of the optimization problem that is being solved as the search progresses [9]. Another item that must be addressed is at what temperature cooling should be ended and the optimization search terminated. This can be triggered by a combination of lower temperature limit, maximum number of histories, and lack of improvement in the objective function value.

Another item that enters is how perturbations for a history are going to be made. One could randomly perturb a single decision variable, several decision variables, or all the decision variables. The nature of SA is such that it is better to creep towards the vicinity of the global optimum decisions, implying only a few decision variables should be perturbed for each history case. If many are simultaneously perturbed and the feasible decision space is small versus the total decision space, and disjointed, it will prove difficult to locate and once located, stay within the feasible decision space. One now is left with deciding whether unitary, binary, tertiary, or higher number of decision variables should be perturbed each history. This can be decided randomly based upon a probability of the order of the perturbation, this probability distribution determined by trial-and-error hence application specific, e.g. LP optimization.

So far we have not discussed how constraints are to be imposed during the search process for the family of near-optimum decisions. As mentioned earlier, they may be imposed as either hard or soft constraints. Hard constraints normally apply to those that can be directly imposed on decision variables, thereby reducing the decision space and hence computational effort to solve the optimization problem. A PWR example of this is to not allow discrete burnable poison rods to be inserted in fuel assemblies under control rod locations. By contrast, soft constraints normally apply to constraints that require the response of the system being optimized, e.g. lattice or core, to be evaluated to determine whether a constraint violation has occurred. A BWR example is any of the thermal margins or cold shutdown margin. Since the constraints can result in a disjointed feasible space and since as noted above SA normally only perturbs a limited number of decision variables each history, one needs to transverse the infeasible decision space to locate the islands of feasible decision space within the total decision space. To accomplish this when soft penalty functions are utilized, they are added to the true objective function to obtain the augmented objective function as now indicated,

$$\widetilde{F} = F + \sum_{n=1}^{N} \lambda_n(T)\Theta_n \qquad (2)$$

where Θ_n denotes the penalty function for constraint n, which has zero value when no constraint violation occurs and increases when a constraint violation does occur as the magnitude of the violation increases in accordance with the mathematical definition of the penalty function [10]. There are a number of penalty functions from which to select that will impact the robustness and efficiency of SA. The "lambda" multipliers on the penalty functions increase as the cooling temperature decreases. By the appropriate relationship of cooling temperature and "lambda" multiplier and cooling schedule, early in the optimization search the entire decision space will be searched until the feasible space about the global minimum is determined, as which time the "lambda" multipliers and temperature should be such that any history thereafter that results in constraint violations has a low probability of being accepted via Eq. (1). This results in restricting the search to the feasible subspace containing the global minimum. Fortunately, the rate that the "lambda" multipliers should increase can be determined adaptively based upon the behavior of the optimization search as it proceeds.

Because of the stochastic nature of the SA search, many near-optimum solutions are generated, providing the designer the opportunity to review these solutions and down select based upon additional criteria not captured in the optimization. This implies that an archive of these solutions should be constructed as the SA search proceeds. A number of the SA solutions are very similar, since recall SA perturbs only a few decision variables in moving from one history to the next history. An example for an LP optimization would be the sole difference being a rotation of one fuel assembly (really a major axis crossing of a shuffled fuel assembly). To introduce diversity in the archived solutions, various metrics of diversity between loading patterns have been defined and used to build a diverse archive.

To illustrate the nature of SA, Figure 1 through

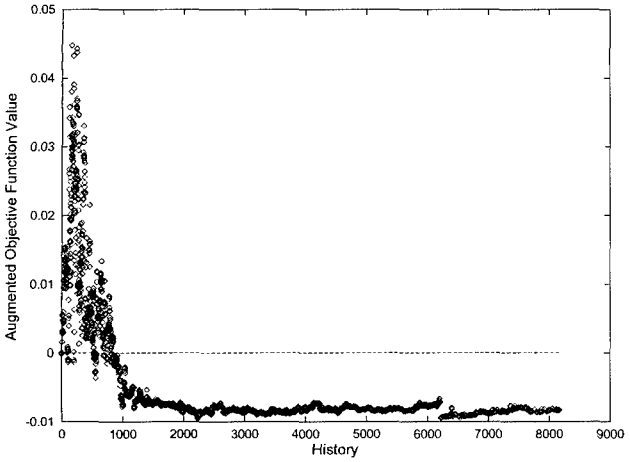

Fig. 1. Augmented Objective Function Behavior per Accepted History for BWR

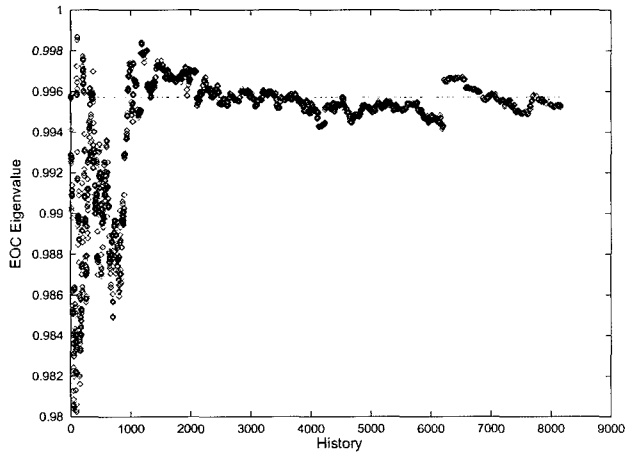

Fig. 2. End of Cycle k_{eff} Values per Accepted History for BWR

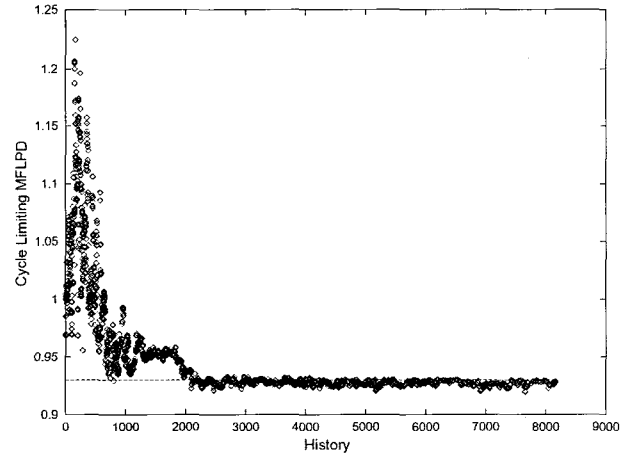

Fig. 3. Cycle Limiting MFLPD Values per Accepted History for BWR

Figure 3 present an SA optimization to determine the LP for a BWR with the objective of maximizing end-of-cycle core reactivity, taken from the work of Karve, Moore and Turinsky [11-13]. Figure 1 and Figure 2 show the augmented and true objective functions, respectively, for only those histories that result in acceptance. Note how local minima are found and then escaped. Figure 3 shows the MFLPD constraint value as the optimization search progresses. Near the end of the search, the constraints are or are nearly satisfied, allowing the search to concentrate of minimizing the real objectives value. For each history, the two-group, three dimensional neutron diffusion equation and associated two-phase flow fluids equations, must be solved at discrete burnup steps over the reload cycle, hence the computational burden is very high. Application of approximate core models, e.g. linearization via direct perturbations or generalized perturbation theory determined sensitivity coefficients, can minimize the computational burden but perhaps misdirect the search because of first-order accuracy in predicting the objective function and constraints' values. Alternatively, multi-processor computers can be employed. Recognizing that the SA algorithm is recursive, parallelization needs to be introduced in both the neutron diffusion equation and two-phase flow fluids equations solutions [14], or by employing a derivative of the SA algorithm with parallel constructs [15].

As noted earlier, real engineering problems generally involve multiple objectives [16]. A simple way to address multiple objectives is to assign weights to each objective and sum the weighted objectives to obtain a single objective function. Unfortunately, the near-optimum solutions determined will be dependent upon the values of the weights, which are not known. A much more appealing approach would be to define the trade-off surface of the decision variables. For example, if the objectives of interest are the feed fuel enrichment required to achieve a specified cycle energy requirement and the radial peaking factor, a measure of thermal margin, one would like to know for each value of the radial peaking factor what is the lowest feed fuel enrichment that can be utilized, where the decision being optimized is the LP. One could do an SA optimization minimizing the feed fuel enrichment and imposing radial peaking factor as a constraint, repeating this optimization for different values of the radial peaking factor constraint. This would require considerable computational time. Alternatively, one could seek a method that determines the tradeoff surface in a single optimization run such that the associated computational time is substantially reduced from the single objective, multiple run approach just noted. Multi-objective SA (MOSA) is one approach to attempt to accomplish this. Park's [17], refining the idea of Engrand [18], defines a multi-objective function as follows,

$$F = \sum_{i=1}^{N} w_i f_i \qquad (3)$$

where w_i and f_i denote for the i^{th} objective the weight and objective function, respectively. The weights are determined adaptively based upon the optimization problem's attributes as the search progresses. Using this in Eq. (1), SA is employed. However to determine the trade-off surface, which would be an N dimensional surface for N objectives, one must do something else. Otherwise, one would tend to not span the trade-off surface. This something else is to introduce the concept of a non-dominated solution, which is defined as follows,

A history X is non-dominated by history Y if $F_i(X) > F_i(Y)$ *for* $\forall i = 1, N$.

Figure 4 shows surfaces corresponding to non-dominated solutions. Now in contrast to having a single history that corresponds to the current history from

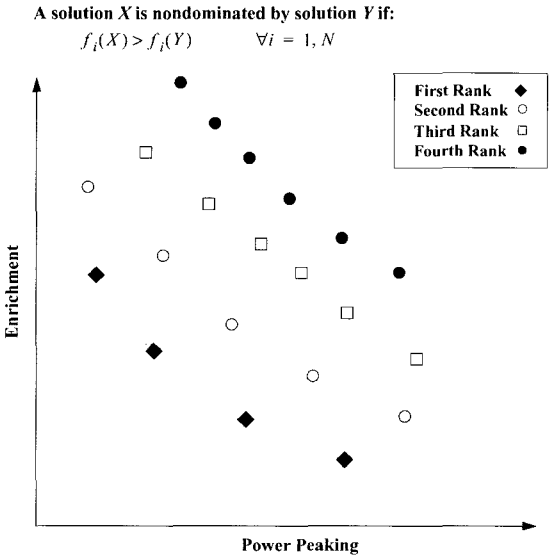

Fig. 4. Nondominated Solutions Identification for Two Objectives

which perturbations to the decisions variables are applied, all non-dominated solutions at any point during the optimization search can be considered for perturbation. Which non-dominated solution is selected is done randomly. In this manner the trade-off surface as depicted in Figure 4 moves from about the northeast to spanning the range from the southeast to northwest, composed of the family of near-optimum, non-dominated solutions that define the trade-off surface. Since our experience with loading pattern optimization is that MOSA is not able to span a very wide search space, particularly, it is not able to search across multiple feed fuel LPs, we will defer to presenting computational results until we discuss multi-objective GA (MOGA).

GA is based upon the biological principle of evolution, which originates by survival of the fittest [19]. In contrast to SA which has one current history where a few decision variables are perturbed, GA consists of a population of solutions. The original population may be generated by successively applying SA perturbations and accepting all associated histories. Once a population is established, breeding occurs within the population to determine the next generation of population. Using LP as an example, one defines a geno-type associated with characterizing an LP. By breaking and linking two geno-types, the offspring LP is generated. There are numerous ways of breaking and linking the geno-types, so computational experimentation is necessary to determine which approach is appropriate for the application at hand. The process that does this is referred to as the crossover operator. Clearly the crossover operator has the potential to perturb many more decision variables than the few perturbations that are associated with SA, therefore, one's expectation is that GA will be more efficient, i.e. require fewer histories, to span the decision space. By retaining genetic attributes of the parents, which if they possess desirable attributes, e.g. low objective function value in feasible decision space, one expects the successive generations of offspring to have improving attributes due to inheritance even though the crossover operator can perturb many decision variables simultaneously. The crossover operator also has the potential of destroying the physical inventory, e.g. for LP the same fuel assembly will appear in the geno-type string of both parents, resulting in two of these assemblies in the offspring even though there is only one such fuel assembly. Ways to restore the physical inventory, a tie breaker operator, then needs to be introduced [17]. How the parents are selected is dependent on their fitness, which could be determined by the modified objective function value. To introduce population diversity, one does not always wish to breed the most fit parents, so inferior solutions retain the potential to breed but with lower probability. Just as in the biological breeding process, mutations are introduced into the geno-type to introduce population diversity that did not exist in the original population. The mutation operator can be selected to be the SA perturbations operator if so desired. Reviewing the above, we see that the developer of GA must define the crossover operator, mutation operator, tie breaker operator, frequency of crossover versus mutation, parent fitness metric, selection process for parents, population size, and generations to follow. When done correctly, GA can prove more efficient and robust than SA in determining the vicinity of the global, optimum solution. There is some thought that a combination of GA to determine this vicinity followed by SA to complete a local decision space search to better determine the family of near-optimum decision variables may be an effective strategy, but limited computational experiments for fuel management problems have been completed to confirm

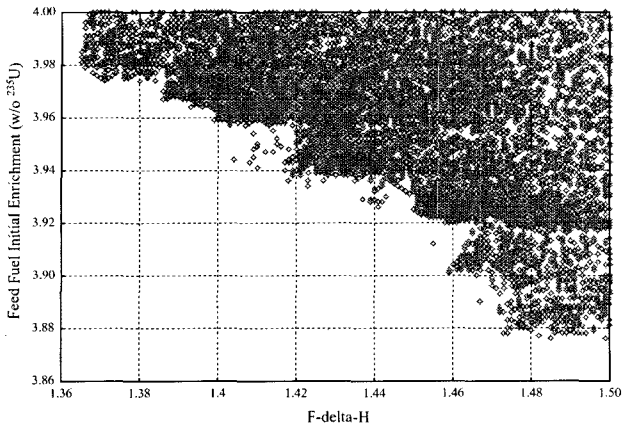

Fig. 5. MOGA Result for Feed Fuel Enrichment versus Radial Peaking Factor for PWR

this. Note that GA is embarrassingly parallel in that breeding can occur simultaneously, so it can be implemented on multi-processor computers without modifying the GA algorithm.

Just as there is a MOSA derivative for SA, there is a MOGA derivative for GA. The manner of how MOGA achieves multi-objective optimization follows the same approach as MOSA except that the underlying optimization method is GA versus SA. Figure 5 presents MOGA results generated by Keller [20] which shows the feasible solutions found for the multi-objective of feed fuel enrichment minimization while constrained to satisfy the cycle energy requirement and radial peaking factor minimization for a PWR with LP being the decision variable. Note the dense number of near-optimum solutions that lie on the tradeoff surface. The saw-tooth nature of the surface is associated with different fresh fuel location patterns, with the immediate surrounding solutions being alterations of the burnt fuel locations. The computational effort to complete this MOGA evaluation was found to be approximately 50% greater than a single GA evaluation, so clearly is preferred over multiple GA evaluations for different radial peaking factor constraints values.

So what can we do in practice today regarding nuclear fuel management optimization using mathematical optimization capabilities. We can optimize the fresh fuel' s lattice designs [21-24], but unfortunately if this is done in isolation of the core design it is hard to define what the objective function and constraints to be imposed should be. Given fresh fuel lattice designs, there is some capability to determine the axial elevation spans that the lattices should be assigned to [6]. Most recently, Kropaczek and Jessee [25] have optimized BWR lattice and bundle designs based upon a fixed inventory of fuel pin designs, i.e. fuel enrichment and gadolinia loading as function of elevation, for a fixed LP and CRP/CF. This work quantified the benefits of increasing the number of streams, i.e. different fresh bundle designs, while still constraining the manufacturing to a limited number of fuel pin designs. Regarding LP determination, we can determine the optimum LP for either a single non-equilibrium reload cycle [10,26-31] or the ideal equilibrium cycle [32,33]. This includes placement of the burnable poison in the fresh fuel assemblies. For a BWR, the LP optimization can be done in coincidence with the CRP/CF search [12,34]; however, the later tends to be completed utilizing heuristic rules versus a mathematical optimization approach. Fortunately, PWR LP objectives and constraints generally are of a 0-D or 2-D radial nature, so computational effort to complete the LP optimization are acceptable if a high fidelity 2-D radial model can be established, which the consistent collapse methodology [35] has been shown capable of producing. For the BWR LP problem [2,11-13], objectives and constraints require 3-D modeling due to axial heterogeneities due to the fuel, voiding, and partial control rod insertions. So one is left with the options of approximating the evaluation of the objective function and constraints as a function of decision variables, e.g. surface response model, accepting very long running optimizations, e.g. several days on a single processor, high end PC, or implementing on multi-processor computers.

So what remains to be done? Today we cannot solve the coupled lattice, bundle, LP and for a BWR CRP/CF decision making optimization problem. Once we achieve this capability, we still have to address the multi-cycle nature of the incore nuclear fuel management problem, which today is ignored since we are limited to single-stage, i.e. single cycle, successive optimization. Once the multi-cycle nature of the optimization problem is addressed, we still have to address the uncertainties that enter the optimization, due to input date, e.g. cross sections, modeling, economics data, e.g. ore price, plant operations, e.g. actual versus planed cycle energy productions, and future fuel design changes. Indeed, it would be nice to know the overall core uncertainties as a function of different sets of near-optimum decision variables. Finally, given the increasing attention to closed fuel cycles to minimize long-lived radioactive waste streams and the associated issues of proliferation resistance, our multi-objective needs to be expanded to include metrics for these two attributes in addition to the currently treated attributes related to economics, and operating and safety margins. As one can see, those involved with developing nuclear fuel management optimization capability have much work remaining to be completed.

3. REACTOR OPERATIONAL AID TOOLS

Reactor operational aid tools assist the reactor operator and engineer in understanding the observed behavior of

the core and what actions are required to obtain the desired behavior of the reactor core. Today these tools largely reside on computers versus the paper plots and tables of the past. Obviously written procedures can be considered the most basic and perhaps most important of reactor operational aid tools. Acknowledging this, we shall move on to consider other reactor operational aid tools. With regard to understanding the observed behavior of the core, it is important to recognize deviations from expected behavior as predicted by core physics simulation software. Zero power startup physics tests, special tests during the cycle, and plant follow data via incore instrumentation and core critical conditions all provide valuable information to improve future prediction capability either via improvements to input data and models, or bias adjustments. To assist the operator in deciding on what actions should be initiated to obtain a desired response of the reactor core, pre-calculated core attributes (e.g. reload core nuclear design report), and off-line and on-line core simulations of future evolutions can be utilized. In this section, we shall mainly concentrate on the on-line core simulation capabilities that are now or shortly are likely to be available, but first we return to describing the tools that are used to interpret and understand observed core behaviors.

Core follow has always been utilized to understand observed core behaviors. The challenge is to utilize the data obtained from core observables, e.g. incore detector currents, and convert them to the attributes of interest, e.g. power peaking factors. To do this, core simulator predictions are utilized to complete the data processing [36]. Needless to say, a valid concern is that since we are using core simulator predictions in processing the data, the results of which are used to verify the fidelity of the core simulator, we may be masking inadequacies in the fidelity of the core simulator. More direct observables, such as gamma scans, can be used to overcome this but at great cost. The change that has taken place from the past is that these conversion factors are more-and-more being calculated for the current core condition versus pre-calculated based upon some assumed core condition. On-line core follow via a core simulator allows this more accurate approach to be taken [37,38].

Regarding startup physics testing that many times is mandated by the licensing authority, the main goal in recent times is to minimize the time to complete these tests since they are normally on the critical path to reactor escalation to power. Needless to say, the most direct manner of reducing the startup physics test time is to reduce the number of startup physics tests required. This has been met with some success with licensing bodies based upon the reasoning that the excellent agreement between measured and predicted core attributes as displayed from past startup physics tests justifies not continuing to perform these tests. Complementing this approach to reduce the time for startup physics tests are alternate test approaches. For PWRs, the Dynamic Rod Worth Measurement [39] approach to measuring rod worths has for certain utilities reduced the associated measurement time versus the Rod Swap and Born End Point measurement approaches. To support Dynamic Rod Worth Measurement processing of data, in addition to the common dependence on the inverse point kinetic equations, 3-D steady-state and transient simulations of the core and transport calculations of the excore detectors' responses are required to account for prompt and delayed neutrons' spatial redistribution during rod insertion. Here again we see heavy reliance on simulation models in reducing the experimental data so that it provides useful information to validate these same simulation models. Needless to say, the concern is simulation modeling errors that enter processing of the experimental data will result in masking the errors in predicted versus measured rod reactivity worths, which was a focus of the regulatory approval process.

Turning now to the topic of predicting what actions are required to obtain the future desired behavior of the reactor core, this is an area that has always been important to BWRs and subsequently has gained in importance for PWRs. The earlier importance to BWRs originates because all possible core states over a cycle cannot be easily bounded by pre-calculated assumed conditions. This follows given the various CRP/CF pairings that are possible and the poorer prediction accuracy of core simulators for BWRs. By contrast for PWRs, with their practice of operating at higher powers with control rods nearly all the way out at their bite positions and the higher fidelity of their associated core simulators, pre-calculated bounding scenarios with reference to operational freedom allowed by Technical Specifications and operating procedures can be defined and evaluated apriori. However, this is achieved by restrictions on the operating freedom, e.g. Constant Axial Offset Control limits. So today for both BWRs and PWRs, more reliance on predictive tools to operate the reactor is evolving.

To accurately predict the future, e.g. power maneuver, one must be able to predict the past and current with fidelity. How past and current predictions and measurements are used to predict the future with greater fidelity has been an area of sustained development, with some very recent encouraging indications of substantial improvement on the horizon. Future predictions can be done on-line, i.e. core simulator integral to plant process computer or other computer available to operators and reactor engineers, or off-line, i.e. via design basis core simulators available to reload core designers. Do note that it is now common that the on-line and off-line core simulators are one and the same, with the difference being accessibility, ease of usage and integration into measured plant data processing. It is fair to say that all BWRs and increasingly more so for PWRs, that on-line core

simulators are being utilized to do core follow and future predictions. Various techniques have been employed to improve the fidelity of future predictions by adjusting the core simulator's predictions to improve agreement with past and current measurements. Most times this is done by applying correction factors, e.g. biases, to the predicted results, using techniques such as surface response. The premise of this approach is that the past trend or current values of the correction factors will continue into the future. Occasionally the core simulator model will be adjusted, such as via utilizing a Fourier axial overtone expansion of Xe^{135} concentration to improve agreement between predicted and measured axial flux distribution or axial offset. For simple nodal models whose nuclear data consists on node-wise k_{inf} and migration areas, the values of these parameters have also been adjusted to improve agreement between predicted and measured incore detector signals. The hope is that by adjusting the core simulator model a more robust prediction capability of future core behavior can be achieved versus employing a correction factor approach.

Mathematically, when we use past and current data to improve the fidelity of future predictions, we can think of this as solving an inverse problem. Thanks to advances in medical imaging and geophysics exploration, improvements have been made in solving the inverse problem via the application of mathematical inverse theory [40-44]. We will now choose to concentrate our discussion on the application of inverse theory to the core simulation problem by briefly reviewing recent work by Abdel-Khalik and Turinsky [45-47] on using inverse theory to build an adaptive core simulator. We start with the premise that the lack of fidelity originates due to the input data, including correlations describing physical phenomena incorporated within computer software, and not due to models. Their related work on evaluating the uncertainties of key core attributes, e.g. power distribution and core reactivity, due to uncertainties in the evaluated nuclear data file give some credence to this assumption since the predicted uncertainties of key core observables are found to be of the same order of magnitude as the observed differences between measured and predicted values [48]. What we wish to do is adapt the input data to improve agreement between measured and predicted core attributes, e.g. incore detector readings and core reactivity. We wish to do this in such a manner that the adjustments to the input data are restricted to be within the known or expert judgment of the input data uncertainties. We also need to recognize the uncertainties of the measured observables values due to noise and drift. If all these factors are correctly addressed, the adapted core simulator will likely be robust, i.e. able to predict with fidelity future core conditions that differ from past and current conditions. Mathematical regularization methods used in inverse theory capture the manners of adaptation noted above by minimizing the differences between experimentally measured and predicted core observables, e.g. incore detector readings, by adjustment of the parameters characterizing the input data constrained to be within their know uncertainties. Expressed mathematically, the predicted core observables $\vec{d}_o^{\,P}$ are given in terms of parameters $\vec{p}_o$ by

$$\vec{d}_o^{\,P} = \vec{O}(\vec{p}_o) \tag{4}$$

The adapted core observables are then determined solving the following minimization problem

$$\min_{\vec{p}} \left\| \left(\overline{\overline{C}}^{M} \right)^{-0.5} (\vec{d}^{\,M} - \vec{O}(\vec{p})) \right\|^2 \text{ subject to } \left\| \left(\overline{\overline{C}}^{P} \right)^{-0.5} (\vec{p} - \vec{p}_o) \right\| \le \varepsilon \tag{5}$$

where $\vec{d}^{\,M}$ denotes the measured values and the quantity ε denotes the range of adjustment allowed. This constrained minimization problem can be recast into an unconstrained minimization problem as

$$\min_{\vec{p}} \left[\left\| \left(\overline{\overline{C}}^{M} \right)^{-0.5} (\vec{d}^{\,M} - \vec{O}(\vec{p})) \right\|^2 + \alpha^2 \left\| \left(\overline{\overline{C}}^{P} \right)^{-0.5} (\vec{p} - \vec{p}_o) \right\|^2 \right] \tag{6}$$

where the value of α is selected to assure the ε constraint noted in Eq. (5) is satisfied. In practice, the value of α is selected to be close to the knee of the characteristic L curve [49] obtained when plotting the first term of Eq. (6), the misfit term, versus the term multiplied by α^2 in Eq. (6), the regularization term. The two covariance matrices appearing in Eq. (6), $\overline{\overline{C}}^{M}$ and $\overline{\overline{C}}^{P}$, denote the covariance matrices of the measured core observables and input core parameters, respectively.

Now let us consider what is involved to obtain the predicted core parameters and covariance matrix of the input core parameters. The predicted core parameters can be such quantities as incore detector signals and core reactivity, e.g. critical state. Current practice starts with an evaluated nuclear data file, e.g. ENDF/B. A processor code, e.g. NJOY and AMPX, reads this data file and generates the many-group cross section library. This library serves as the input to lattice physics codes, e.g. CASMO and HELIOS, which determines the few-group, spatially homogenized nodal cross-section set. This set is then characterized in some fashion to allow interpolation to local core conditions, e.g. burnup, fuel temperature and coolant density, the coefficients involved in this characterization being the input core parameters mentioned above. Finally, the core parameters are used by the core simulator to predict the core observables, in addition to the core attributes not measurable but of interest, e.g. power distribution, control rod worths and reactivity coefficients. The above computational steps are complex, computationally intensive, and involve nonlinear operators. A similar set of linked computational steps, minus the core simulator step, is required to evaluate the core parameters' covariance matrix, starting with the covari-

ance information provided in the evaluated nuclear data file.

Our interest is in adjusting the input core parameters via the solution of the constrained, minimization problem. To do this, the problem is linearized, that is, we wish to determine the sensitivity of the core observables to the core parameters, mathematical represented in terms of the Jacobian matrix given by

$$\overline{\overline{\Delta O}} = \left.\frac{\delta \vec{O}(\vec{p})}{\delta \vec{p}}\right|_{\vec{p}_o} \tag{7}$$

allowing us to predict the core observables to first-order accuracy using

$$\vec{d} = \vec{d}_o + \overline{\overline{\Delta O}}\Delta\vec{p} \tag{8}$$

where $\Delta\vec{p} \equiv \vec{p} - \vec{p}_o$ Substituting Eq. (8) into Eq. (6) permits us to solve for the adapted core parameters by taking the derivative of the resulting equation with respect to core parameters and setting the result equal to zero, which produces the modified normal equation

$$\vec{p} = \vec{p}_o + \left[\overline{\overline{\Delta O}}^T\left(\overline{\overline{C}}^M\right)^{-1}\overline{\overline{\Delta O}} + \alpha^2\left(\overline{\overline{C}}^P\right)^{-1}\right]^{-1}\left[\overline{\overline{\Delta O}}^T\left(\overline{\overline{C}}^M\right)^{-1}\Delta\vec{d}\right] \tag{9}$$

where $\Delta\vec{d} = \vec{d}^M - \vec{d}^P$ denotes the difference between measured and calculated core observables. Since Eq. (8) is only first-order accurate, one can reevaluate the Jacobian matrix about $\vec{p}$, the adapted core parameter values, and repeat the process utilizing a slightly modified version of Eq. (9). This relinearization process can iteratively continue until convergence in the core parameter values is reached.

To evaluate the left hand side of Eq. (9) requires us to determine the sensitivities and core parameters' covariance matrix, and to evaluate the action of an inverse matrix operation. Given that there are typically about 10^5 incore detector readings over the course of a reload cycle, and that a design quality core simulator may have 10^6 core parameters input, this implies a total of 10^{11} sensitivity coefficients must be evaluated and stored, both not realistic to consider achieving. Likewise, the core parameters covariance matrix would have 10^{12} entries, also not realistic to evaluate and store. To overcome this problem an Efficient Subspace Method (ESM) [47] has been developed to reduce the sizes of the Jacobian and covariance matrices.

Figure 6 through Figure 10 present adaptation results for a BWR reload core. In this case, since this research topic is still early in development, what is being adapted is one core simulator to another core simulator, these two core simulators differing substantially in their prediction of core attribute values. The core observables being fitted to are core reactivity and spatial nodal powers as a function of cycle exposure. Note that similar results are obtained

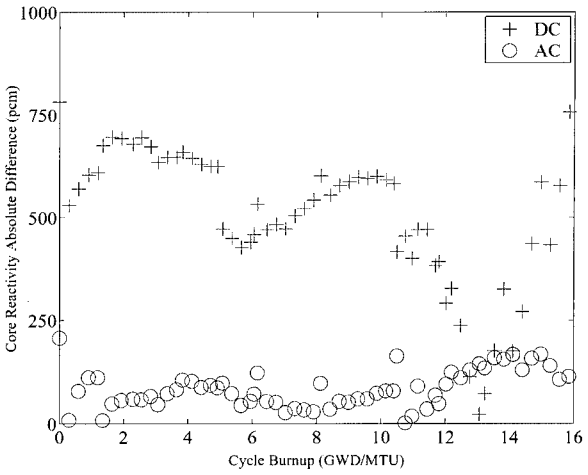

Fig. 6. Core Reactivity Differences Pre (DC) and Post (AC) Adaptation

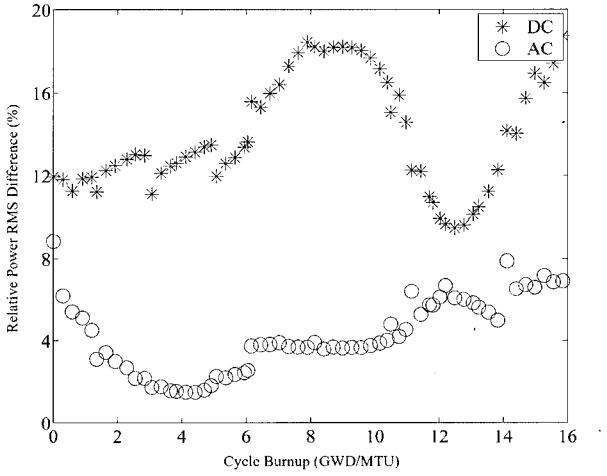

Fig. 7. Axial Core Power RMS Differences Pre (DC) and Post (AC) Adaptation

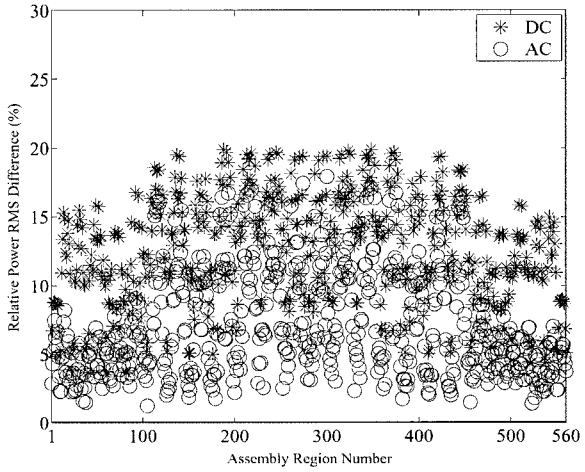

Fig. 8. Assemblies Powers RMS Differences Pre (DC) and Post (AC) Adaptation Towards Beginning of Cycle

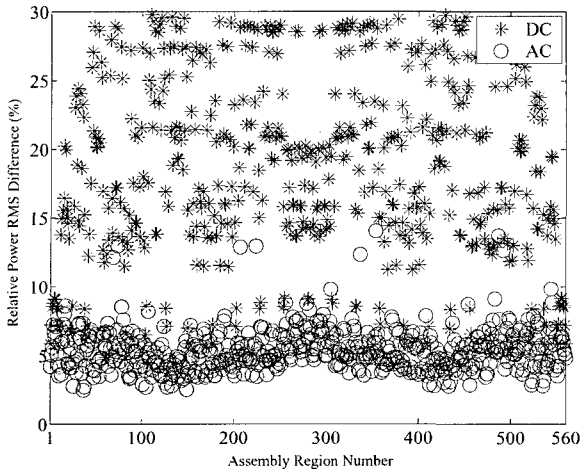

Fig. 9. Assemblies Powers RMS Differences Pre (DC) and Post (AC) Adaptation Towards Middle of Cycle

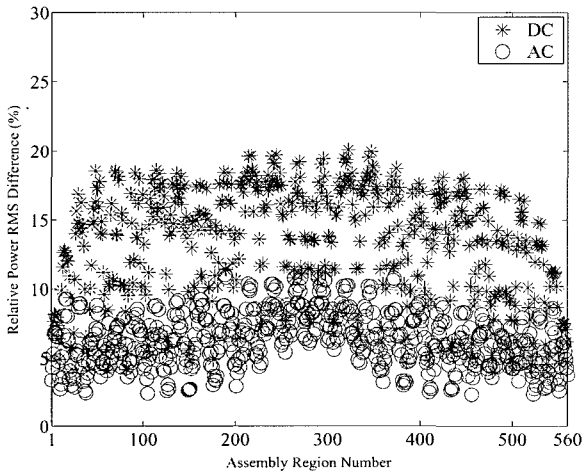

Fig. 10. Assemblies Powers RMS Differences Pre (DC) and Post (AC) Adaptation Towards End of Cycle

when incore detector readings are used in place of spatial nodal powers, even though there are many less incore locations than spatial nodes. The core parameters being adapted are the fitting coefficients to the few-group, homogenized cross sections mentioned above. The notation DC and AC refer to pre and post adaptation predictions, respectively. Figure 6 shows the core reactivity prediction difference. Likewise, Figure 7 through Figure 10 show similar results now for various characterizations of the core power distribution. Given that modeling differences between the two computer codes are not addressed in the adaptation, and that the core parameters are constrained to be adapted within their known uncertainties, the improvement in agreement post adaptation is very encouraging.

Needless to say, adapting one core simulator to another core simulator may have some practical applicability if multiple core simulators are utilized, e.g. design core, plant on-line core monitoring, and incore nuclear fuel management core simulators, but this is not adapting to experimentally measured core observables. Measured core observables involve detectors that have noise, drift and may fail. Simulations have been completed to show that Gaussian noise can be treated [50,51], but the issues of detector drift and failure have not been addressed in this work to date. Plant operating data also involves uncertainties in reactor state, e.g. power level, control rod positions and assumed equilibrium, steady state conditions, and fuel fabrication uncertainties. In addition, nonlinear feedbacks such as due to thermal-hydraulics exist, implying that their associated core parameters, e.g. void-quality correlation, need to be considered for adaptation. Finally, adaptation implies a change in neutron interaction rates, which in turn implies a change in isotopic depletion, which in turn implies that the beginning of cycle isotopic number densities for the shuffled fuel should be altered consistent with the adaptation. This nonlinear, multi-cycle feedback effect was not treated in the results presented above. What one concludes from this discussion is that early results from research on developing an adaptive core simulator are promising, but that there remains much more work to complete to address the challenges just noted.

4. SUMMARY

Significant advances have been made in addressing the incore nuclear fuel management optimization problem. These advances evolved to a level starting about one decade ago where automated tools utilizing mathematical optimization methods have been of benefit to the reload core design engineer. However, much work remains to be completed to integrate the lattice design/bundle design/core LP (with CRP/CF for BWR) optimization problem, and the true multi-cycle nature of nuclear fuel management.

For on-line operator aides, the utilization of the same core simulators for this application and those used in the reload design process has improved fidelity in interpreting measured core observables and reduced engineering effort. Preliminary results on developing an adaptive core simulator which improves fidelity and is robust are encouraging. However to achieve the state of practical application, which nuclear fuel management optimization now enjoys, will require substantial additional research and development to overcome the challenges noted above.

REFERENCES

[1] P. J. Turinsky and G. T. Parks, Chapter: Advances in Nuclear Fuel Management Optimization for Light Water Reactors, Advances in Nuclear Science and Technology, 21, 137, Kluwer Academic/Plenum Publishers, NYC, 1999.

[2] C. C. Oyarzun, D. J. Kropaczek, S. B. Sutton and W. E.

Russell, "The Global Nuclear Fuel Optimization System for BWR Fuel Cycle Management," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[3] H. Q. Lam, D. Sato, D.C. Little and F. D. Popa, "LP-FUN Applications to Reload Design," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[4] I. A. Ben Hmaida, J. N. Carter, C. R. E. de Oliviera. A. J. H. Goddard and G. T. Parks, "Nuclear In-Core Fuel Management Optimization using the Tabu Search Method," Proceedings of Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications, Madrid, 1658, 1999.
[5] F. Glover, "Tabu Search Part I," ORSA Journal on Computing, 1, 190, 1989.
[6] C. Martin del Campo and J. L. Fracois, "A Two Steps Optimization Process Using Genetic Algorithms and TABU Search Methods for the BWR Axial Fuel Assembly Design," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[7] P. J. M. van Laarhoveen and E. H. L. Aarts, Simulated Annealing: Theory and Applications, D. Reidel Publishing Company, Dordrecht, Holland, 1987.
[8] E. Aarts and J. Korst, Simulated Annealing and Boltzmann Machines – A Stochastic Approach to Combinatorial Optimization and Neural Computing, John Wiley and Sons, New York, 1989.
[9] D. J. Kropaczek, P. J. Turinsky, G. I. Maldonado and G. T. Parks, "The Efficiency and Fidelity of the In-Core Fuel Management Code FROMOSA-P," International Conference on Reactor Physics and Reactor Computations, Tel Aviv, 572, INS/ENS, 1994.
[10] D. J. Kropaczek and P. J. Turinsky, "In-Core Nuclear Fuel Management Optimization for PWRs Utilizing Simulated Annealing," Nuclear Technology, 95, 9, 1991.
[11] B. R. Moore, P. J. Turinsky and A. A. Karve, "FORMOSA - B: A Boiling Water Reactor In-Core Fuel Management Optimization Package," Nuclear Technology, 126, 153, 1999.
[12] A. A. Karve and P. J. Turinsky, "FORMOSA-B: A Boiling Water Reactor Incore Fuel Management Optimization Package II," Nuclear Technology, 131, 48, 2000.
[13] A. A. Karve and P. J. Turinsky, "FORMOSA-B: A Boiling Water Reactor Incore Fuel Management Optimization Package III," Nuclear Technology, 135, 241, 2001.
[14] G. T. Parks, P. J. Turinsky and G. I. Maldonado, "Solving the PWR Reload Core Optimization Problem," Paper CIN-90-0095-I, 1991 IBM Supercomputing Competition, Third Prize- Engineering Category, 281, 1991.
[15] A. Yamamoto and H. Hashimoto, "Applications of Temperature Parallel Simulated Annealing to Loading Pattern Optimizations of Pressurized Water Reactors," Proceedings of Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications, Madrid, 1445, 1999.
[16] I. M. Stancu-Minasian, Stochastic Programming with Multiple Objective Functions, D. Reidel Publishing Company, Hingham, MA, 1984.
[17] G. T. Parks and A. Suppapitnarm, "Multiobjective Optimization of PWR Reload Core Designs using Simulated Annealing," Proceedings of Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications, Madrid, 1435, 1999.
[18] P. Engrand, "A Multi-Objective Approach Based on Simulated Annealing and its Application to Nuclear Fuel Management," Proceedings of 5th International Conference on Nuclear Engineering, Nice, France, 1997.
[19] D. E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley Publishing Company, New York, 1989.
[20] P. M. Keller, "Adaptive Constrained Multiobjective Genetic Algorithm for Incore Fuel Management Optimization," Transactions of the American Nuclear Society, 92, 2005.
[21] A. K. Ziver, J. N. Carter, C. C. Pain, C. R. E. de Oliveira, A. J. H. Goddard and R. S. Overton, "An Automated Tool Based on Genetic Algorithms to Optimize the Design of Individual Fuel Assemblies," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[22] E. W. Klee, T. Gou and G. I. Maldonado, "A Coupling of Within-Bundle Loading Optimization to Lattice Physics Calculations," Proceedings of Advances In Nuclear Fuel Management II Topical Meeting, 17-47, Myrtle Beach, SC, 1997.
[23] Y. Hirano, K. Hida, K. Sakurada and M. Yamamoto, "Optimization of Fuel Rod Enrichment Distribution for BWR Fuel Assembly," PHYSOR'96 Topical Meeting, Mito, Japan (1996).
[24] C. A. Daubert, D. J. Kropaczek and W. E. Russell, "Fuel Rod Optimization for Coupled BWR Fuel Bundle and Core Design," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[25] M. A. Jessee, Coupled Bundle-Core Design Using Fuel Rod Optimization for Boiling Water Reactors, Master of Nuclear Engineering Project Report, Department of Nuclear Engineering, North Carolina State University, 2005.
[26] G. I. Maldonado and P. J. Turinsky, "Application of Non-Linear Nodal Diffusion Generalized Perturbation Theory to Nuclear Fuel Reload Optimization," Nuclear Technology, 110, 198, 1995.
[27] G. I. Maldonado, P. J. Turinsky and D. J. Kropaczek, "Constrained Minimization of Fresh Reload Fuel Enrichment Within the FORMOSA Codes Nodal GPT Optimization Framework," Nuclear Science and Engineering, 121, 312, 1995.
[28] X. Mouney, "ARPEGE, The EDF Tool for Core Reload Design," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[29] F. C. M. Verhagen and P. H. Wakker, "ROSA, A Flexible Loading Pattern Optimization Tool for PWRs," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[30] P. M. Keller, "FORMOSA-P Advanced GA Methodology," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[31] J. A. Van de Velde, J. K. Axmann and H.-W. Bolloni, "Recent Issues in Automated Fuel Management Optimization with Adaptive Evolutionary Algorithms," Proceedings of Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications, Madrid,

1479, 1999.
[32] Y. P. Mahlers, "Core Reload Optimization for Equilibrium Cycles Using Simulated Annealing and Successive Linear Programming," Annals of Nuclear Energy, 22(11), 1327, 2002.
[33] J. E. Hoogenboom, R. van Geemert, A. J. Quist, T. Terlaky and C. Roos, "New Ways for Optimizing Equilibrium Reactor Reload Patterns," Proceedings of Mathematics and Computation, Reactor Physics and Environmental Analysis in Nuclear Applications, Madrid, 1469, 1999
[34] S. Jagawa, "Improved Control Rod Adjustment Model in the Loading Pattern Optimization Code FINELOAD-3," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[35] P. M. Keller and P. J. Turinsky, "FORMOSA-P 3-D/2-D Geometry Collapse Methodology," Nuclear Science and Engineering, 139, 235, 2001.
[36] H. Zhang, "GNF BWR Core Simulator Improved Instrument Adaptive and Rejection Method for On-line Core Monitoring," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[37] A. S. DiGiovine and A. Noel, "GARDEL-PWR: Studsvik's Online Monitoring and Reactivity Management System," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[38] L. J. Covington and A. Noel, "GARDEL-BWR: Advanced BWR Online Core Monitoring," Proceedings of Advances In Nuclear Fuel Management III Topical Meeting, Hilton Head, SC, 2003.
[39] Y. A. Chao, D. M. Chapman, D. J. Hill and L. R. Grobmyer, "Dynamic Rod Worth Measurement," Nuclear Technology, 132, 403, 2000.
[40] H. W. Engl, M. Hanke and A. Neubauer, Regularization of Inverse Problems, Kluwer Academic Publishers, 1996.
[41] A. N. Tikhonoiv, "Regularization of incorrectly posed problems," Soviet Mathematics Doklady, 4, 1624, 1963.
[42] A. Tarantola, Inverse Problem Theory, Methods for Data Fitting and Model Parameter Estimation, Elsevier Science Publishers B. V., 1998.
[43] A. N. Tikhonov, Numerical Methods for the Solution of Ill-Posed Problems, Kluwer Academic Publishers, 1995.
[44] M. Bertero and P. Boccacci, Introduction to Inverse Problems in Imaging, Institute of Physics Publishing, Bristol, 1998.
[45] H. S. Abdel-Khalik and P. J. Turinsky, "Adaptive Core Simulation, Part I: Theory," Nuclear Technology, 150, 2005.
[46] H. S. Abdel-Khalik and P. J. Turinsky, "Adaptive Core Simulation, Part II: Numerical Experiments," Nuclear Technology, **150**, 2005.
[47] H. S. Abdel-Khalik, Adaptive Core Simulation, Ph.D. Dissertation, Department of Nuclear Engineering, North Carolina State University, 2004.
[48] H. S. Abdel-Khalik and P. J. Turinsky, "Evaluation of Core Attributes Uncertainties due to Input Data Uncertainties," Transactions of the American Nuclear Society, **92**, 2005.
[49] H. W. Engl and W. Grever, "Using the L-Curve for determining optimal regularization parameters," Numerische Mathematik, 69, 25, 1994.
[50] H. S. Abdel-Khalik and P. J. Turinsky, "Inverse Method Applied to Adaptive Core Simulation," Mathematics and Computations 2003: Nuclear Mathematical and Computational Sciences: A Century in Review, A Century Anew, Gatlinburg, 064, 2003.
[51] H. S. Abdel-Khalik and P. J. Turinsky, "Adaptive Core Simulation: Efficient Sensitivity Analysis," Proceedings of Advances in Nuclear Fuel Management-III Topical Meeting, ANS, Hilton Head, 19-08, 2003.