International Journal of CAD/CAM Vol. 5, No. 1, pp. 51 ~ 58 (2005)

INTERNATIONAL JOURNAL OF CAD/CAM
www.ijcc.org

Problems For Line Labelling: A Test Set of Drawings of Objects with Higher-Valency Vertices

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Abstract − **Interpreting a natural line drawing as a solid object requires simplifying assumptions in order to make the problem more tractable. Unfortunately, some of the assumptions made in the past have overly simplified the problem. Restricting the valency of vertices, and in particular allowing only trihedral vertices, distorts the problem, since algorithms which are satisfactory for the simplified problem are not satisfactory in the general case. This paper presents a test set of drawings of objects with higher-valency vertices. The intention in creating this test set is that it may be used to determine how effective various algorithms are in dealing with general (i.e. unrestricted) valency vertices.**

Keywords: **Line labelling, K-Vertices, Rotationally-symmetric vertices, Creative design, Software engineering issues**

1. Introduction

The problem of interpreting a natural line drawing as a solid object is, in principle, impossible: for any natural line drawing, there are an infinite number of solid objects which could, if viewed from the correct viewpoint, result in that drawing. In order to make the problem more tractable, some simplifying assumptions are required.

Unfortunately, as [2, 15] point out, some of the assumptions made in the past have overly simplified the problem. Algorithms which work very well if these assumptions are met are far less successful (if they work at all) in the general case. One well-known example of these is Clowes-Huffman line labelling [1, 3], a procedure which is very effective indeed for drawings of objects containing only trihedral vertices, but which is less effective (when it works at all) for drawings of objects containing higher-valency vertices.

A trihedral vertex is a vertex at which exactly three edges meet. When viewed in a line drawing, each edge could result in a line which is visibly convex, visibly concave, or occluding (and hence convex in the solid object); in the case of occluding lines, there are two conceivable labels, since the region on one side of the line is occluding and that on the other is occluded.

By exhaustive analysis of the possibilities, Clowes [1] and Huffman [3] showed that in any valid drawing of a trihedral object, there are only 6 possible L-junctions (not 16), only 3 possible W-junctions (not 64), and only 5 possible Y-junctions (not 64). These, together with the 4 possibilities at occluding T-junctions, make up the Clowes-Huffman catalogue.

By applying the 1-node constraint that each junction in the drawing must be labelled with a label in the catalogue, and the 2-node constraint that each line in the drawing must have the same label at either end, the possible ways of labelling the drawing can be enumerated. There are many algorithms for implementing Clowes-Huffman labelling - the simplest to implement is Kanatani's algorithm [5], shown below.

[Initialisation]
For each junction, *candidate label set* = all valid labels for that junction type
For each boundary line, *candidate label set* = {occluding such that outside is occluded}
For each non-boundary line, *candidate label set* = {occluding to left, occluding to right, convex, concave}
Set of junctions to be processed S_J = {all junctions}
Set of edges to be processed S_L = {all edges}
[Processing]
Loop
For each junction in S_J
Eliminate from the *candidate label sets* for neighbouring lines any line labels inconsistent with the remaining candidate labels for this junction
If the junction label is unique, remove the junction from S_J
For each line in S_L
Eliminate from the candidate label sets for the neighbouring junctions any junction labels inconsistent with the remaining candidate labels for this line
If the line label is unique, remove the line from S_L
Exit the loop if S_J and S_L are both empty (a unique

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labelling has been obtained)
Exit the loop if the candidate label set for any junction or line is empty (no valid labelling can be obtained given the starting conditions)
Exit the loop if no candidate labels were eliminated in this iteration
End Loop

Kanatani's algorithm only returns a unique labelling if there is only one valid way of labelling the drawing, but this is commonly the case with trihedral drawings. For this reason, although line-labelling algorithms are $O(e^n)$ in theory, they are usually O(n) in practice, as [10] has confirmed experimentally.

Extension of these algorithms to allow extended trihedral vertices does not materially alter this success [10] (an *extended trihedral vertex* is one at which exactly three face planes meet; there may be 4 or 6 edges).

However, extension to allow tetrahedral vertices does materially alter the result (a tetrahedral vertex is one at which exactly four edges meet). The junction catalogue of views of tetrahedral vertices is no longer sparse [14]. For example, there are 27 possible tetrahedral Y-junctions, so (including both trihedral and tetrahedral labels) 32 of 64 Y-junction labels are possible. As a result, drawings of tetrahedral objects usually have many possible labellings; enumerating them all is very slow (and often impractical), and there is the additional problem of selecting the best of these interpretations [14].

The problem becomes even worse when higher-valency vertices are allowed. The Clowes-Huffman approach [1, 3] analysed 256 configurations to obtain a catalogue of 14 junction labels. The similar approach for tetrahedral vertices [14] analysed 69632 configurations (65536 general tetrahedral vertex configurations, plus 4096 K-vertex configurations-see Section 2 for a description of K-vertices) to obtain a catalogue of 109 junction labels. Extending this approach to produce a full catalogue of (for example) general 8-hedral vertices is clearly impractical.

Huffman [4] describes a test which can determine whether or not a particular junction label is valid for higher-valency vertices, but not a fast algorithm for performing this test. Even if such an algorithm existed, the problems of speed (enumerating all the possible labellings is slow) and choice (heuristics are required to choose between the possible labellings) remain.

Huffman [4] also illustrates how catalogue-based labelling methods can lead to labellings which have no geometric interpretation, a problem which has been considered further in [16].

In summary, Clowes-Huffman labelling, although excellent for labelling drawings of trihedral objects, is not a good choice for more general natural line drawings.

It is not the purpose of this paper to describe alternative algorithms which may supersede Clowes-Huffman labelling. Work on developing such algorithms continues, e.g. [17]. Nor does it examine alternative approaches which attempt to do without labelling when processing natural line drawings [9] or wireframe drawings [6, 7, 13].

Instead, this paper provides a test set of drawings by means of which the performance of such algorithms can be evaluated. It concentrates on two particular types of higher-valency vertices. Section 3 describes some simple extended-K-vertices (vertices which can appear in objects containing only cuboids and axis-aligned wedges). Section 4 describes some rotationally -symmetric vertices. Section 5 shows drawings intended to represent realistic engineering objects which include such vertices. Section 6 presents some conclusions.

Together, the drawings in Sections 3, 4 and 5 perform a number of functions. They show that higher-order vertices are a realistic feature of engineering design, not a rare curiosity which can be ignored in practice. They illustrate the scope of the labelling problem for general polyhedra, drawing attention to some subproblems which do not occur in restricted subsets of the problem such as trihedral polyhedra. Practically, they comprise a test set which can be used for evaluating new approaches to interpreting line drawings, to determine whether or not these approaches can deal effectively with higher-valency vertices.

To facilitate the use of these drawings as a test set, they are available for download in three formats (list of junctions and lines, Postscript, and GIF) at http://ralph.cs.cf.ac.uk/Data/sketch.html (the Fourth Test Set).

2. K-Vertices

A K-vertex is a tetrahedral vertex formed entirely from cuboids and axis-aligned wedges; when fully-visible, the shape is reminiscent of a capital K. There are four types of K-vertex, illustrated in Fig. 1. The K-vertices are shaded.

The complete catalogue of K-vertex labels was obtained by the following procedure [14]. Split the

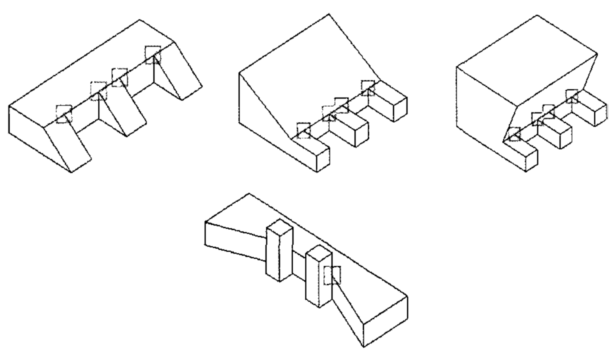

Fig. 1. The Four Types of K-Vertex.

Gaussian sphere into regions by creating four planes through the origin. Three of the planes should be perpendicular; the fourth should be a linear combination of two others. For each combination of solid and empty regions which meet the criteria for a valid single manifold polyhedral object, (I) view the central junction from one viewpoint located in each empty region, and for each view in which the central junction is not occluded by solid (a) count the number of visible lines (if the ray from the intersection of the line with the Gaussian sphere to the viewpoint passes through a solid region, the line is not visible); (b) determine the orientation of the visible lines, and order them clockwise; (c) for each visible line, determine whether it is concave, convex, clockwise-occluding or anticlockwise -occluding; (d) derive the junction label from the number of visible lines and the line types; (II) output the set of junction labels for the different viewpoints of this vertex as a single group.

A combination of solid and empty regions is valid providing it meets the following criteria [14]: (a) at least one region must be solid; (b) at least one region must be empty; (c) the solid regions must be contiguous, in order for the solid to be manifold; (d) the empty regions must also be contiguous; (e) points and lines may not be degenerate (e.g. the four regions viewed cyclically about a line may not be solid-empty-solid-empty, as this would produce two degenerate lines); (f) none of the planes may divide the sphere into an entirely solid part and an entirely empty part (there would be no junction to see).

Note that this is, essentially, an extension of the procedure used for the Clowes-Huffman catalogue.

3. Extending K-Vertices

As seen in the previous section, K-vertices are produced when one axis-aligned wedge meets one or more cuboids. They are particularly common in engineering objects since most engineering objects can be considered as unions of cuboids and axis-aligned wedges [11].

The concept of K-vertices can be extended to a more general class by considering all vertices which can be formed by unions of cuboids and axis-aligned wedges.

However, exhaustively cataloguing these extended-K-vertices is impractical. While there are only 4 basic types of K-vertex, there are many more basic types of extended-K-vertex (the number is probably in the thousands). Fortunately, it may also be noted that since, as shown in Section 1, catalogue-based methods are no longer considered appropriate, it is not necessary to produce a full catalogue of such vertices. Instead, this section shows only the simplest combinations of cuboids and wedges, starting with unions of two such primitive solids and progressing only as far as unions of three primitive solids.

Note that at least two of the primitive solids must be wedges - if there are no wedges, the vertex is trihedral or extended trihedral. Additionally, if there is only one cube and one wedge, the vertex is a K-vertex. Note also that the objects in this section are not mirror-symmetric-the reflected version of each has been omitted for brevity.

3.1. Two Wedges

Fig. 2 shows an object constructed from the union of two wedges. Although only four lines meet at the emphasised junction in the figure (shaded), this junction clearly corresponds to a 5-hedral vertex since the two "underneath" faces cannot be coplanar.

3.2. Three Wedges

Fig. 3 shows an object constructed from the union of three wedges. The central junction is visibly 5-hedral.

3.3. Two Wedges, One Cuboid

In several of the figures in this section there is a two-line cross configuration which does not appear in tetrahedral or extended-trihedral junctions. This configuration (where two lines cross at a point where one ore more other lines terminate) can present data representation problems if a data representation based on tetrahedral or extended trihedral junctions is extended to allow for more general vertices.

3.3.1. Both Wedges Adjacent to Cuboid

Fig. 4 shows objects constructed from the union of two wedges and a cuboid, with both wedges adjacent to the cuboid. Note that some combinations produce

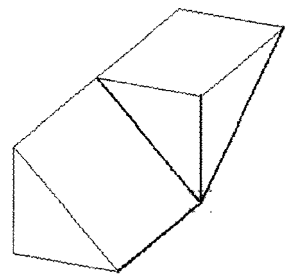

Fig. 2. Object from Two Wedges.

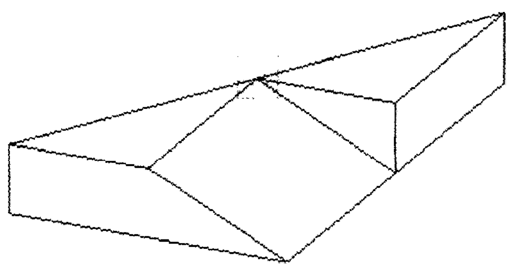

Fig. 3. Object from Three Wedges.

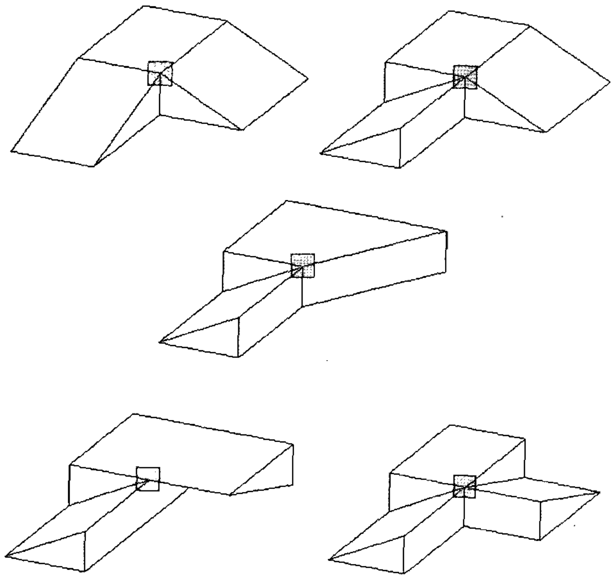

Fig. 4. Two Wedges Adjacent to Cuboid.

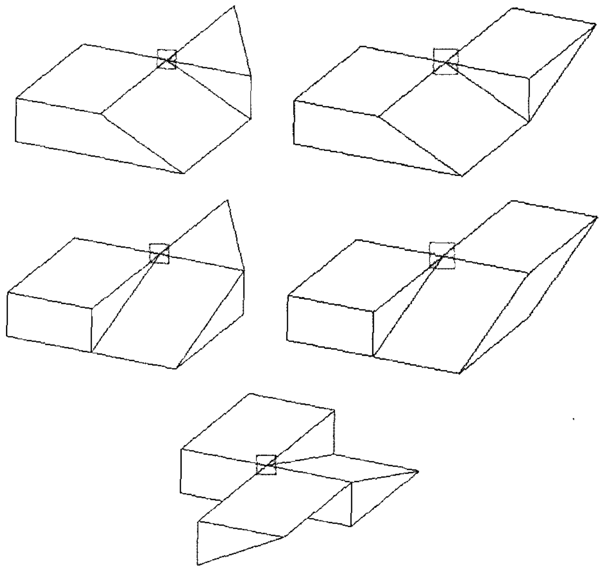

Fig. 6. Two Wedges Adjacent to One Another.

general tetrahedral vertices - since these have already been catalogued elsewhere [14], they are omitted.

3.3.2. Wedges Adjacent to One Another

Fig. 5 and Fig. 6 show objects constructed from the union of two wedges and a cuboid, with the wedges adjacent to one another.

Fig. 5 shows unique vertex types. In the drawings in Fig. 6, the non-aligned plane of the wedge does not pass through the central point; the resulting vertex types can also occur as unions of one wedge and two cuboids, and so are equivalent to the central vertex in Section 3.4.

3.4. One Wedge, Two Cuboids

Fig. 7 shows the only higher-valency vertex type constructed from one wedge and two cuboids-although only 5 lines are visible, it is clearly 6-hedral.

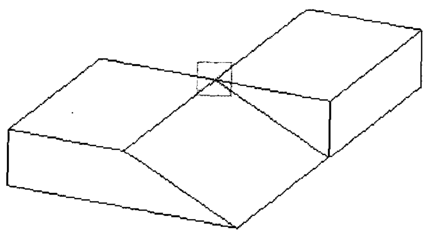

Fig. 7. One Wedge Adjacent to Two Cuboids.

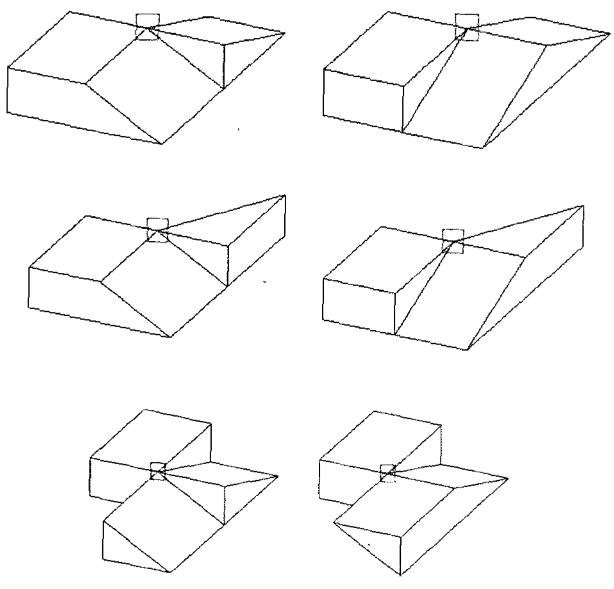

Fig. 5. Two Wedges Adjacent to One Another.

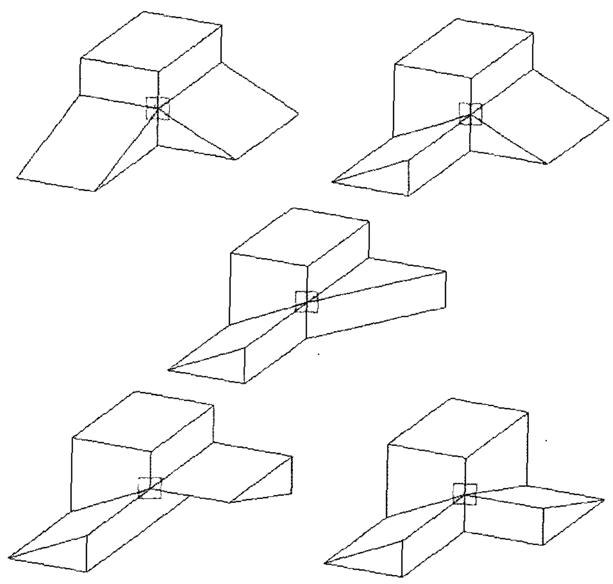

Fig. 8. Two Wedges Adjacent to Tall Cuboid.

3.5. Two Wedges, One Tall Cuboid

This subsection is not an exhaustive catalogue. Instead, it illustrates only those vertices which are obtained if the cuboids in Section 3.3 are extended upwards. If the exhaustive space-dividing search approach were followed, these would be regarded as unions of two wedges and two cuboids.

3.5.1. Both Wedges Adjacent to Cuboid

It is interesting that when the cuboids in Fig. 4 are extended upwards, as in Fig. 8, all five vertices become similar: 6-hedral, with alternating convex and concave edges.

3.5.2. Wedges Adjacent to One Another

Fig. 9 shows the results when the cuboids of Fig. 5 are extended upwards.

3.6. Two Wedges, One Long Cuboid

This subsection is not an exhaustive catalogue. Instead, Fig. 10 illustrates only those vertices which are obtained if the cuboids in Section 3.3 are extended to touch both wedges. Note that some of the resulting solids are non-manifold; these are excluded. If the exhaustive space-dividing search approach were followed, these would be regarded as unions of two wedges and two cuboids.

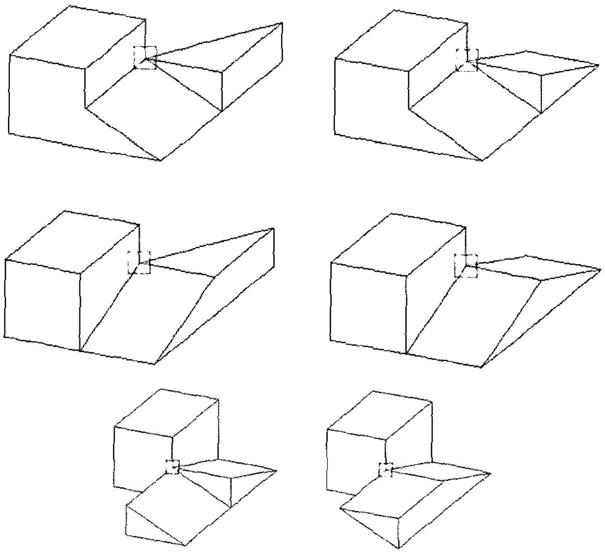

Fig. 9. Two Wedges Adjacent to One Another.

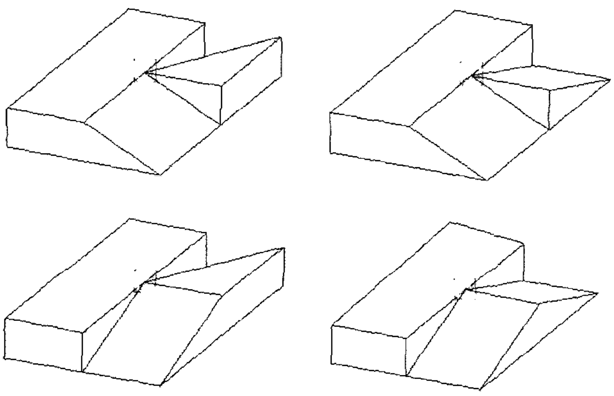

Fig. 10. Two Wedges Adjacent to Long Cuboid.

4. Rotationally-Symmetric Vertices

In practice, perhaps the most common objects in hich higher-valency vertices are routinely observed are screws and screwdrivers. For ease of presentation, this section illustrates convex screwdriver heads rather than concave screw heads. Some of the screwdriver heads illustrated here have been observed in real objects; the remainder (the majority) are natural extensions of the same idea. This is not a complete catalogue: only 6-hedral and 8-hedral heads with rotational symmetry are considered.

4.1. 6-Hedral Heads

The centres of rotation in Fig. 11 are, respectively, all-convex, alternating convex and concave, and alternating convex and concave with the convex edges being coplanar. The first object has C_6 (sixfold rotational) symmetry; the other two have C_3 (threefold rotational) symmetry. The rotationally-symmetric vertices are shaded.

The first two drawings in Fig. 12 have four convex and two concave edges; in the second drawing, the convex edges are coplanar. The third drawing has two convex and four concave edges. All three objects have C_2 symmetry.

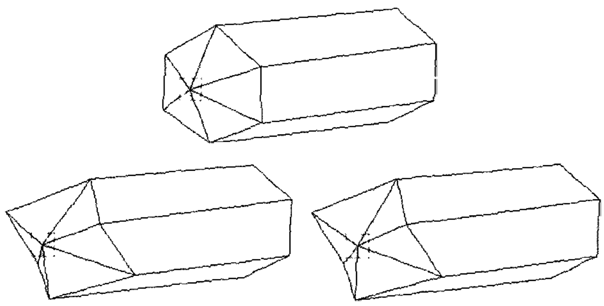

Fig. 11. 6-hedral Vertices.

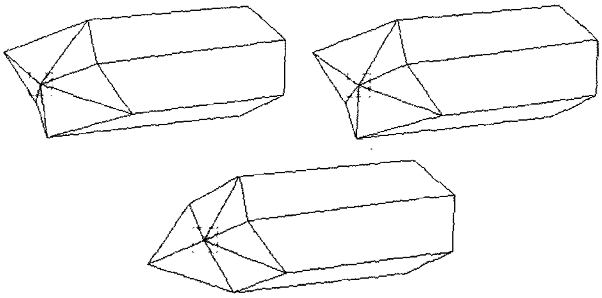

Fig. 12. 6-hedral Vertices.

4.2. 8-Hedral Heads

The centre of rotation in Fig. 13 is all-convex. The object has C_4 symmetry (objects with C_8 symmetry are of course also possible).

The centres of rotation in Fig. 14 are all alternating convex and concave. In the second drawing, one pair of convex edges is collinear; in the third drawing, both pairs of convex edges are collinear. The second object has C_2 symmetry; the other two have C_4 symmetry. The crossing-lines configuration, noted for extended-K-vertices, can be seen in the third drawing.

The preferred interpretation of the central vertex in the first drawing of Fig. 15 has two opposed edges which are concave, the remainder being convex. It has C_2 symmetry. It is possible to construct similar objects in which two opposed convex edges are collinear, or two pairs of opposed convex edges are collinear (forming two coplanar faces), while retaining the C_2 symmetry (these objects are not illustrated). The second drawing shows a central vertex in which pairs of convex edges alternate with pairs of concave edges. It too has C_2 symmetry. The third drawing shows a similar central vertex at which all of the convex edges are coplanar. It too has C_2 symmetry (as do objects, not illustrated, where only one pair of opposed convex edges are collinear). The crossing-lines configuration, noted for extended-K-vertices, can again be seen here.

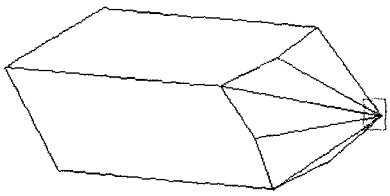

Fig. 13. 8-hedral Vertices.

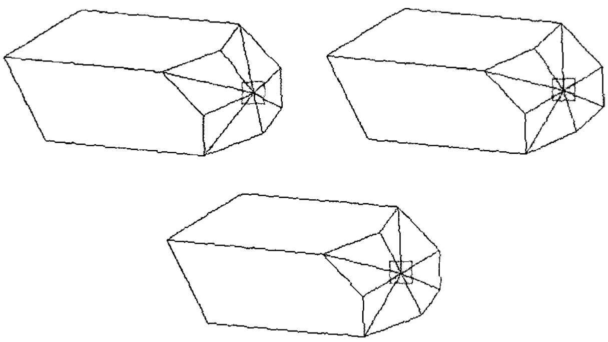

Fig. 14. 8-hedral Vertices.

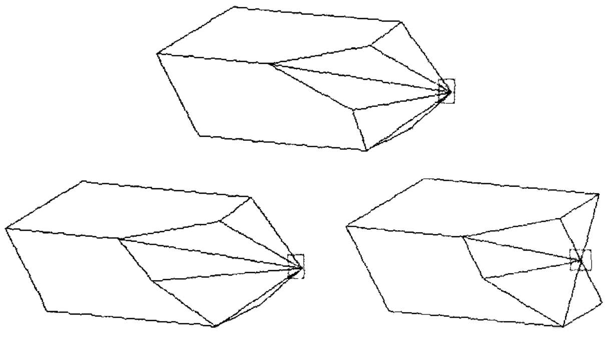

Fig. 15. 8-hedral Vertices.

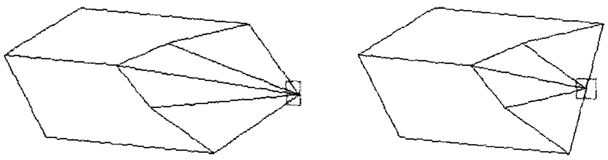

Fig. 16. 8-hedral Vertices.

By comparison Fig. 15, Fig. 16 can reasonably be interpreted as showing objects whose centres of rotation have two opposed convex edges, the remaining edges being concave (although it is far from clear that these should be the preferred interpretations, particularly in the case of the second drawing). In these interpretations, both objects have C_2 symmetry; in the second object, the opposed convex edges are collinear.

5. Realistic Engineering Objects

Practical engineering objects frequently contain symmetry: an axis of rotational and/or a plane of mirror symmetry (the reasons, both aesthetic and practical, for this are described in [8]). In attempting to create a convincing set of objects which look as if they might be concept designs for new engineering objects, I have applied symmetry to a selection of the vertex types presented in the two previous Sections.

The two drawings in Fig. 17 illustrate a specific problem: a junction label normally associated with

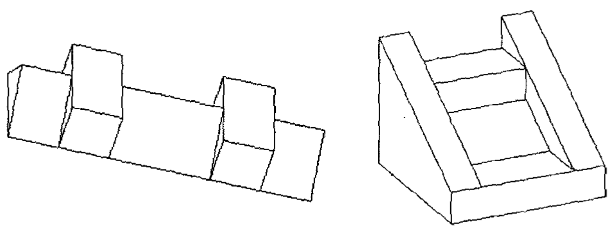

Fig. 17. Problematic T-Junctions.

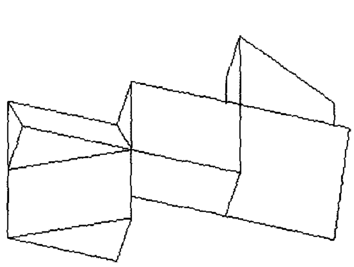

Fig. 18. Problematic Cross-Junctions.

occluding T-junctions here represents an extended-K-junction (the problematic junctions are shaded). ([12] includes a drawing similar to the second of these, but without the mirror symmetry and without the problematic T-junction).

Fig. 18 illustrates a further problem associated with the cross-configuration. At one, there are only the two crossing lines. At the other, the two lines cross at the termination point of two other lines.

Figs 19-24 do not illustrate any particular point other than that engineering concept drawings may reasonably include higher-valency vertices.

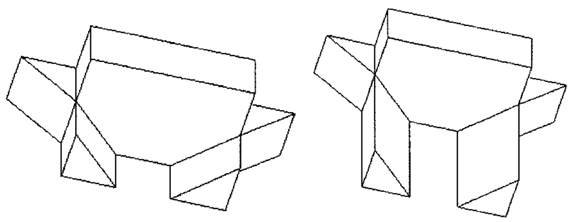

Fig. 19. Example Objects with Higher-Valency Vertices.

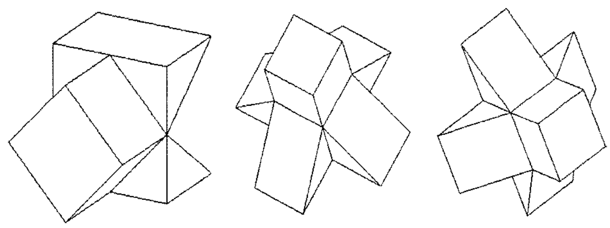

Fig. 20. Example Objects with Higher-Valency Vertices.

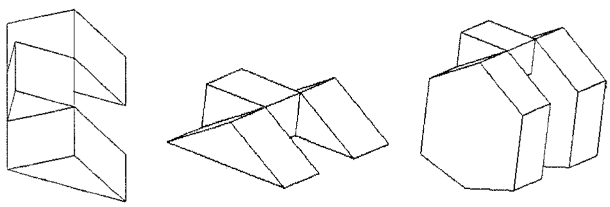

Fig. 21. Example Objects with Higher-Valency Vertices.

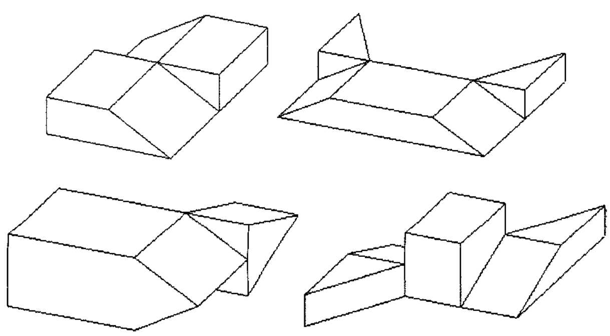

Fig. 22. Example Objects with Higher-Valency Vertices.

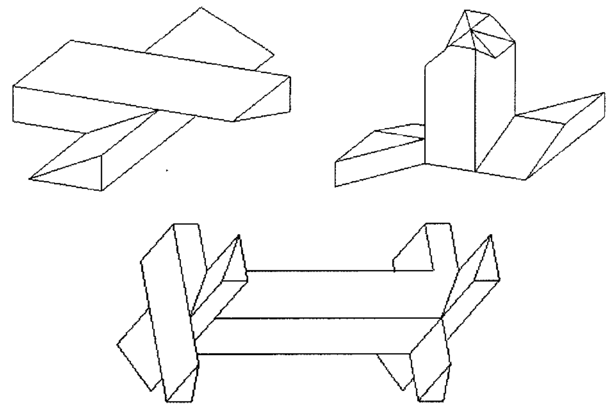

Fig. 23. Example Objects with Higher-Valency Vertices.

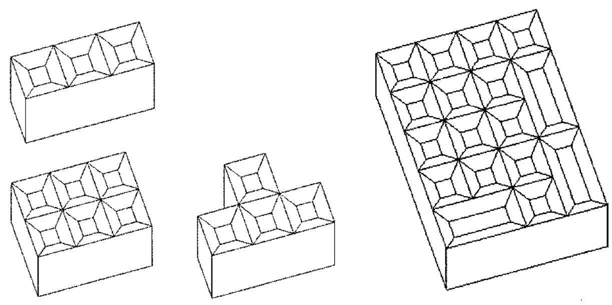

Fig. 24. Example Objects with Higher-Valency Vertices.

6. Conclusions

This paper has presented drawings of polyhedral objects containing higher-valency vertices, showing that although these are not common in engineering objects, they are not so rare that they can be ignored. As and when new methods for producing frontal geometries from line drawings, capable of processing higher-valency vertices, are developed, these drawings can be used as a test set.

We have not attempted to develop any mathematical theory of higher-order vertices. Such theory as already exists [4] has not proven useful in practice as it has not led to a general labelling algorithm. It remains possible that an alternative theoretical approach would be more productive, and this is one possible route towards developing a general algorithm.

Acknowledgements

Funding for this investigation was provided by Japan Society for the Promotion of Science Fellowship number P03717; this support is acknowledged with gratitude.

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