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Influence of Tip mass on Dynamic Behavior of Cracked Cantilever Pipe Conveying Fluid with Moving Mass

Han-Ik Yoon*
Corresponding author. Division of Mechanical Engineering, Dong-eui University, 995 Eomgwangno, Busanjin-Gu, Busan 614-714, Korea
In-Soo Son
The Center for Industrial Technology, Dong-eui University, 995 Eomgwangno, Busanjin-Gu 614-714, Korea

In this paper, we studied about the effect of the open crack and a tip mass on the dynamic behavior of a cantilever pipe conveying fluid with a moving mass. The equation of motion is derived by using Lagrange's equation and analyzed by numerical method. The cantilever pipe is modelled by the Euler-Bernoulli beam theory. The crack section is represented by a local flexibility matrix connecting two undamaged pipe segments. The influences of the crack, the moving mass, the tip mass and its moment of inertia, the velocity of fluid, and the coupling of these factors on the vibration mode, the frequency, and the tip-displacement of the cantilever pipe are analytically clarified.

Key Words : Dynamic Behavior, Open Crack, Cantilever Pipe Conveying Fluid, Moving Mass, Tip Mass, Euler-Bernoulli Beam, Flexibility Matrix

Nomenclature

a_c : Maximum depth of crack
A : Cross-sectional area
b : Half-length of crack
C : Flexibility matrix
d : Tip-displacement of pipe, dimensionless
F_f : Tangential follower force
J_o^* : Moment of inertia of tip mass, dimensionless
K_I : Stress intensity factor (fracture mode I)
K_R : Rotating spring coefficient
k : Number of segment
m : Mass per unit length of pipe
m_f : Fluid mass per unit length of pipe
m_m : Moving mass
m_p : Tip mass
q : Deflection of pipe
u : Velocity of fluid
U : Velocity of fluid, dimensionless
v : Velocity of moving mass
V : Velocity of moving mass, dimensionless
μ_m : Ratio of tip mass to its moment of inertia
θ^* : Half-angle of crack, dimensionless
ξ : Distance measured along pipe, dimensionless
ξ_c : Crack position, dimensionless

* Corresponding Author,
E-mail : hiyoon@deu.ac.kr
TEL : +82-51-890-1645; **FAX** : +82-51-890-2232
Corresponding author. Division of Mechanical Engineering, Dong-eui University, 995 Eomgwangno, Busanjin-Gu, Busan 614-714, Korea. (Manuscript **Received** February 21, 2005; **Revised** August 18, 2005)

1. Introduction

The effect of cracks on the dynamic behavior of structure elements is an interesting subject of investigation. When a structure is subjected to damage its dynamic response is varied due to the change of its mechanical characteristics. And the effect of moving mass on the structures and the machines is an important problem both in the field of transportation and on the design of machining processes. The fluid flowing inside the pipe acts as the concentrated tangential follower force at the tip of the pipe, and exerts a lot of influences on the dynamic characteristics of a pipe. Therefore, it is worth while to study these factors influencing in the dynamic characteristics of structure. The transfer of energy between the flowing fluid and the pipe was discussed by Benjamin (1961). Langthjem and Sugiyama (1999) studied the dynamic stability of a cantilevered two-pipe system conveying different fluids. A lot of studies about the dynamic behavior of a beam structure under moving load and moving mass were reported (Stanisic, 1985 ; Lee, 1996 ; Yoon et al., 2004). Recently, Mahmoud and Zaid (2002) used an equivalent static load approach to determine the stress intensity factors for a single or double-edge crack in a beam subjected to a moving load. Lim et al. (2003) executed the nonlinear dynamic analysis of a cantilever tube conveying fluid with system identification. Chondros and Dimarogonas (1989, 1998) studied the effect of the crack depth on the dynamic behavior of a cantilevered beam. They showed that the increase of the crack depth reduces the natural frequency of a beam. Also, they used energy method and a continuous cracked beam theory for analyzing the transverse vibration of cracked beams. Ostachowicz and Krawczuk (1991) investigated the influence of the position and the depth of two open cracks upon the fundamental frequency of the natural flexural vibrations of a cantilever beam. To model the effect of the local stress in the crack, they introduced two different functions according to the symmetry of the crack. Closing or breathing cracks have been investigated by Carlson (1974) and Gudmundson (1983), who studied the effects of closing cracks on the dynamical characteristics of an edge-cracked cantilever beam. Gudmunston found that the relative increase in natural frequencies caused by a closing crack is much smaller than the decrease due to an open crack. An equation of bending motion for Euler-Bernoulli beam containing pairs of the symmetrical open cracks was derived by Christides and Barr (1984). The cracks were considered to be normal to the beam's neutral axis and symmetrical about the plane of bending. Dado and Abuzeid (2003) studied the modeling and analysis algorithm for cracked Euler-Bernoulli beams by considering the coupling between the bending and axial modes of vibration. This algorithm is applied to the analysis of the vibration behavior of the cracked beam and the natural frequency and mode shapes under the effect of added mass and rotary inertia at the free end. Liu et al. (2003) examined the suitability of using coupled responses to detect damage in thin-walled tubular structures. By coupled response they referred to the ability of a structural member with a circumferential crack to experience composite vibration modes (axial and bending) when excited purely laterally. Recently, Yoon and Son (2004) investigated the effects of the open crack and the moving mass on the dynamic behavior of simply supported pipe conveying fluid. They studied about the influences of the crack, the moving mass and its velocity, the velocity of fluid, and the coupling of these factors on the Timoshenko beam.

In this study, the crack effects on the dynamic behavior of the cracked cantilever pipe conveying fluid with a moving mass and tip mass are investigated. The influences of a crack, a fluid and the velocity of moving mass have been studied on the dynamic behavior of a cantilever pipe system conveying fluid. In addition, the influences of a tip mass have studied on the dynamic characteristics of a cracked cantilever pipe. The cantilever pipe conveying fluid has a circular hollow cross-section. The crack is assumed to be always open during vibrations.

2. Theory and Formulations

The system with a moving mass on the cracked cantilever pipe conveying fluid with a tip mass is shown in Fig. 1, where m_m is a moving mass, v is the velocity of the moving mass, u is the velocity of fluid flow, L is the total length of the pipe, m_p is a tip mass, and x_c is the position of the crack from the left-hand clamped end. F_f is the tangential follower force due to the fluid. Figure 2 shows a circular hollow cross-section of the cracked section. θ_c and $2b$ are the crack depth (severity) and the length of a crack, respectively. Two equations of motion are derived for the two

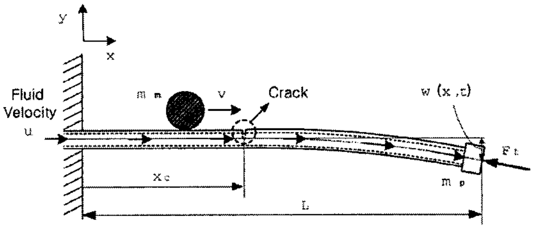


Fig. 1 Geometry of the cracked cantilever pipe conveying fluid with a moving mass and tip mass

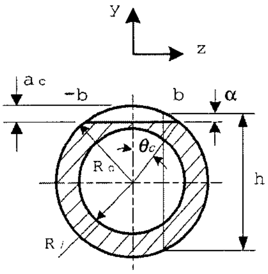


Fig. 2 Cross section of the cracked pipe

parts of the pipe located on the left and on the right of the cracked section.

2.1 Energy of cantilever pipe and moving mass

By using the assumed mode method, the transverse displacement $w(x, t)$ of a cracked cantilever pipe can be assumed to be

$$w_k(x, t)=\sum_{n=1}^{\mu}\phi_{nk}(x)\,q_n(t) \tag{1}$$

where $q_n(t)$ are generalized coordinates which is time dependent, μ is the total number of the generalized coordinates, and $\phi_{nk}(x)$ are spatial mode functions of a cantilever pipe with a tip mass missing the fluid and a moving mass. k is the number of the segments. $\phi_{nk}(x)$ can be described to be as follows :

I) segment 1 $(0\leq x\leq x_c)$;

$$\begin{aligned}\phi_{n1}(x)=&A_1\cos(\lambda_n x)+A_2\sin(\lambda_n x)\\&+A_3\cosh(\lambda_n x)+A_4\sinh(\lambda_n x)\end{aligned} \tag{2}$$

II) segment 2 $(x_c\leq x\leq L)$;

$$\begin{aligned}\phi_{n2}(x)=&A_5\cos(\lambda_n x)+A_6\sin(\lambda_n x)\\&+A_7\cosh(\lambda_n x)+A_8\sinh(\lambda_n x)\end{aligned} \tag{3}$$

where λ_n is the frequency parameter, which is easily calculated using the frequency equation of a cantilever beam (Rao, 1995). The constants A_1, A_2, ···, A_8 can be found from the boundary conditions. The boundary conditions of a cracked cantilever pipe are

$$\begin{aligned}&\text{at } x=0\,;\ w_1(x, t)=0 \text{ and } \frac{\partial w_1(x, t)}{\partial x}=0\\&\text{at } x=L\,;\ EI\frac{\partial^2 w_2(x, t)}{\partial x^2}=-J_o\frac{\partial^3 w_2(x, t)}{\partial x\partial t^2}\\&\text{and } \frac{\partial}{\partial x}\left(EI\frac{\partial^2 w_2(x, t)}{\partial x^2}\right)=-m_p\frac{\partial^2 w_2(x, t)}{\partial t^2}\end{aligned} \tag{4}$$

where m_p and J_o are the tip mass and the moment of inertia of the tip mass, respectively. And E is the modulus of elasticity of the pipe and I means the moment of inertia of the pipe cross-section.

The boundary conditions for the transverse deflection, bending moment, shear force and slope at the cracked section $(x=x_c)$ are

$$\begin{aligned}&\phi_{n1}(x_c)=\phi_{n2}(x_c)\\&\frac{\partial^2\phi_{n1}(x_c)}{\partial x^2}=\frac{\partial^2\phi_{n2}(x_c)}{\partial x^2}\\&\frac{\partial^3\phi_{n1}(x_c)}{\partial x^3}=\frac{\partial^3\phi_{n2}(x_c)}{\partial x^3}\\&\frac{\partial\phi_{n2}(x_c)}{\partial x}-\frac{\partial\phi_{n1}(x_c)}{\partial x}=\frac{EI}{K_R}\frac{\partial^2\phi_{n2}(x_c)}{\partial x^2}\end{aligned} \tag{5}$$

where K_R is the bending stiffness. In Fig. 1, the energy of the cracked cantilever pipe with a tip mass can be written as

$$\begin{aligned}T_p=&\frac{1}{2}m\sum_{n=1}^{\mu}\Big[\int_0^{x_c}\{\phi_{n1}(x)\,\dot{q}_n(t)\}^2dx\\&+\int_{x_c}^{L}\{\phi_{n2}(x)\,\dot{q}_n(t)\}^2dx\Big]\\&+\frac{1}{2}m_p\sum_{n=1}^{\mu}\{\phi_{n2}(L)\,\dot{q}_n(t)\}^2\\&+\frac{1}{2}J_o\sum_{n=1}^{\mu}\Big\{\frac{d}{dt}(\phi'_{n2}(L)\,q_n(t))\Big\}\end{aligned} \tag{6}$$

$$\begin{aligned}V_p=&\frac{1}{2}\sum_{n=1}^{\mu}\sum_{k=1}^{2}\Big[EI\int_0^{L_k}\{\phi''_{nk}(x)\,q_n(t)\}^2dx\Big]\\&+\frac{1}{2}K_R(\Delta y'_c)^2\end{aligned} \tag{7}$$

where $(\cdot)$ denotes $\partial/\partial t$, and $(')$ represents $\partial/\partial x$.

In addition, m is the mass per unit length of the cantilever pipe. In Eq. (7), the quantity

$$\Delta y_c' = \frac{dw_2(x,\ t)}{dx}\bigg|_{x_2=0} - \frac{dw_1(x,\ t)}{dx}\bigg|_{x_1=x_c} \quad (8)$$

represents the jumps in the rotation. The kinetic energy of a moving mass can be expressed as

$$\begin{aligned} T_m = \frac{1}{2} m_m \sum_{n=1}^{\mu} \sum_{k=1}^{2} \{ & v^2 q_n^2(t)\, \phi_{nk}'^2(x_m) \\ & + 2v q_n(t)\, \dot{q}_n(t)\, \phi_{nk}(x_m)\, \phi_{nk}'(x_m) \\ & + \dot{q}_n^2(t)\, \phi_{nk}^2(x_m) + v^2 \} \end{aligned} \quad (9)$$

Since the horizontal velocity of a moving mass is v, the horizontal displacement of a moving mass x_m is

$$x_m = f_m(t) = \int_0^t v\ dt\ (0 \le x_m \le L) \quad (10)$$

2.2 Work and energy due to the fluid flow

The kinetic energy of the fluid flow inside the pipe can be expressed as

$$\begin{aligned} T_f = \frac{1}{2} m_f \sum_{n=1}^{\mu} \sum_{k=1}^{2} \Big[\int_0^{L_k} \{ & u^2 + 2u\phi_{nk}(x_f)\, \dot{q}_n(t)\, \phi_{nk}(x_f)\, q_n(t) \\ & + \{\phi_{nk}(x_f)\, \dot{q}_n(t)\}\} dx_f \Big]\ (x_f = ut,\ 0 \le x_f \le L) \end{aligned} \quad (11)$$

where m_f is the fluid mass per unit length of a pipe. The work of a follower force due to the fluid discharge is divided into two kinds of work, one is the work done by conservative force component, and the other is the work done by non-conservative force component. The work W_c due to the conservative component of a tangential follower force is

$$W_c = \frac{1}{2} \sum_{n=1}^{\mu} \sum_{k=1}^{2} \int_0^{L_k} m_f u^2 \{ \phi_{nk}'(x_f)\, q_n(t) \}^2 dx_f \quad (12)$$

The work δW_{nc} due to the non-conservative component of a follower force is

$$\delta W_{nc} = -\sum_{n=1}^{\mu} m_f u^2 \{ \phi_{n2}'(L)\, \phi_{n2}(L) \}\, q_n(t)\, \delta q_n(t) \quad (13)$$

2.3 Crack modeling

Consider the bending vibrations of a uniform Euler-Bernoulli beam in the $x-y$ plane, which is assumed to be a plane of symmetry for any cross-section. The crack is assumed to be always open during vibrations. The additional strain energy due to the crack can be considered in the form of a flexibility coefficient expressed in terms of the stress intensity factor, which can be derived by Castigliano's theorem in the linear elastic range. Therefore the local flexibility in the presence of the width $2b$ of a crack is defined by

$$C_{ij} = \frac{\partial u_{ij}}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j} \left(\int_{-b}^{b} \int_0^{a_c} J(a)\, da dz \right) \quad (14)$$

where P_i is the load in the same direction as the displacement and $J(a)$ is the strain energy density function. The function is

$$J(a) = \frac{1}{E^*} (K_{IP} + K_{IM})^2 \quad (15)$$

where $E^* = E/(1-\nu_p^2)$ for the plane strain, and ν_p is Poisson's ratio. K_{IP} and K_{IM} are the stress intensity factor for the fracture mode (I) due to force P and moment M, respectively. The stress intensity factors are given by

$$\begin{aligned} K_{IP} &= \frac{P}{2\pi R t_p} \sqrt{\pi R \theta_c}\, F_t(\theta_c) \\ K_{IM} &= \frac{M}{\pi R^2 t_p} \sqrt{\pi R \theta_c}\, F_b(\theta_c) \end{aligned} \quad (16)$$

where $R = (R_o + R_i)/2$ is the mean radius. θ_c is the half-angle of the total through-wall crack, and

$$\begin{aligned} F_t(\theta_c) &= 1 + A_t \left[5.3303 \left(\frac{\theta_c}{\pi} \right)^{1.5} + 18.773 \left(\frac{\theta_c}{\pi} \right)^{4.24} \right] \\ F_b(\theta_c) &= 1 + A_t \left[4.5967 \left(\frac{\theta_c}{\pi} \right)^{1.5} + 2.6422 \left(\frac{\theta_c}{\pi} \right)^{4.24} \right] \end{aligned} \quad (17)$$

where

$$\begin{aligned} A_t &= \left(0.125 \frac{R}{t_p} - 0.25 \right)^{0.25} \quad \text{for } 5 \le \frac{R}{t_p} \le 10 \\ A_t &= \left(0.4 \frac{R}{t_p} - 3.0 \right)^{0.25} \quad \text{for } 10 \le \frac{R}{t_p} \le 20 \end{aligned} \quad (18)$$

where t_p is the thickness of the pipe. Substituting equations (15)-(18) into equation (14), the flexible matrix due to the crack can be obtained.

2.4 Equation of motion

2.4.1 Dimensionless equation of motion

The equation of motion of the system is obtained by substituting the above work and energy functions into the Lagrange's equation.

For simplicity, the following dimensionless parameters are introduced :

$$\xi=\frac{x}{L},\ \xi_f=\frac{x_f}{L}=uL\sqrt{\frac{m}{EI}}\tau,\ \tau=\frac{t}{L^2}\sqrt{\frac{EI}{m}}$$

$$M_m=\frac{m_m}{mL},\ U=uL\sqrt{\frac{m_f}{EI}},\ \xi_c=\frac{x_c}{L}$$

$$V=v\sqrt{\frac{m_m L}{EI}},\ M_f=\frac{m_f}{m},\ M_p=\frac{m_p}{mL}$$

$$\xi_m=\frac{x_m}{L}=vL\sqrt{\frac{m}{EI.}}\tau,\ K_R^*=\frac{K_R L}{EI},\ J_o^*=\frac{J_o}{mL^2}$$

$$\theta^*=\frac{\theta_c}{\pi},\ \mu_m=\frac{M_p}{J_o^*}=\frac{m_p L^2}{J_o} \tag{19}$$

where m is the mass per unit length of the cantilever pipe. The dimensionless $w(x,\ t)$ is given by

$$\bar{w}=\frac{w}{L}=\sum_{n=1}^{\mu} d_n(\tau)\,\phi_n(\xi) \tag{20}$$

Therefore, the dimensionless equation of motion is obtained matrix form as follows :

$$\mathbf{M}\ddot{\mathbf{d}}+\mathbf{C}\dot{\mathbf{d}}+\mathbf{K}\mathbf{d}=\mathbf{0} \tag{21}$$

where $(\cdot)$ denotes $\partial/\partial\tau$. The matrices of the equation (21) can be written as follows :

$$\mathbf{M}=\sum_{n=1}^{\mu}\sum_{k=1}^{2}\Big\{\int_0^{L_k^*}\phi_{nk}^2(\xi)\,d\xi +M_f\int_0^{L_k^*}\phi_{nk}^2(\xi_f)\,d\xi_f+M_m\phi_{nk}^2(\xi_m) +M_p\phi_{n2}^2(1)+J_o^*\{\phi'_{n2}(1)\}^2\Big\} \tag{22a}$$

$$\mathbf{C}=\sum_{n=1}^{\mu}\sum_{k=1}^{2}\Big\{M_f\int_0^{L_k^*}\frac{d}{d\tau}\{\phi_{nk}^2(\xi_f)\}\,d\xi_f +M_m\frac{d}{d\tau}\{\phi_{nk}^2(\xi_m)\}\Big\} \tag{22b}$$

$$\begin{aligned}\mathbf{K}=\sum_{n=1}^{\mu}\sum_{k=1}^{2}\Big[&\int_0^{L_k^*}\{\phi''_{nk}(\xi)\}^2 d\xi+\sqrt{M_m}\,V\Big\{\frac{d}{d\tau}\{\phi'_{nk}(\xi_m)\}\phi_{nk}(\xi_m)\\&+\frac{d}{d\tau}\{\phi_{nk}(\xi_m)\}\phi'_{nk}(\xi_m)\Big\}-V^2\{\phi'_{nk}(\xi_m)\}^2\\&+\sqrt{M_f}\,U\int_0^{L_k^*}\Big\{\frac{d}{d\tau}\{\phi'_{nk}(\xi_f)\}\phi_{nk}(\xi_f)\\&+\frac{d}{d\tau}\{\phi_{nk}(\xi_f)\}\phi'_{nk}(\xi_f)\Big\}\,d\xi_f\\&-U^2\int_0^{L_k^*}\{\phi'_{nk}(\xi_f)\}^2 d\xi_f+U^2\phi'_{n2}(1)\,\phi_{n2}(1)\\&+K_R^*\{\phi'_{n2}(\xi_2=0)-\phi'_{n1}(\xi_1=\xi_c)\}^2\Big]\end{aligned} \tag{22c}$$

where $(')$ stands for $\partial/\partial\xi$, $L_1^*=\xi_c$ and $L_2^*=1$.

2.4.2 Modal formulation

The equation (21) can be transformed into the following equation :

$$\mathbf{M}^*\dot{\boldsymbol{\eta}}+\mathbf{K}^*\boldsymbol{\eta}=\mathbf{0} \tag{23}$$

where

$$\mathbf{M}^*=\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix},\ \mathbf{K}^*=\begin{bmatrix}\mathbf{C} & \mathbf{K}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix},\ \boldsymbol{\eta}=[\dot{\mathbf{d}}\ \mathbf{d}]^T \tag{24}$$

where $\mathbf{I}$ represents a unit matrix. For the complex modal analysis, it is assumed that $\boldsymbol{\eta}$ is a harmonic function of τ expressed as

$$\boldsymbol{\eta}=e^{\lambda\tau}\boldsymbol{\Theta} \tag{25}$$

where λ is the eigenvalue, and $\boldsymbol{\Theta}$ is the corresponding mode shape. From the eigenvalues in the equations (23)-(25), the frequencies can be obtained.

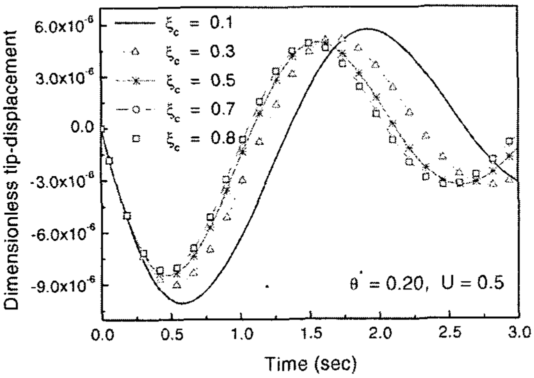
3. Numerical Results and Discussion

In this study, the dynamic behavior of the cracked cantilever pipe conveying fluid inluenced by the moving mass, the crack severity ratio θ^* $(=\phi/\pi)$, tip mass and the position ratio of the crack $\xi_c(=x_c/L)$. It is computed by the forth order Runge-Kutta method. To illustrate this response, the length of the pipe $L=1$ m, out- radius $R_o=0.03$ m and in-radius $R_i=0.02$ m were considered (Young's modulus=210 GPa, material density=7860 kg/m^3).

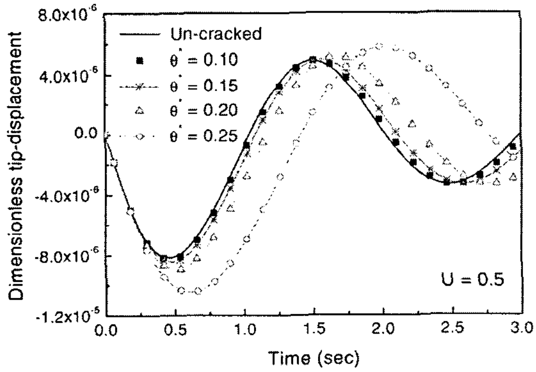
In this study, we have studied the dynamic behavior of the cracked cantilever pipe conveying fluid for the first mode of vibration. And the dimensionless velocity of the moving mass is constant as 6.8E-3($v=1$ m/s).

3.1 Results for without tip mass

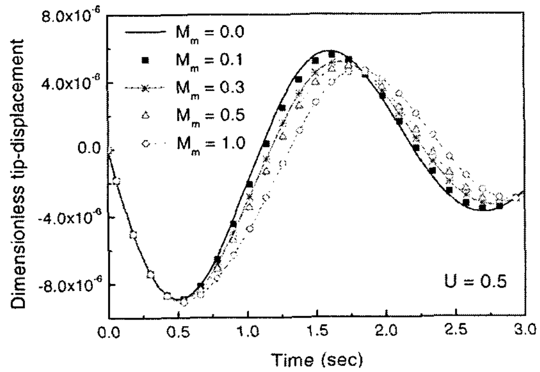
Figure 3 shows the dimensionless tip-displacement of a cracked cantilever pipe conveying fluid with $U=0.5$. The dimensionless velocity of moving mass is 6.8E-3. The horizontal axis is the scale of time, and the axis of the ordinates are the scale of dimensionless tip-displacement of the cracked cantilever pipe. Figure 3 (a) represents the effect of crack position on the tip-displacement of the cantilever pipe for $M_m=0.3$ and $\theta^*=0.2$. The crack position from the left-hand clamped end gradually moved to free end of the pipe with decreasing of the tip-displacement of the cantilever pipe conveying fluid. The difference of maximum tip-displacement of the cracked pipe in the two case of the crack position $\xi_c=0.1$ and $\xi_c=0.5$ is about 14.7%. The variation of tip-displacement of the cracked pipe according to the crack severity is shown in Fig. 3(b). In Fig. 3(b), the crack position is 0.3. Generally, the tip-diaplacement of the cracked pipe is proportional to the crack severity. As the crack severity increases, the time that makes the maximum tip-



(a) Effect of crack position ($M_m=0.3$, $\theta^*=0.2$)

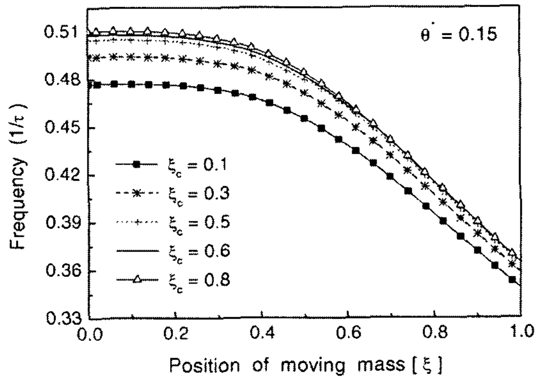


(b) Effect of crack severity ($M_m=0.3$, $\xi_c=0.3$)

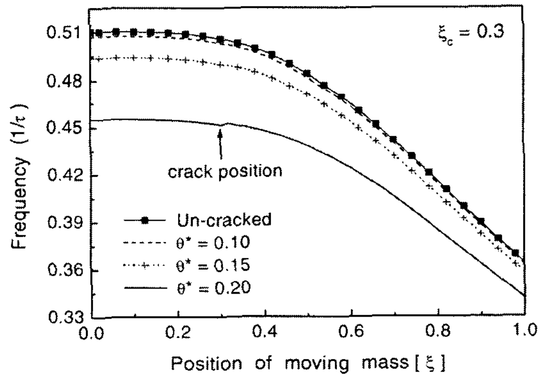


(c) Effect of moving mass ($\theta^*=0.15$, $\xi_c=0.3$)

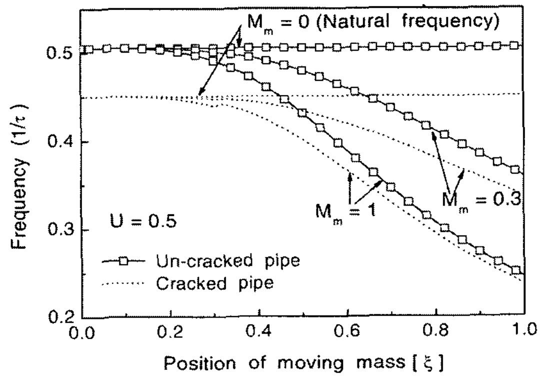
Fig. 3 Dimensionless tip-displacement of cantilever pipe conveying fluid ($V=6.8E-3$, $U=0.5$)



(a) Effect of crack position ($M_m=0.3$, $\theta^*=0.15$)



(b) Effect of crack severity ($M_m=0.3$, $\xi_c=0.3$)



(c) Effect of moving mass ($\xi_c=0.3$, $\theta^*=0.2$)

Fig. 4 Frequency of cantilever pipe conveying fluid ($V=6.8E-3$, $U=0.5$)

displacement of the cracked pipe is delayed. In the two case of $\theta^*=0.10$ and $\theta^*=0.25$, the time that appears the maximum tip-displacement of the cracked cantilever pipe is 1.51 sec and 2.13 sec, respectively. Figure 3(c) shows the dimensionless tip-displacement of a cracked cantilever pipe conveying fluid according to the moving mass. In Fig. 3(c), the cracke position and crack severity are 0.3 and 0.15, respectively. In this curves, as the moving mass increases, the maximum tip-deflection of the cracked cantilever pipe conveying fluid is increased. Figure 4 represents the frequencies of a cracked pipe coneying fluid with the moving mass for the first mode. The horizontal axis is the scale of position of moving mass, and the axis of the ordinates are the scale of frequency of the cracked cantilever pipe. Figure 4(a) shows the frequency of the cantilever pipe according to the crack position for $M_m=0.3$ and $\theta^*=0.15$.

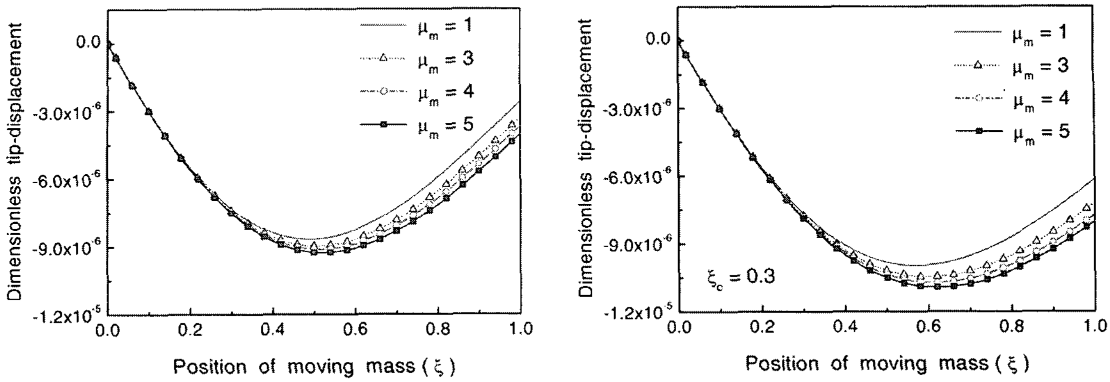
When the crack severity is constant, the frequency of the cantilever pipe conveying fluid is getting small as the moving mass moves to the free end of the pipe. The crack position from the left-hand clamped end gradually existed to the free end of the pipe with increasing the frequency of the cantilever pipe conveying fluid. Figure 4 (b) shows the effect of the crack severity on the frequency of the cracked cantilever pipe. The crack position is 0.3. The frequency of the cantilever pipe conveying fluid is in inverse proportion to the crack severity. The difference of natural frequencies of the cantilever pipe in the two case of $\theta^*=0$ and $\theta^*=0.2$ is about 12.2%. And when position of the moving mass exists in the free end of cantilever pipe conveying fluid, the differecne of frequencies of the cantilever pipe is about 6.72%. The variation of frequency of the cracked cantilever pipe conveying fluid according to the moving mass is shown in Fig. 4(c). In curves, the position and severity of a crack are 0.3 and 0.15, respectively. When $M_m=0$, the difference of the natural frequency of the cantilever pipe between cracked pipe ($\theta^*=0.2$) and uncracked pipe is about 11.15%. When the position of the moving mass exists in the free end of cantilever pipe conveying fluid, the frequency of the cantilever pipe conveying fluid is more sensitive to the moving mass than to the effect of the crack. Table 1 represents a comparison between the present results and others for natural frequency ratio of a cantilever beam without a moving mass and a fluid flow. In order to compare the present results and others, the dimensions of the beam with the rectangular cross-section are $L=0.2$, the height of beam $h=0.0078$ and $d=0.025$ m.

3.2 Results for with tip mass

Figures 5 and 6 show the dimensionless tip-displacement of a cracked cantilever pipe conveying fluid with a tip mass for $U=0.5$. The horizontal axis scale is the position of a moving mass, and the axis of the ordinates are the scale of dimensionless tip-displacement of the cracked cantilever pipe. Figures 5(a) and (b) represent the tip-displacement of the cantilever pipe conveying fluid with a moving mass according to the ratio of the tip mass to the its moment of inertia. When the crack position is constant, as the crack

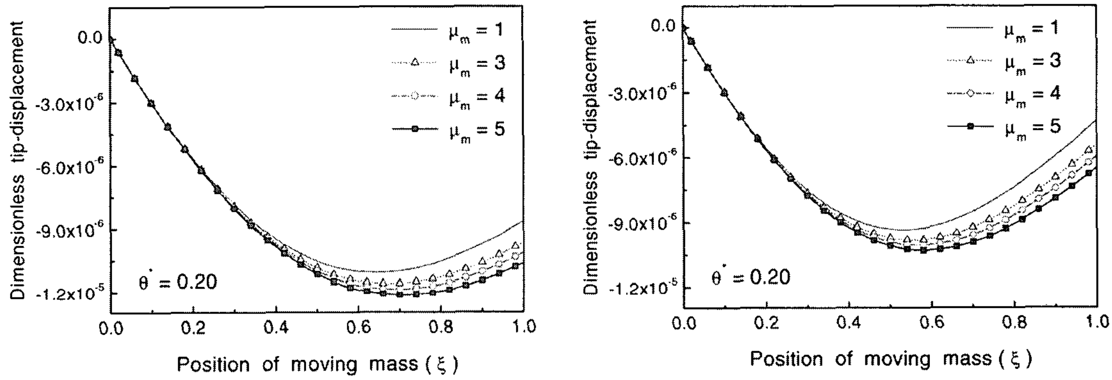
Table 1 Comparison of present results and others for natural frequeny ratio (cracked/uncracked) of a cantilever beam

	Crack position (ξ_c)	Crack depth $\left(\frac{a_c}{h}\right)$			
		0.2	0.25	0.4	0.6
Present result	0.2	0.99276	0.98803	0.96633	0.80523
	0.4	0.99707	0.99512	0.98530	0.91074
Kisa et al. (2000)	0.2	0.9837	–	0.9614	0.8122
	0.4	0.9933		0.9709	0.9091
Shen et al. (1990)	0.2	–	0.9817	0.9520	0.8213



(a) Un-cracked pipe (b) $\theta^*=0.20$

Fig. 5 Dimensionless tip-displacement of craked cantilever pipe according to tip mass and crack severity (V=6.8E-3, U=0.5)



(a) $\xi_c=0.1$ (b) $\xi_c=0.5$

Fig. 6 Dimensionless tip-displacement of craked cantilever pipe according to tip mass and crack position (V=6.8E-3, U=0.5)

severity increases, the dimensionless tip-displacement of the cantilever pipe conveying fluid is increased. Generally, the tip-diaplacement of the cantilever pipe is proportional to the ratio μ_m. As the ratio μ_m increases, the position of the moving mass that makes the maximum tip-displacement of the cantilever pipe is moved to the free end of the cnatilever pipe. In Fig. 6(a) for $\mu_m=1$ and $\mu_m=4$, the maximum tip-displacement of the cracked cantilever pipe occurs at $\xi=0.53$ and $\xi=0.59$, a distance from the left-hand clamped end, respectively. Figures 7 and 8 show the frequency of the cracked cantilever pipe according to the tip mass. The effect of the ratio μ_m on the frequency of the cracked cantilever pipe for $\xi_c=0.3$ and $\theta^*=0.1$ is shown in Fig. 7. Totally, the frequencies of the cracked cantilever pipe are in inverse

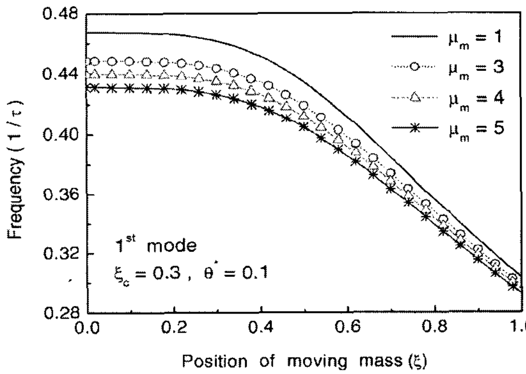
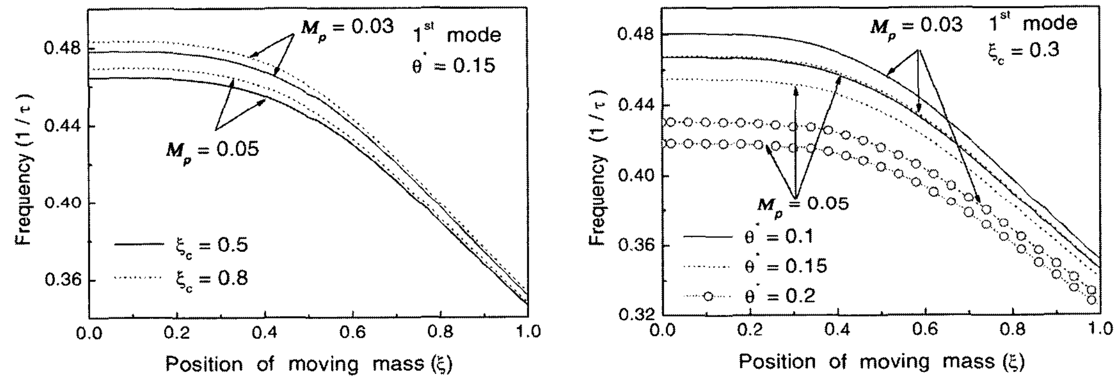


Fig. 7 Frequency of cracked cantilever pipe according to the ratio (μ_m) for first mode (V=6.8E-3, U=0.5)

proportion to the ratio μ_m. The difference of frequencies of the cracked cantilever pipe in the two

Table 2 Frequency of cracked cantilever pipe conveying fluid with tip mass and its moment of inertia for first mode (U=0.5, M_m=0.3, ξ=0.5)

J_o^*	M_p	θ^*=0.15			ξ_c=0.3		
		ξ=0.1	ξ_c=0.5	ξ_c=0.8	θ^*=0.10	θ^*=0.15	θ^*=0.20
0	0.03	0.4322	0.4558	0.4602	0.4581	0.4477	0.4159
	0.05	0.4212	0.4440	0.4485	0.4464	0.4361	0.4049
	0.10	0.3968	0.4183	0.4225	0.4206	0.4107	0.3809
0.01	0.03	0.4219	0.4445	0.4490	0.4470	0.4367	0.4054
	0.05	0.4115	0.4336	0.4380	0.4362	0.4259	0.3952
	0.10	0.3887	0.4095	0.4138	0.4119	0.4021	0.3728
0.05	0.03	0.3867	0.4066	0.4109	0.4092	0.3994	0.3701
	0.05	0.3787	0.3982	0.4025	0.4007	0.3911	0.3623
	0.10	0.3607	0.3793	0.3834	0.3817	0.3725	0.3448



(a) Effect of crack position (b) Effect of crack severity

Fig. 8 frequency of cracked cantilever pipe according to tip mass

case of μ_m=3 and μ_m=5 is about 6.25%. Figures 8(a) and (b) show the effect of crack position and crack severity on the frequency of the cracked cantilever pipe, respectively. In Fig. 8, the velocity of fluid is 0.5 and a moving mass M_m is 0.3. In Fig. 8, the frequencies of the cracked cantilever pipe are in inverse proportion to the tip mass. Table 2 represents the frequencies of the cracked cantilever pipe conveying fluid with the tip mass for M_m=0.3, U=0.5 and ξ=0.5. In Table 2, the unit of frequency is $1/\tau$.

4. Conclusions

In this paper, the influences of the crack severity and the tip mass have been studied on the dynamic behavior of the cracked cantilever pipe conveying fluid with a moving mass by the numerical method. The cantilever pipe is modeled by the Euler-Bernoulli beam theory. The equation of motion is derived by using Lagrange's equation. The cracked pipe has been treated as two undamaged segments connected by a rotational elastic spring at the cracked section. The stiffness of the spring depends on the crack severity and the geometry of the cracked section. When the velocity of a moving mass is constant, the influences of a moving mass, the velocity of fluid flow, the crack, the tip mass and its moment of inertia and the coupling of these factors on the frequencies and tip-displacement of the cantilever pipe are depicted. The main results of this study are summarized as follows :

(1) The tip-displacement of the cracked pipe is proportional to the crack severity. As the crack

severity increases, the time that makes the maximum tip-displacement of the cracked pipe is delayed.

(2) When the crack severity is constant, the frequency of the cantilever pipe is getting small as the moving mass moves to the free end of the cantilever pipe.

(3) When the position of the moving mass exists in the free end of cantilever pipe, the frequency of the cantilever pipe conveying fluid is more sensitive to the moving mass than to the effect of the crack.

(4) As the ratio of tip mass to its moment of inertia μ_m is increased, the position of the moving mass that makes the maximum tip-displacement of the pipe is moved to the free end of the cantilever pipe.

(5) The frequencies of the cracked cantilever pipe are in inverse proportion to the ratio μ_m.

(6) These study results will contribute to the safety test and stability estimation of structures including a cracked pipe conveying fluid with a moving mass and a tip mass.

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