International Journal of CAD/CAM Vol. 5, No. 1, pp. 91 ~ 98 (2005)

INTERNATIONAL JOURNAL OF CAD/CAM
www.ijcc.org

# A Generalized Scheme for Constructing Polyhedral Meshes of Catmull-Clark Subdivision Surfaces Interpolating Networks of Curves

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**Abstract – This paper presents a scheme for interpolating intersecting uniform cubic B-spline curves by Catmull-Clark subdivision surfaces. The curves are represented by polygonal complexes and the neighborhoods of intersection points are modeled by X-Configurations. When these structures are embedded within a control polyhedron, the corresponding curves will automatically be interpolated by the surface limit of subdivision of the polyhedron. The paper supplies a construction which clearly shows that interpolation can still be guaranteed even in the absence of symmetry at the X-configurations. In this sense, this scheme generalizes an already existing technique by the same authors, thereby allowing more freedom to designers.**

***Keywords*: Subdivision surfaces, Interpolation, Polygonal complexes**

## 1. Introduction

The term *lofting* has its historical origins in the days when ship hulls were designed manually. Nowadays, this is used, in the context of geometric modeling and computer-aided design, more with reference to the task of generating a surface interpolating a sequence of non intersecting curves [4, 5, 6]. This is somewhat generalized in [12] for the interpolation process to work even when it starts with an arbitrarily-connected network of polygons representing arbitrarily intersecting smooth curves.

It goes without mentioning, of course, that the degree of arbitrariness of the connection of the input curves is bounded by the compatibility of these curves with an adequately smooth interpolating surface. One can say, for instance, that smoothness adequacy is reached when the tangents to these curves, at the intersection point, are coplanar. At the same time, the underlying process should not be hindered by the fact that a multiplicity of surfaces can satisfy the interpolation constraint.

The scheme described in this paper may be utilized in two possible scenarios: (1) when the control polygons are given without any additional information, and (2) when these polygons are tagged edges on an existing control polyhedron, with many of these having common intersection points.

Our approach [2] to the solution of the above interpolation problem relies on the following elements:

- The tagged polygons are re-represented by equivalent polygonal complexes [10] constructed as an integral part of the control polyhedron representing the surface.
- The regions, where the complexes meet in this network, are called X-Configurations. As have originally been envisaged, these configurations are designed to have specific symmetry and planarity properties necessary to guarantee interpolation at the limit of the subdivision process by a surface with an adequate degree of smoothness.
- Since interpolation can never be achieved at extraordinary points (see [9] and [12]) with the standard subdivision coefficients, the coefficients are slightly modified at and in the vicinity of the extraordinary points. Yet, in spite of modification, analysis of the corresponding subdivision matrix [3] shows that the resulting surface is still smooth, even at the extraordinary points.

The final gap that still exists in the above picture is to show how to re-represent the initial network of polygons within the same control polyhedron. This should be done in such a way that, when subdivided, this polyhedron leads to an adequately smooth surface interpolating the initial curve network. Among other things, the construction suggested in this paper attempts to do without the symmetry and co-planarity constraints proposed in [2].

## 2. Catmull-Clark Subdivision and Polygonal Complexes

The material presented in this section is necessary to make subsequent material better understood. Here, the reader might find that the particular labeling of vertices

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in the following subsection somewhat counter-intuitive. However, these labels are deliberately designed to match other labels for other vertices in subsequent sections of the paper in support for the point we are trying to make.

### 2.1. Uniform Cubic Subdivision of a Polygon

In a single subdivision step (see Fig. 1), the control polygon P = [$A_0$, $E_0$, $F_0$, ...] is subdivided into a polygon Q = [$A_0$, $e_0$, $m_0$, $n_0$, $p_0$, ...] as follows:

- $e_0$, $n_0$ and $q_0$ are the midpoints of the edges $A_0E_0$, $E_0F_0$ and $F_0G_0$ respectively, etc.
- $m_0$ and $p_0$ are the midpoints of the edges joining the midpoints of $E_0n_0$ and $E_0e_0$ and the midpoints of $F_0n_0$, and $F_0q_0$ respectively, etc.

Repeating this process sufficiently often will lead to a cubic B-spline curve. In this process, the extremity $A_0$ of the open polygon is interpolated by the limit curve under the version of this subdivision scheme being adopted here.

### 2.2. Catmull-Clark Subdivision of a Polyhedron

In a single CC subdivision step (see Fig. 2), a control mesh M is subdivided into another control mesh M' (see [7] for more details), as follows:

Each face F of the mesh gives rise to an F-vertex that is the average of the vertices of the face F. Each inner edge E gives rise to an E-vertex that is the average of the vertices of the edge together with the F-vertices of the adjacent faces of E. Each inner vertex v gives rise to a V-vertex specified by the expression ((n-2)*v + (R + S)/n)/n, where

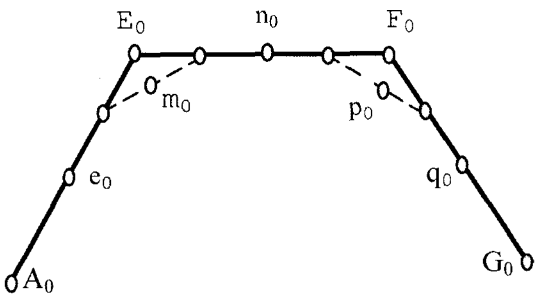


Fig. 1. Uniform Cubic Subdivision of a Polygon.

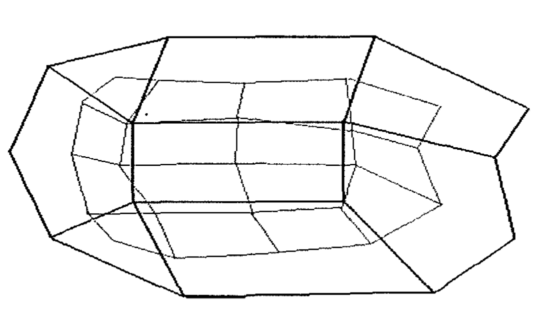

Fig. 2. Catmull-Clark Subdivision of a Polyhedron.

- n is the number of faces adjacent to v
- $R = \sum_{n}^{i=1} v_i$ , where $v_i$ is the other vertex of an edge

incident on v in the corresponding mesh.

- $S = \sum_{n}^{i=1} V_{fi}$ , where $V_{fi}$ is an F-vertex of a face $f_i$ of the

mesh adjacent to the vertex v.

At the end of this process, each F-vertex is connected to the adjacent E-vertices and each E-vertex is connected to the adjacent V-vertices. The resulting faces will form the new subdivided mesh. Repeated application of this subdivision process will lead to a smooth surface.

Accordingly, the boundary vertices and edges do not contribute any new vertices. In other words, the initial boundary vertices will not be interpolated by the limit surface in the version of the subdivision scheme adopted here.

### 2.3. Polygonal Complexes and Their Limit Curves

A *simple* CC polygonal complex (see Fig. 3) is a 3 × n matrix M of points representing three control polygons: top($t_i$), middle($m_i$) and bottom($b_i$), all having the same number n of vertices. Such a complex may also be seen as a sequence of pairs of rectangular faces; where each pair of faces of the sequence has a common edge and each two consecutive pairs have common respective edges.

A *general* CC polygonal complex (see Fig. 4) is encountered when the control polygons ($t_i$), ($m_i$) and ($b_i$) do not all have the same number of vertices. In other words, the corresponding faces are not all rectangular at the outer edges. However, each inner vertex of a CC complex must still be regular in the

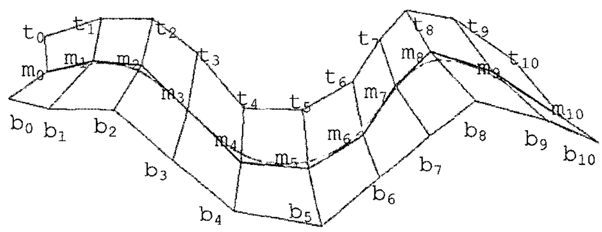


Fig. 3. A Simple Polygonal Complex.

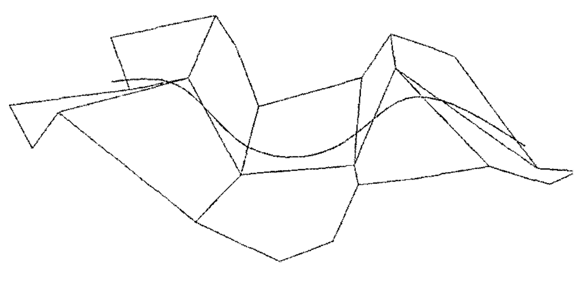

Fig. 4. A General Polygonal Complex.

sense that it connects exactly four edges. Here, a general CC complex reduces to a simple one after a single CC subdivision step.

Under subdivision, a CC complex goes through a sequence of thinner and thinner complexes leading, at the limit, to a smooth curve. Moreover, the limit of a simple CC complex M is a cubic B-spline curve whose control polygon P is given by the following formula (see [10]):

$$(1/6)*[1\ 4\ 1]*M \qquad (1)$$

Thus, when a complex is embodied within a control mesh, its limit curve is automatically interpolated by the limit surface of the mesh. More importantly perhaps, the limit of a CC complex M' is a B-spline curve identical to that of m, when M' is obtained from M by substituting the mid-row m of M by the polygon (see [11]):

$$m' = (1/4)*[-1\ 6\ -1]*M \qquad (2)$$

This way a curve defined by a control polygon ($m_i$) can be turned into a polygonal complex M by adding to it two more rows of points ($t_i$) and ($b_i$). This way, transformation (2) guarantees that any mesh embodying M' will interpolate the original curve of ($m_i$).

**Property 1:** when a CC complex M is subdivided one step into M' and when the limit polygon P of M is also subdivided one step into P', the property stated in equation (1) will be preserved under subdivision. That is, P' will be identical to (1/6)*[1 4 1]*M'.

This property is worth remembering, as it is the source of the intuition that motivated the development of the solution to the interpolation problem presented in this paper.

### 2.4. X-Configurations

The previous section established the correspondence between CC complexes and cubic B-spline curves and the usefulness of that for interpolating of isolated (non-intersecting) curves. In this section, this notion will be further developed to deal with situations where the given curves can intersect.

As an initial motivation, note that when two complexes meet end-to-end, their limit curves will meet but might not remain the same in the immediate neighborhood of their meeting point (see Fig. 5). This is to be expected, since the middle vertex there is no longer a border vertex. However, this vertex will be

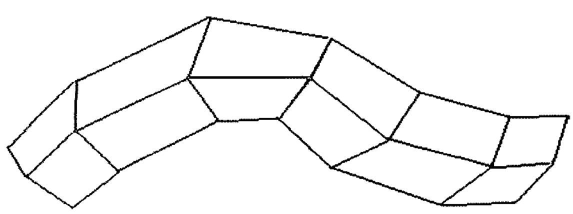

**Fig. 5.** Intersection of Two Complexes.

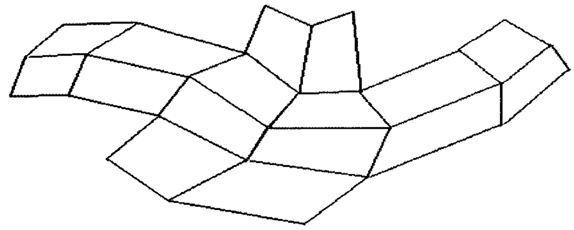

**Fig .6.** Intersection of Four Complexes.

regular. The same observation holds when the number of complexes is four (see Fig. 6).

In both cases, *it would be interesting to have some degree of control over the way these complexes behave at their meeting point* so that their corresponding curves meet at this same point exactly without the need for any further manipulation.

In the same context, when the number of complexes is not two or four, the centre vertex will not be regular. This also says that each connecting complex will not be regular around the centre vertex in the sense discussed above, because all polygons of various complexes will have this vertex as their meeting point. This provides the second motivation for determining a structure with a predictable behavior at the limit of the subdivision. Hence, the following definitions:

**Definition 1:** an X-Slice is a closed polygon with one of its vertices marked as its starting point.

**Definition 2:** an X-Configuration is composed of an *even* number n ($n \geq 4$) of X-Slices, all adjacent (one to the next and the last to the first) around the same starting point.

### 2.5. Symmetric X-Configurations

A symmetry condition can be formulated so that, when satisfied by an X-Configuration, will leave the centre of the X-Configuration undisturbed under CC subdivision. This observation follows immediately from the symmetric formulation of the CC subdivision rules.

This condition can be stated as follows (see Fig. 8): the 2k X-Slices go around the centre of the X-

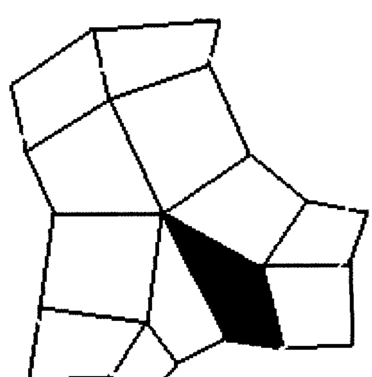

**Fig. 7.** An X-Configuration (One Shaded X-Slice).

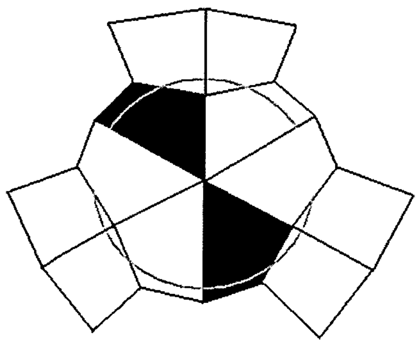

**Fig. 8.** A Symmetric X-Configuration.

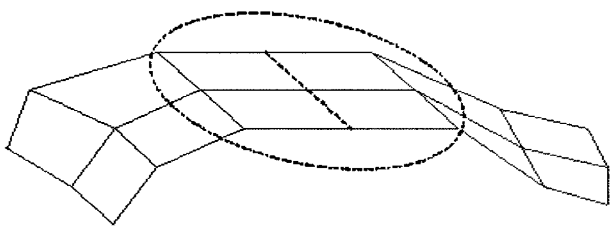

**Fig. 9.** Intersection at a Symmetric and Planar X-Configuration.

Configuration in consecutive pairs. Each component of a pair of X-Slices is a reflection of its counterpart with respect to their common edge. Furthermore, each X-Slice is a reflection of the X-Slice directly opposite to it with respect to the centre of the X-Configuration.

This symmetry condition does not in any way imply that all the vertices of the X-Configuration involved are necessarily coplanar. Moreover, we do not exclude the possibility that a more general (but more subtle) version of this condition can be formulated.

It is straightforward to see that a one-step subdivision, of a symmetric X-Configuration, results in a symmetric X-Configuration and the limit of subdivision of a symmetric X-Configuration is a point that is precisely its innermost vertex.

Consequently, when any number n ($n \geq 2$) of complexes meet at a symmetric and planar X-Configuration of the kind discussed above, one would be justified in expecting that the corresponding limit curves will meet at the centre of this X-Configuration and that the surface will be tangent-plane continuous there.

For instance, if two complexes meet at a *symmetric* X-Configuration, then the curves corresponding to these two complexes will meet at the centre of this X-Configuration and will be smooth there (see Fig. 9). However, this is not generally true in the absence of symmetry. Symmetry also permits the interpolation of a curve that carries through the centre of an X-Configuration. This added feature comes at no extra cost.

## 3. The Curve-Network Interpolation Task

We view the interpolation process as composed of the following steps:

- Each curve of the tagged control polygons is represented by a polygonal complex that corresponds to the same curve at the limit of subdivision [11].
- An X-Configuration with the desired planarity and symmetry characteristics is constructed at each region where two or more of these complexes meet. However, in view of the fact that the symmetry constraints are not so easily realizable in all situations, we will explore the possibility of whether these properties can be waived in order to reduce the difficulty of the construction. We stress that this can be done only at the expense of smoothness at the corresponding extraordinary points.

In the regions delimited in the mesh by various polygonal complexes, additional control points may be introduced in order to close the wider gaps that might otherwise still appear in the mesh. This process is often referred to as skinning [11, 12, 13]. These extra points will allow the designer the ability to satisfy further requirements such as local normal and curvature constraints [1].

### 3.1. Constructing X-Configurations Without Symmetry and Co-Planarity Properties

The construction of an X-Configuration is illustrated by an example of 3 curves meeting at a point (see Fig. 10). Assume that $A_0B_2$, $A_0B_3$ and $A_0E_1$ are the tagged (marked as grey) edges of three control polygons meeting at $A_0$. This is done in the absence of any assumption concerning symmetry and planarity around the meeting point. An X-Configuration may be constructed around this meeting point as follows.

First, the extremities of the neighboring end edges are joined together so as to obtain the virtual edges $B_2B_3$, $B_2E_1$ and $B_3E_1$. Second, $A_0$ is joined with each of the middles of these virtual edges and the corresponding edge is stretched beyond the middle with an equal distance, thus obtaining the points $A_1$, $D_2$ and $D_3$, respectively. Finally, $A_2$ (for example) is constructed in such a way that the face $A_0A_1A_2B_2$ is a parallelogram. The other points $A_3$, $C_2$, $C_3$, $E_2$ and $E_3$ are constructed similarly.

The thickness of the emerging polygonal complexes can be controlled through controlling the length of the edges $A_0A_1$, $A_0D_2$ and $A_0D_3$. This helps in avoiding any undesirable intersection with other edges of the mesh.

The co-planarity of the initial end edges is by itself a sufficient condition for the planarity of the resulting X-Configuration. The same applies very well for the symmetry property. Moreover, in the case where $B_2$, $A_0$ and $B_3$ are co-linear, for instance, $A_1$ can simply be selected in the plane orthogonal to that line at $A_0$ and the construction carries on from there. There might be too much freedom to handle here, but the following paragraph provides for additional justifications.

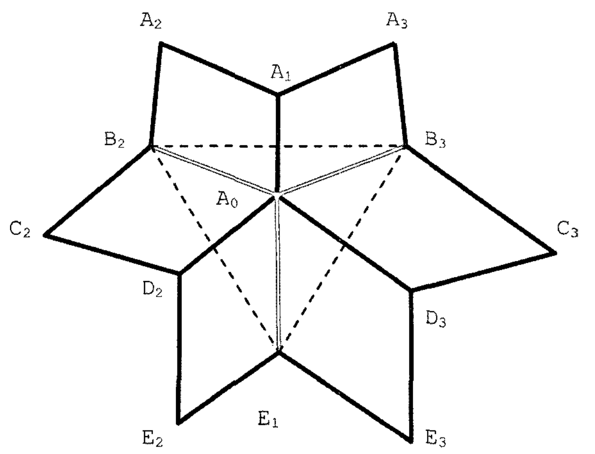


**Fig. 10.** X-Configuration Construction in the Case of Three Polygons Meeting at the Same Point.

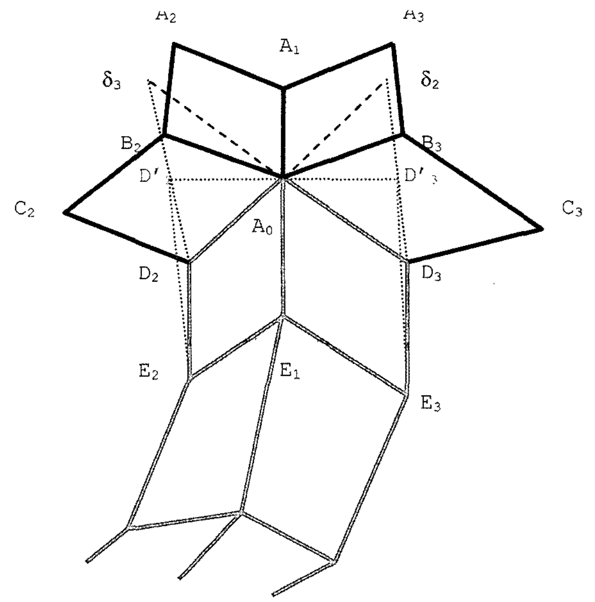


**Fig. 11.** Virtual Face Construction.

Even though the construction of these extra points looks close to arbitrary, it is already shown in details in [2] that the exact choice of these points do not affect the interpolation constraints, because repositioning the initial control points with respect to the new points is going to be done before subdivision begins. However, this choice will obviously affect the quality of the interpolating surface. Again, it is shown in [1] how this extra degree of freedom can be utilized for the benefit of the design.

This construction process can be cast under the form of a general algorithm (see Fig. 10). Indeed, an X-Configuration can be defined by $2 \times N$ (N here represents the valence of the meeting point) X-Slices meeting at a common point, say $A_0$, with the property that each X-Slice is a parallelogram with:

- One side (e.g. $A_0B_2$) is a tagged edge passing through $A_0$.
- The next side (i.e. $A_0A_1$) passes through the middle of the line joining the extremities $B_2$ and $B_3$ of the tagged edges $A_0B_2$ and $A_0B_3$ joining at $A_0$.

### 3.2. Virtual Face Construction

The underlying insight for achieving interpolation is that the vertices of the polyhedron should constantly correspond to their counterparts in the curve being interpolated at every step of the subdivision process right to the limit of the subdivision. This correspondence is very well summarized in **Property 1**. Accordingly, the subdivision coefficients of an X-Configuration will be modified following the approach suggested in [2]. This again makes use of the notion of virtual faces for providing a geometric justification of the new subdivision coefficients (see Fig. 11).

In this construction, the points $\delta_2$ and $\delta_3$ are the reflections with respect to $A_0$ of $D_2$ and $D_3$ respectively. This way, $D'_2$ and $D'_3$ are the midpoint of $D_2\delta_3$ and $\delta_2D_3$ respectively. Accordingly, $A_0E_1E_2D'_2$ and $A_0E_1E_3D'_3$ are the virtual faces corresponding to $A_0E_1E_2D_2$ and $A_0E_1E_3D_3$ respectively. Accordingly, the following identities are established:

$$D'_2 = (D_2 + 2A_0 - D_3)/2$$
$$D'_3 = (D_3 + 2A_0 - D_2)/2$$

These identities are invariant in all situations without reliance on any specific property (such as symmetry or co-planarity) of the X-configuration. Clearly, $A_0$ is the midpoint of the segment $D'_2D'_3$ which, as shown in [2], constitutes a sufficient condition guaranteeing interpolation at the corresponding extraordinary points.

Following the same approach of [2], the Catmull-Clark subdivision scheme is modified in the following ways:

- The V-vertex of the extraordinary point $A_0$ is itself.
- Any X-Slice F of an X-Configuration is substituted by its corresponding virtual face F' when calculating its F-vertex.

The CC subdivision is not modified anywhere else.

### 3.3. The Subdivision Matrix

The above virtual face construction reflects itself by a subdivision matrix to be applied only at X-Configurations and nowhere else. The subdivision matrix is the same as the one given in [2] but after replacing the matrix parameter t by 1/2.

The coefficients of the subdivision matrix are just a particular case of the coefficients of the matrix presented in [2], even though the construction used to get the coefficients is radically different. This factor may be used in support of the validity of the subdivision scheme.

The application of this subdivision matrix guarantees the smoothness of the interpolating limit surfaces only when the corresponding X-Configurations are symmetric and co-planar. In this case, the eigenvalue of the matrix

$$\frac{1}{288} \times \begin{bmatrix} 216 & 12 & 12 & 12 & 8 & 8 & 8 & 2 & 2 & 2 & 2 & 2 & 2 \\ 144 & 108 & 0 & 0 & 0 & 0 & 0 & 18 & 18 & 0 & 0 & 0 & 0 \\ 144 & 0 & 108 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 18 & 0 & 0 \\ 144 & 0 & 0 & 108 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 18 \\ 144 & 18 & 0 & 18 & 72+36t & -18t & -18t & 18 & 0 & 0 & 0 & 0 & 18 \\ 144 & 18 & 18 & 0 & -18t & 72+36t & -18t & 0 & 18 & 18 & 0 & 0 & 0 \\ 144 & 0 & 18 & 18 & -18t & -18t & 72+36t & 0 & 0 & 0 & 18 & 18 & 0 \\ 144 & 72 & 0 & 0 & 72t & -72t & 0 & 72 & 0 & 0 & 0 & 0 & 0 \\ 144 & 72 & 0 & 0 & -72t & 72t & 0 & 0 & 72 & 0 & 0 & 0 & 0 \\ 144 & 0 & 72 & 0 & 0 & 72t & -72t & 0 & 0 & 72 & 0 & 0 & 0 \\ 144 & 0 & 72 & 0 & 0 & -72t & 72t & 0 & 0 & 0 & 72 & 0 & 0 \\ 144 & 0 & 0 & 72 & -72t & 0 & 72t & 0 & 0 & 0 & 0 & 72 & 0 \\ 144 & 0 & 0 & 72 & 72t & 0 & -72t & 0 & 0 & 0 & 0 & 0 & 72 \end{bmatrix}$$

**Fig. 12.** The Subdivision Matrix.

satisfy the conditions for smoothness suggested in [3].

In all other cases, the smoothness of the surface in the regions where the matrix is applied depends on the relative positions of the corresponding vertices of the control mesh. That is, the surface might have features such as bumps, creases, etc. However, the interpolation constraints will be respected in all situations. In such cases, the subdivision matrix is the same as the one given above except that t is replaced by 1/2 and the first row by

[288 0 0 0 0 0 0 0 0 0 0 0 0]

### 3.4. X-Complexes

An X-Complex *unit* is defined to be an open polygonal complex terminated at either or both ends by an X-Configuration (see Fig. 13). An X-Complex is also defined to be a network of X-Complex units that meet at common X-Configurations.

### 3.5. The Repositioning Process

The subdivision of an X-Complex unit is an X-Complex unit and the limit of the subdivision of an X-Complex unit is a curve delimited at either or both ends by the centre of the X-Configuration of the corresponding X-Complex unit. The limit curves possess exact expressions through multiplication by the matrix [1 4 1]/6 (see equation (1)). Similarly, the curve corresponding to the middle polygon of this X-Complex unit is interpolated through multiplication by [-1 6 -1]/4 (see equation (2)). The full details of how this is obtained can be found in [2].

That is, away from the intersection point, the vertices of the tagged edges are repositioned as follows. Assuming P to be a tagged vertex, let O and N be the vertices of the polygonal complex adjacent to P from either side (see Fig. 14):

- The centre G of the X-Configuration remains invariant
- The vertex P is repositioned by multiplying the matrix [O P N] by [-1 6 -1]/4.

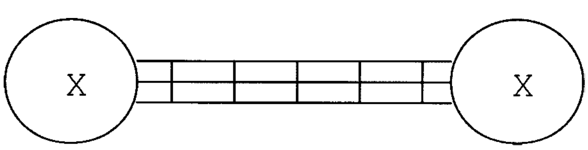


**Fig. 13.** An X-Complex Unit.

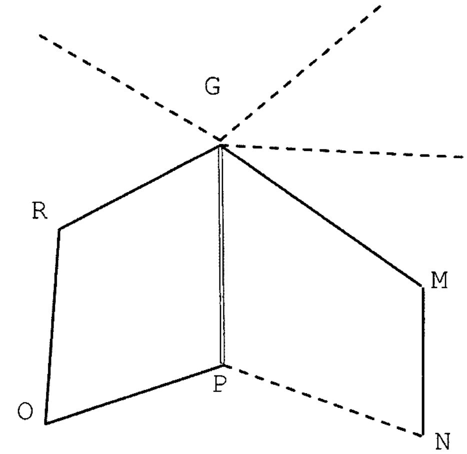


**Fig. 14.** Two X-Slices around an Extraordinary Vertex.

### 3.6. An Illustrative Example

Given a curve network embedded within a mesh, when each curve of the network tagged for interpolation corresponds to an X-Complex unit and each meeting point in the network corresponds to an X-Configuration, and when the mesh is subdivided according to the coefficient specified by the matrix in Fig. 12, the resulting limit surface from the mesh will interpolate the curve network.

Moreover, the degree of smoothness of the surface

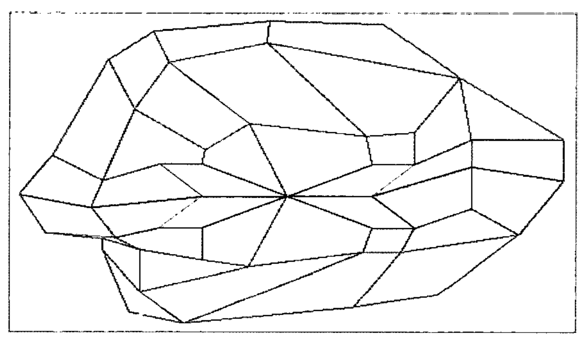

**Fig. 15.** Initial X-Complex and Polyhedron.

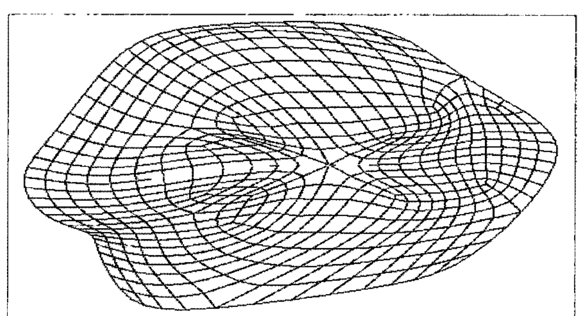

**Fig. 16.** Interpolation of Intersecting Curves.

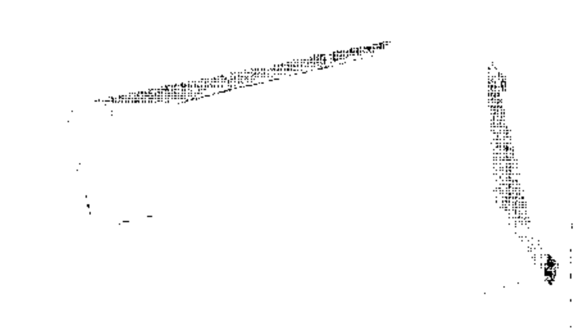

**Fig. 17.** Another X-Complex and Polyhedron.

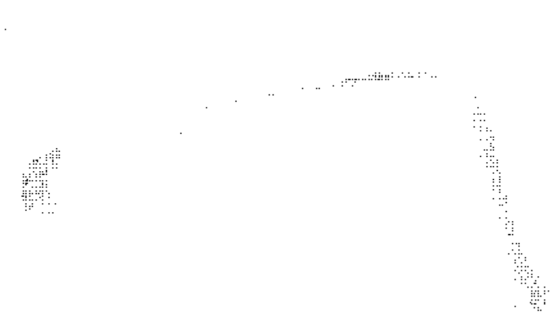

**Fig. 18.** Interpolation of More than One Intersection Points.

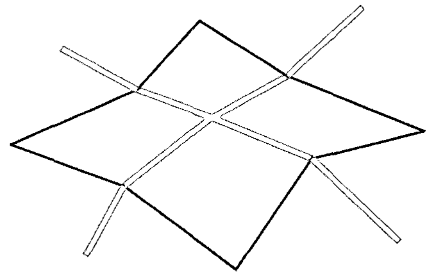

**Fig. 19.** Arbitrarily Connected Initial Polyhedron Red Edges are Tagged for Interpolation.

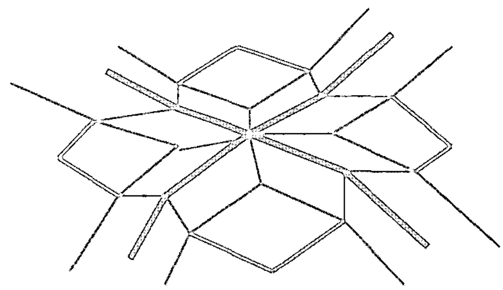

**Fig. 20.** Adjusting Polyhedron Connections so as to Embody the New X-Configuration.

will depend on the position of each constituent curve relative to each other at their respective meeting points.

To illustrate the working of this scheme, Fig. 15 shows an initial polyhedron with an embedded X-Complex. Fig. 16 shows the limit intersecting curves of this complex being interpolated by the limit surface of this polyhedron. The X-Configuration at the centre of this complex is deliberately deprived of any symmetry or planarity characteristics. This network also illustrates an additional useful feature. In fact, the initial network is designed in such a way that the two horizontal curve components of the network actually form a single smooth curve that carries through the intersection point of this network.

Fig. 17 and Fig. 18 provide another illustration embodying a network of three X-Configurations.

## 4. Applications

One immediate application of this approach is to modify an existing control polyhedron so as to guarantee the interpolation of some of its tagged edges by the corresponding limit surface. The process consists of constructing an X-Configuration from the vertices and edges at each point common to many curves. Some additional vertices/faces might need to be added as illustrated in Fig. 19 and Fig. 20.

Another application of the above scheme would be to construct a control polyhedron embodying a given network of control polygons targeted for interpolation. Such an approach is suggested in [8] where a Coon's patch of the first degree can be used to generate the mesh between the curves.

## 5. Conclusions

The solution to the interpolation problem presented in this paper is very general, uniform (i.e. no awkward particular cases), easy to understand and implement. It is a generalization to the approach suggested in [2], in the sense that it can be made use of in a wider context. Moreover, it has several important applications.

## Acknowledgement

This work was supported by a URB research grant from the American University of Beirut number #DDF 111130-688129.

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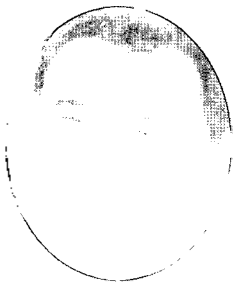

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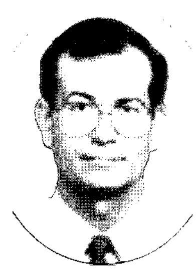

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