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Time Optimal Attitude Maneuver Strategies for the Agile Spacecraft with Reaction Wheels and Thrusters

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Reaction wheels and thrusters are commonly used for the satellite attitude control. Since satellites frequently need fast maneuvers, the minimum time maneuvers have been extensively studied. When the speed of attitude maneuver is restricted due to the wheel torque capacity of low level, the combinational use of wheel and thruster is considered. In this paper, minimum time optimal control performances with reaction wheels and thrusters are studied. We first identify the features of the maneuvers of the satellite with reaction wheels only. It is shown that the time-optimal maneuver for the satellite with four reaction wheels in a pyramid configuration occurs on the fashion of single axis rotation. Pseudo control logic for reaction wheels is successfully adopted for smooth and chattering-free time-optimal maneuvers. Secondly, two different thrusting logics for satellite time-optimal attitude maneuver are compared with each other: constant time-sharing thrusting logic and varying time-sharing thrusting logic. The newly suggested varying time-sharing thrusting logic is found to reduce the maneuvering time dramatically. Finally, the hybrid control with reaction wheels and thrusters are considered. The simulation results show that the simultaneous actuation of reaction wheels and thrusters with varying time-sharing logic reduces the maneuvering time enormously. Spacecraft model is KOrea Multi-Purpose SATellite (KOMPSAT)-2 which is being developed in Korea as an agile maneuvering satellite.

Key Words : Time-Optimal Control, Reaction Wheel, Thrusting Logics, Hybrid Control

1. Introduction

Since low-earth orbit observation satellites are supposed to collect a large quantity of data in short time, the satellites frequently perform fast

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maneuvers. The minimum time maneuvers have thus been extensively studied.(Bilimoria and Wie, 1993; Steyn, 1995; Byers and Vadali, 1993; Seywald and Kumar, 1993; Scrivener and Thomson, 1993; Hurtado and Junkins, 1998; Shen and Tsiotras, 1999) Bilimoria and Wie (1993) showed that the time optimal maneuver is of the bang-bang type. That is, the attitude control actuators should always generate their maximum torque for fastest maneuver. Steyn (1995) suggested a control law for the time-optimal maneuver along the eigen-axis.

Reaction wheels and thrusters are commonly used for spacecraft attitude control.(Sidi, 1997; Oh et al., 2001; Oh and Cheon, 2005) While reaction wheels are normally used as primary attitude control devices for the precision attitude control, thrusters are usually adopted for the orbit adjustment maneuver. However, when the satellite needs fast maneuver, thrusters can be used as the assistant devices, and the combinational use of wheel and thruster has been considered. In this paper, time optimal control performances with the combination of reaction wheels and thrusters are studied.

We first identify the features of the maneuvers of the satellite with reaction wheels only. It is well known that time-optimal rotational maneuver with independent actuators on each axis induces the nutational motion deviated from the eigenaxis rotation.(Bilimoria and Wie, 1993) The phenomenon can be explained by the fact that the maximum torque is generated along the axis skewed from the actuator axes. However, in the satellite like KOMPSAT with four reaction wheels in a pyramid configuration, maximum torque is generated along the direction of each actuator axis and the time-optimal maneuver occurs on the fashion of single axis rotation without nutation. On the constraint of the wheel torque jerking, pseudo control of reaction wheels is applied for smooth and chattering-free control in time-optimal maneuvers.

For thrusters, time-optimal performances of two different thrusting logics are compared: one of conventional constant time-sharing thrusting logic and another of newly proposed varying time-sharing thrusting logic. In the minimum time sense, there exists a certain loss of torque in the constant time logic. That is due to the assignment of constant portion of sharing-time on each axis even unnecessary, which results in the loss of torque. To reduce such a loss, the varying time-sharing logic, a new logic, has been invented such as to assign the thruster sharing-time flexibly following the needs of each axis. The performance and the characteristics of the newly suggested varying time-sharing thrusting logic are analyzed intensively in this paper.

Finally, for fast maneuvers we consider the combinational operation of two types of actuators in a hybrid fashion: the simultaneous hybrid control of reaction wheels and thrusters on the varying time-sharing thrusting logic. That is, in order to improve the performance of fast maneuver, thrusters and reaction wheels are operated simultaneously in the hybrid fashion.

The adoption of each logic is tested and verified through some simulations with KOMPSAT-2 satellite model, which is being developed in KARI as an agile maneuvering satellite.

2. Dynamic Characteristics of Attitude Control System

The configuration of the reaction wheel system of KOMPSAT-2 is presented in Fig. 1. The reaction wheel system consists of four reaction wheels mounted on a pyramid configuration.

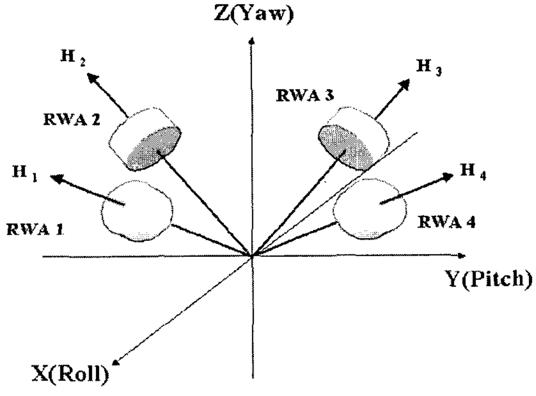
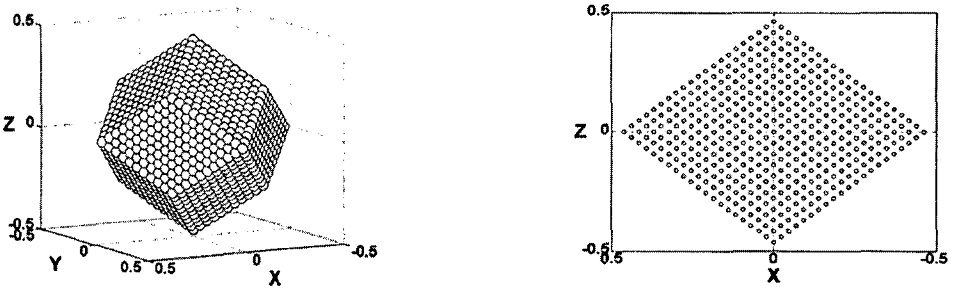
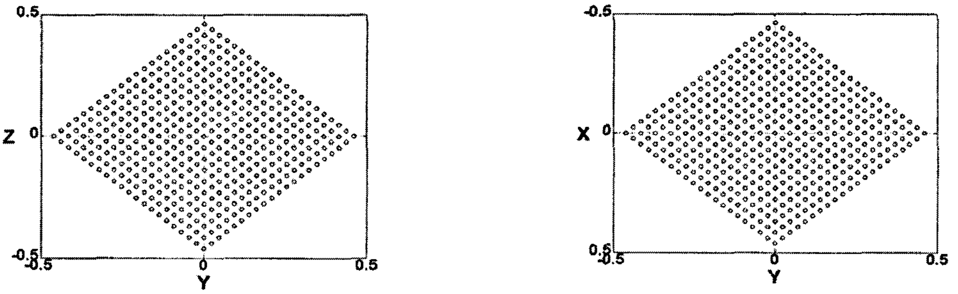


Fig. 1 Installation configuration of reaction wheels



(a) 3D view

(b) X-Z plane view



(c) Y-Z plane view

(d) Y-X plane view

Fig. 2 Torque envelope of reaction wheels in the pyramid configuration

The spacecraft dynamic equations with reaction wheels can be written as

$$I\dot{\omega}=-\omega^{\times}I\omega-\omega^{\times}Ah-Au_w \tag{1}$$

$$\dot{h}=u_w \tag{2}$$

where I is the inertia matrix, ω the angular velocity vector, $\omega^{\times}$ the cross product matrix of ω, A the wheel installation configuration influence matrix, h the wheel angular momentum vector, and $u_w=[u_1\ u_2\ u_3\ u_4]^T$ the control torque vector. Figure 2 shows the torque envelope of the reaction wheel system in the pyramid configuration. Maximum control torque is equally generated on each axis with this configuration.

KOMPSAT-2 is also equipped with four thrusters for orbit and attitude control. Although more than six thrusters are normally used to achieve the three-axis attitude maneuver, it is also possible to control the attitude with only four thrusters. (Sidi, 1997 ; Oh, 1999 ; Oh and Hwang, 1999) Thrusters in KOMPSAT-2 are symmetrically installed as shown in Fig. 3. Each thruster is located at (x_i, y_i, z_i) with the installation cant angle α with respect to the z axis. Any directional torque can be generated by combination of these four thrusting vectors. In order to produce the desired torque, the corresponding pairs of thrusters are fired in roll-pitch-yaw sequence, named "the sequential paired thrusting."

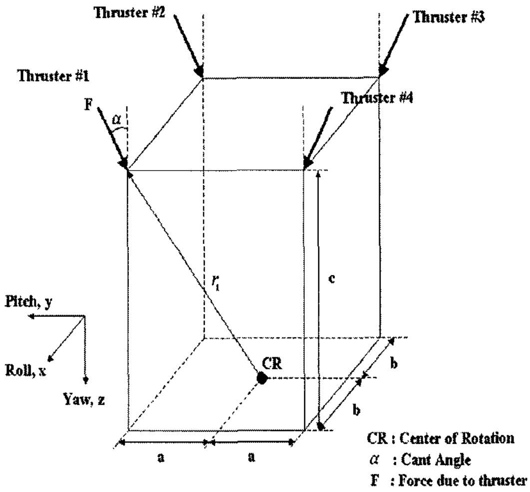


Fig. 3 Thruster installation configuration

The dynamic equation of spacecraft with thrusters only can be written as

$$I\dot{\omega}=-\omega^{\times}I\omega+u_t \tag{3}$$

Table 1 Control design parameters of the KOMPSAT-2

Item	Value
Moment of Inertia I	diag [600 400 400] kgm^2
Maximum wheel momentum $h_{\max}$	15 Nms
Maximum wheel torque $u_{\max}$	0.2 Nm
Thruster force F	4.23 N
Thruster torque $[g_1\ g_2\ g_3]$ at BOL	[1.1 1.48 0.37]Nm

Here u_t denotes the thruster generated torque vector represented as

$$u_t = [g_1 u_{f1}\ g_2 u_{f2}\ g_3 u_{f3}]^T \tag{4}$$

where g_i is the maximum torque generated about the i-th axis and u_{fi} is the on-time duty-ratio of the corresponding pair of thrusters. The design parameters of the reaction wheels and the thrusters system of KOMPSAT-2 are shown in Table 1.

In this paper, the modified Rodrigues parameter is used for the attitude kinematics. The modified Rodrigues parameter σ is defined as

$$\begin{aligned}\sigma &= [\sigma_1\ \sigma_2\ \sigma_3]^T \\ &= \left[e_1 \tan\frac{\phi}{4}\ e_2 \tan\frac{\phi}{4}\ e_3 \tan\frac{\phi}{4}\right]^T\end{aligned} \tag{5}$$

where e_1, e_2 and e_3 are the direction cosines of the principal line and ϕ is the principal rotation angle. (Malcolm, 1993) The kinematic equations using the modified Rodrigues parameter can be written as

$$\dot{\sigma} = \frac{1}{4} G(\sigma)\,\omega \tag{6}$$

where

$$G(\sigma) = \begin{bmatrix} 1+\sigma_1^2-\sigma_2^2-\sigma_3^2 & 2(\sigma_1\sigma_2-\sigma_3) & 2(\sigma_1\sigma_3+\sigma_2) \\ 2(\sigma_1\sigma_2+\sigma_3) & 1-\sigma_1^2+\sigma_2^2-\sigma_3^2 & 2(\sigma_2\sigma_3-\sigma_1) \\ 2(\sigma_1\sigma_3-\sigma_2) & 2(\sigma_2\sigma_3+\sigma_1) & 1-\sigma_1^2-\sigma_2^2+\sigma_3^2 \end{bmatrix}$$

3. Time-Optimal Maneuvering Feature with Pyramid Reaction Wheel System

Consider the minimum time attitude maneuver of the rigid spacecraft equipped with four reaction wheels in a pyramid configuration. The total number of states is then ten (σ_1, σ_2, σ_3, ω_1, ω_2, ω_3, h_1, h_2, h_3, h_4) as shown in Eqs. (1), (2) and (6). To determine the optimal control torque profile for the satellite to maneuver from the initial state to the target state in minimum time, select the cost function as

$$Min\ J = \int_0^{t_f} dt \tag{7}$$

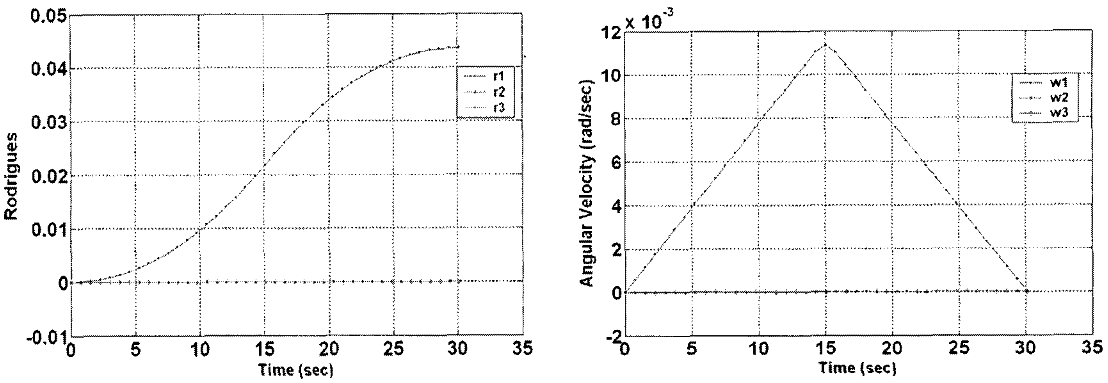
while the wheel control torque is limited by its maximum value $u_{\max}$ as

$$|u_i| \le u_{\max}\ (i=1,\ 4) \tag{8}$$

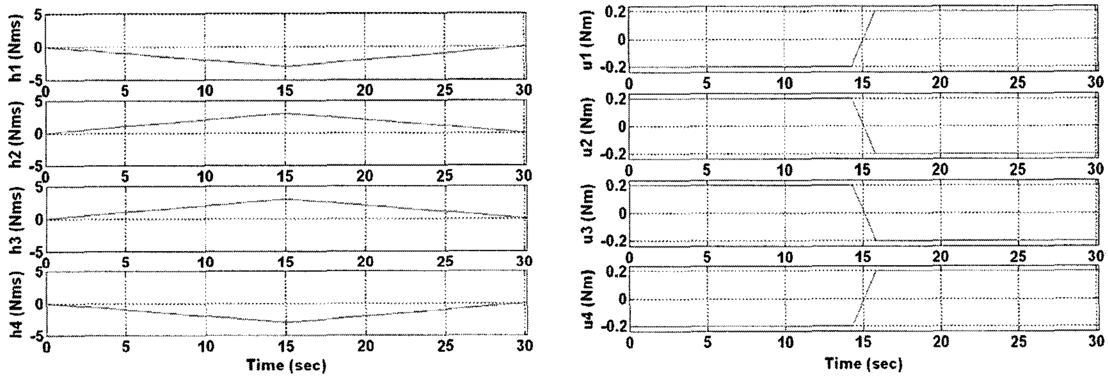
Terminal target states are specified as $\omega(t_f)=0$ and $\sigma(t_f)=\sigma_f$ The optimal control and the required maneuver time have been determined using the sequential quadratic programming for parameter optimization.

3.1 Rotational feature of the time-optimal maneuver

We first consider the rotational feature of the roll axis maneuver in the case of no limit on the wheel angular momentum. It is found that the eigenaxis rotation maneuver is not time-optimal in general, and the time-optimal solutions accompany the nutation deviated from the eigenaxis. (Bilimoria and Wie, 1993) However, when four reaction wheels mounted in a pyramid configuration are used, maximum torque is generated along the direction of each axis as shown in Fig. 2. The maneuver is then performed on the fashion of single axis rotation as shown in Fig. 4. The eigenaxis rotation maneuver is thus time-optimal in this case.



(a) Attitude (b) Angular velocity



(c) Wheel momentum (d) Control torque

Fig. 4 Minimum time maneuver with reaction wheel: Roll 10°

3.2 With the limit on the wheel momentum

Since the time-optimal control is in general of bang-bang type, the momentum of the wheel will reach its limit value rapidly. In order to get a practical optimal control solution with the momentum constraint, the wheel momentum should be constrained in simulations as follows

$$|h_i| \leq h_{\max} \ (i=1, 4) \tag{9}$$

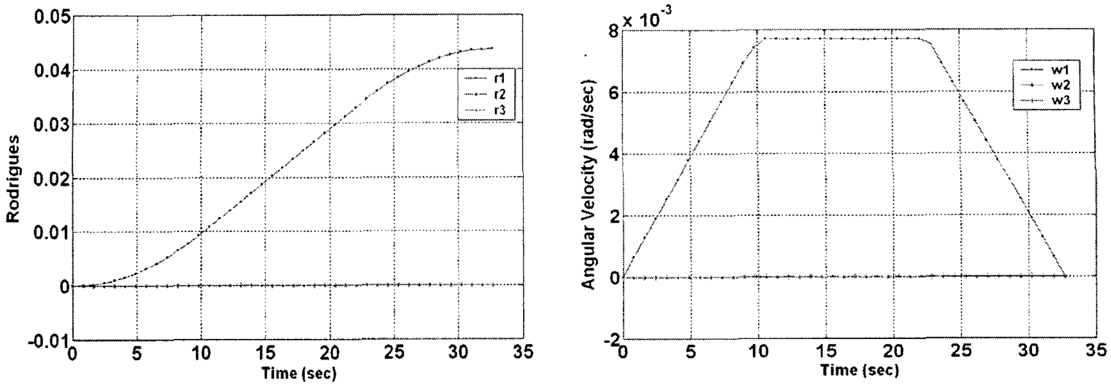
In small angle maneuver, however, the wheels hardly meet the saturation. Therefore, in order to see the effects of saturation on the optimal control, we make a situation of saturation deliberately. Simulations are thus performed with much lower limit value $h_{\max}$ than the real one. Figure 5 shows the results of the time-optimal maneuver of roll 10°. Unfortunately, the chattering phenomenon occurs on the zero-torque phase which is unacceptable control situation.

As mentioned earlier, the solutions to minimum-time optimal control problems are generally of bang-bang control, which is characterized by the abrupt switching of controls between two saturation limits. Instantaneous switching of controls, however, is difficult to implement and might stimulate the flexible modes of spacecraft structure. For this reason, control should be smooth and we thus select the first derivatives of each control torque u_w as new control variables as recommended by Hurtado and Junkins(1998). That is, the new control variable v_w is defined as the rate of change of control as follows:

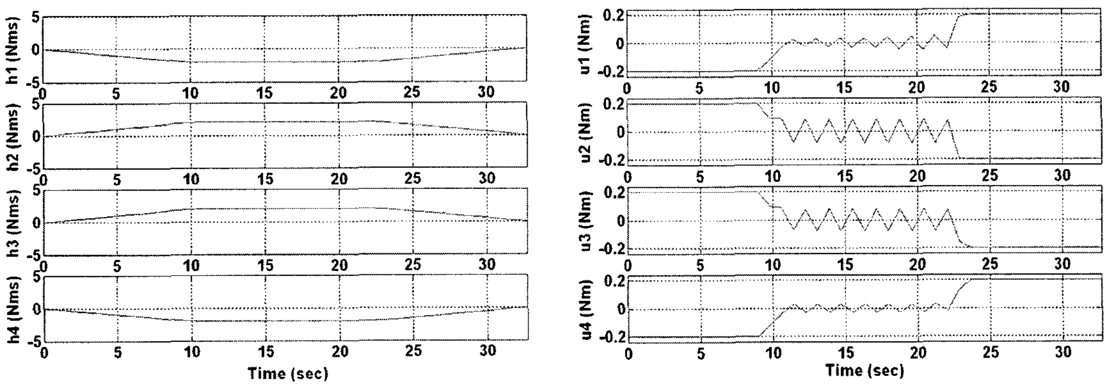
$$\dot{u}_w = v_w \tag{10}$$

The new control v_w might be then set to be constrained by the jerking requirements of spacecraft as

$$|v_{wi}| \leq v_{\max} \ (i=1, 4) \tag{11}$$



(a) Attitude (b) Angular velocity



(c) Wheel momentum (d) Control torque

Fig. 5 Maneuver with the wheel momentum constraint : Roll 10°, $h_{\max}=2$ Nms

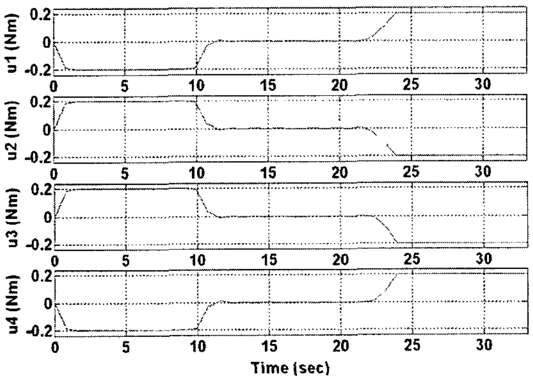


Fig. 6 Control torque with the control slope constraint : Roll 10°, $h_{\max}=2$ Nms, $v_{\max}=0.2$ Nm/s

Figure 6 shows the results of roll 10° maneuver with the new control variable. As shown in this result, the maneuver time increase slightly but the chattering phenomenon of the controls has been completely eliminated and the control profiles become smoother than the results shown in Fig. 5.

4. Time-Optimal Maneuvering Feature with Thrusting Logics

Next consider the minimum time attitude control with thrusters only. The time-optimal control for the thrusters are obtained by minimizing the cost function of Eq. (7) while the thruster torques are constrained in different fashions depending on the thrusting logic used.

4.1 With the conventional constant time-sharing thrusting logic

The thrusting logic of KOMPSAT-2 is one of the conventional sequential paired thrusting logics. In order to produce the required torque about an arbitrary direction, the corresponding thrusters are fired in pairs with a constant time interval (0.25 seconds in KOMPSAT-2 case) in roll-pitch-yaw sequence. In this paper, it is named "constant time-sharing thrusting logic" to distinguish this logic from the newly proposed one in the next section. Since the thrusts are operated by time-sharing mode in the sequence of roll-pitch-yaw with constant time-sharing interval (1/3 each), the maximum thrusting duty ratio for each axis (see Eq. (4) in section II) can not exceed 1/3. Therefore, only one third of torque capacity is actually used in each axis. Figure 7 shows an example of constant time-sharing ratio thrusting logic.

Since the maximum thrusting duty ratio cannot exceed 1/3, the duty ratio in Eq. (4) is constrained in real by

$$|u_{fi}| \leq \frac{1}{3} \tag{12}$$

Figure 8 shows the torque envelope when the constant time-sharing thrusting logic is used. It has the shape of rectangular parallelepiped.

The performance of constant time-sharing thrusting logic is analyzed through simulations. The time-optimal control solutions are obtained by parameter optimization technique, by which the thrusting duty ratio u_{fi} for each axis is determined. For the practical numerical simulations, the number of optimal parameters is determined by

$$N = \frac{\text{Expected Min. Maneuvering Time}}{\text{Firing Interval}} \tag{13}$$

The results of the time-optimal roll maneuver are shown in Fig. 9, where the nutation motion is induced during maneuver. That is similar with the results shown in the previous literatures. (Bilimoria and Wie, 1993; Steyn, 1995) As

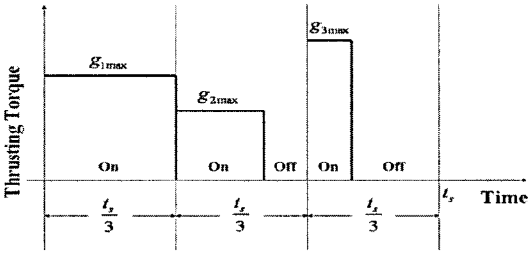
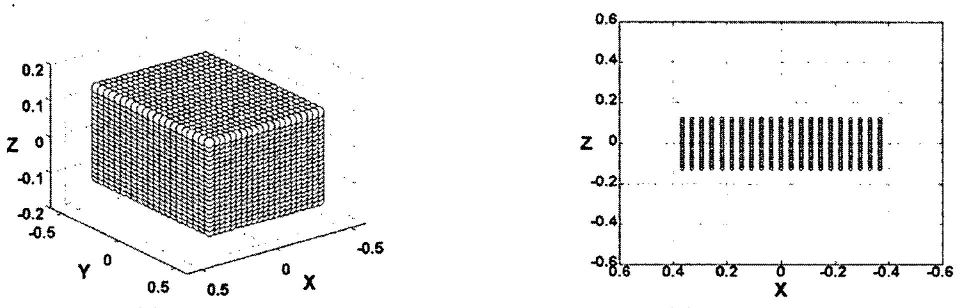
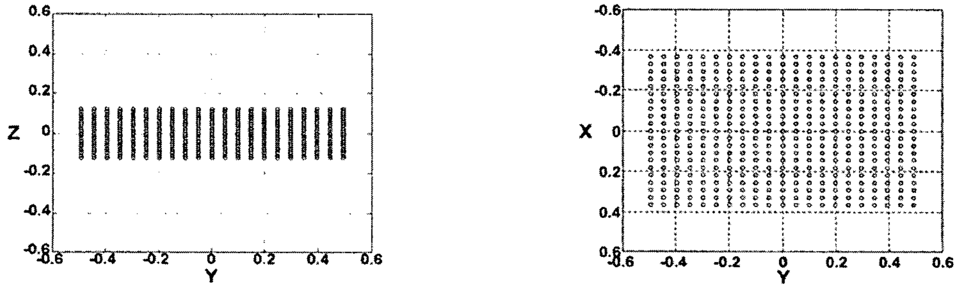


Fig. 7 Example of constant time-sharing thrusting logic

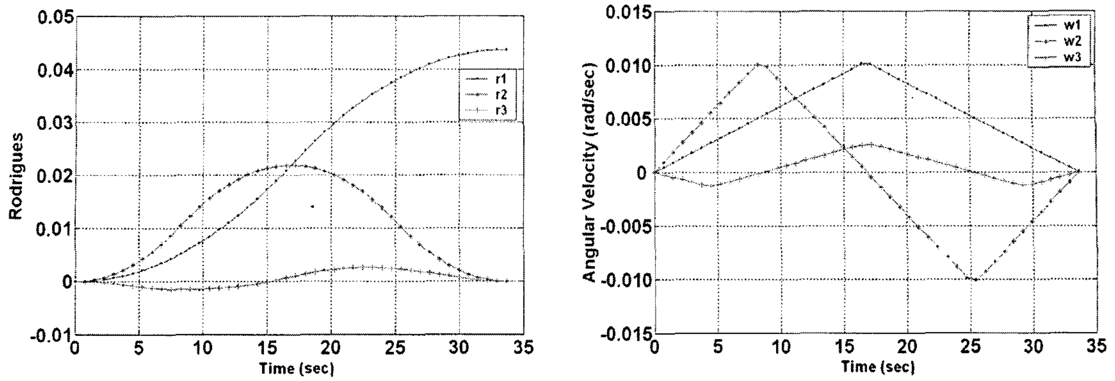


(a) 3D view (b) X-Z plane view



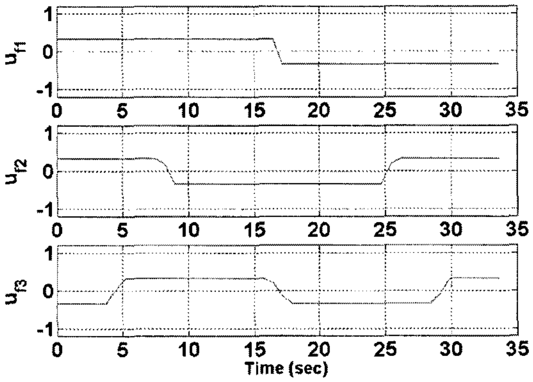
(c) Y-Z plane view (d) Y-X plane view

Fig. 8 Torque envelope with the constant time-sharing ratio thrusting logic



(a) Attitude

(b) Angular velocity



(c) Control duty ratio

Fig. 9 Maneuver with constant time-sharing thrusting logic : Roll 10°

shown in the results of simulation, the constant time-sharing thrusting logic takes a more time than the case of the reaction wheels.

4.2 Varying time-sharing thrusting logic

In the minimum time sense, there exists a certain loss of torque in the constant time-sharing thrusting logic. That is due to the uniform assignment of sharing-time on each axis even in unnecessary case. To reduce such a loss of torque, a new logic is invented such as to assign the sharing-time flexibly. (KARI and HAU, 2003 ; Lee, 2004) In this logic, the sharing-time on a specific axis can be flexibly increased more than 1/3 (up to 100%), while the control constraints is modified as

$$|u_{f1}|+|u_{f2}|+|u_{f3}|\leq 1 \qquad (14)$$

That is, the total sharing-time should not exceed 100%. Figure 10 shows an example of varying time-sharing thrusting logic and Fig. 11 shows the torque envelope of the varying time-sharing thrusting logic. The envelope is much larger than that of constant time-sharing logic.

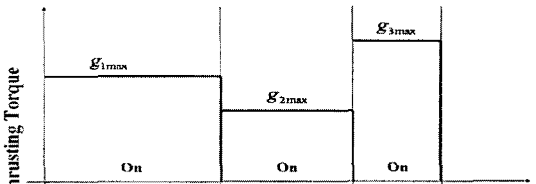
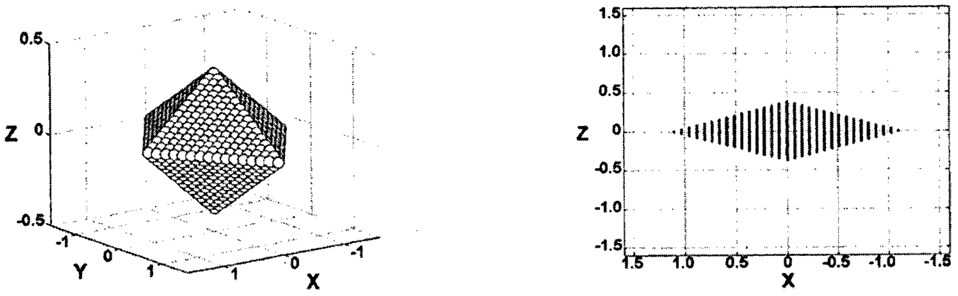
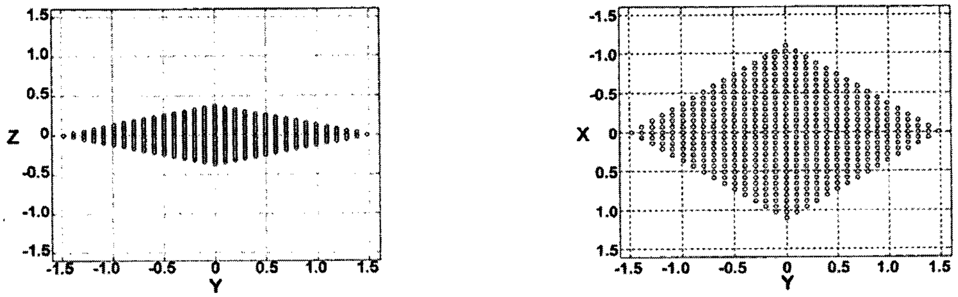


Fig. 10 An example of varying time-sharing thrusting logic

The results of the time-optimal roll maneuver with the varying time logic are shown in Fig. 12. We can see that the maneuver is of a type of single

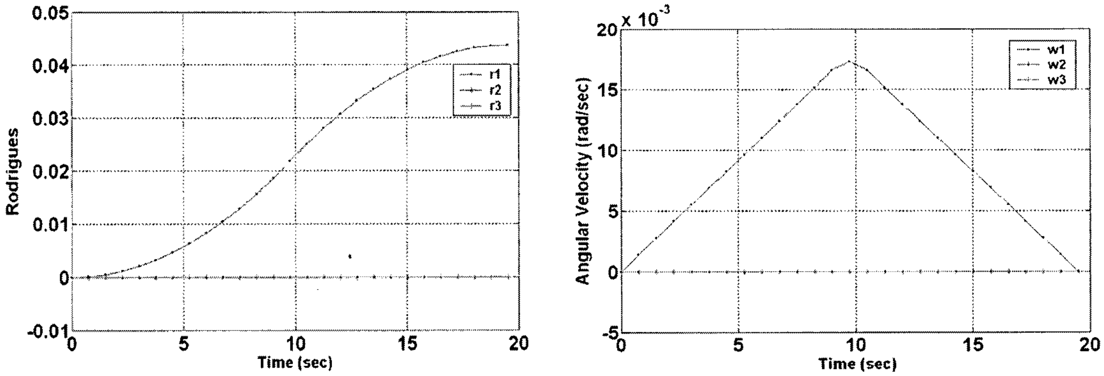


(a) 3D view (b) X-Z plane view

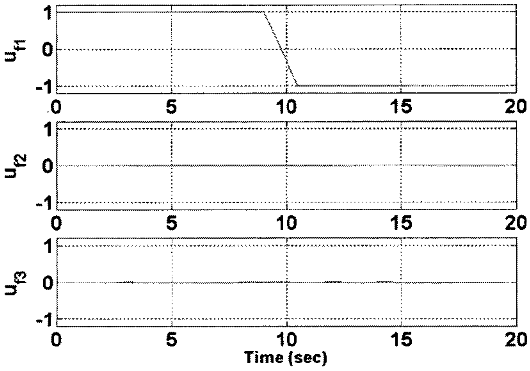


(c) Y-Z plane view (d) Y-X plane view

Fig. 11 Torque envelope for the varying time-sharing thrusting logic



(a) Attitude (b) Angular velocity



(c) Control duty ratio

Fig. 12 Maneuver with varying time-sharing ratio thrusting logic : Roll 10°

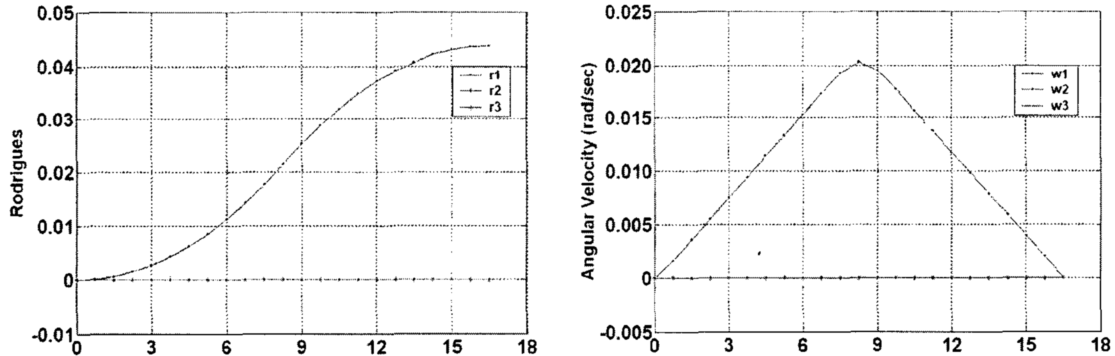
axis rotation without any nutation as shown previously in Fig. 9. In addition, the varying time-sharing thrusting logic reduces the maneuver time about 40% less than that of the constant time-sharing thrusting logic. With slight amendment of conventional constant time-sharing logic to a new logic, the maneuvering time can be reduced dramatically.

5. Time-Optimal Maneuvering Feature with Thrusters and Reaction Wheels in Hybrid Fashion

To improve the performance of fast maneuver more, thrusters and reaction wheels are operated occasionally in the hybrid fashion. It is recommended to use thrusters for coarse control in the first phase of maneuver and to switch at a switching time t_s to reaction wheels for fine control at the terminal phase, i.e., "the sequential hybrid control." The first phase is governed by Eqs.(3) and (6) and the terminal phase by Eqs.(1),(2) and (6). The time-optimal performance depends on the switching time t_s. Another type of hybrid control is "the simultaneous hybrid control," where both the reaction wheels and the thrusters are actuated simultaneously on the following dynamics equation.

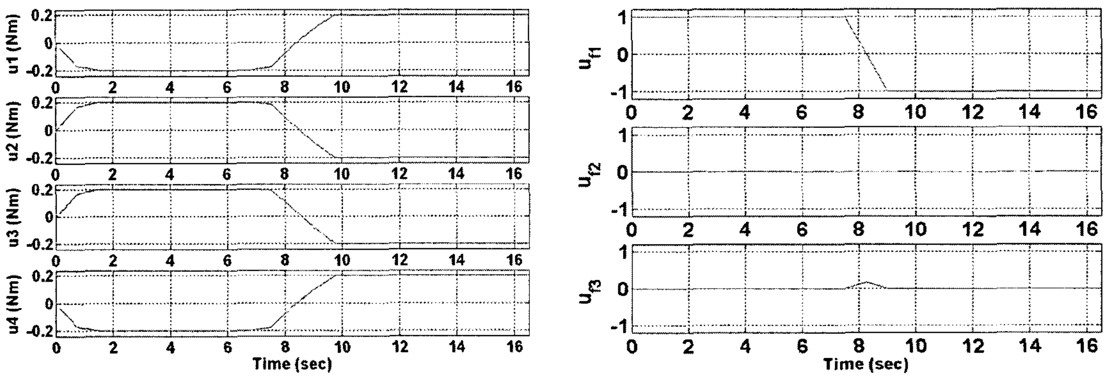
$$I\dot{\omega} = -\omega^{\times} I\omega - \omega^{\times} A h - A w_w + u_t \qquad (15)$$

The performance of the simultaneous hybrid control is much better than that of the sequential hybrid control for faster maneuvers. The best combination of actuators for the fastest maneuvers is the simultaneous hybrid control of reaction wheels and thrusters on the varying time-sharing thrusting logic. As shown in Fig. 13, the maneuvering time is reduced almost 45% less than that of the reaction wheel only case. The



(a) Attitude

(b) Angular velocity



(c) Reaction wheel control torque

(d) Thruster control duty ratio

Fig. 13 Simultaneous hybrid control with varying time-sharing thrusting : Roll 10°

simultaneous control with varying time thrusting logic is the most preferred combination of actuators for achieving agile attitude maneuver.

5. Conclusions

In the satellite with four reaction wheels in a pyramid configuration, the time-optimal maneuver occurs on the fashion of single axis rotation. Pseudo control of reaction wheels are successfully adopted for smooth maneuver and chattering-free solution in time-optimal maneuvers. For thruster control, the newly suggested varying time-sharing thrusting logic can reduce the maneuvering time enormously in comparison with the conventional constant time-sharing logic. The simultaneous actuation of reaction wheels and the thrusters on the varying time-sharing logic is found to be the most efficient combination of actuators for time-optimal maneuvers.

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