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VEHICLE SPEED ESTIMATION BASED ON KALMAN FILTERING OF ACCELEROMETER AND WHEEL SPEED MEASUREMENTS

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ABSTRACT−This paper deals with the algorithm of estimating the longitudinal speed of a braking vehicle using measurements from an accelerometer and a standard wheel speed sensor. We evolve speed estimation algorithms of increasing complexity and accuracy on the basis of experimental tests. A final speed estimation algorithm based on a Kalman filtering is developed to reduce measurement noise of the wheel speed sensor, error of the tire radius, and accelerometer bias. This developed algorithm can give peak errors of less than 3 percent even when the accelerometer signal is significantly biased.

KEY WORDS : Real vehicle speed, Wheel speed, Kalman filter, Fuzzy logic, Slip ratio, ABS, Velocity estimator

1. INTRODUCTION

An anti-lock braking system (ABS) controls wheels not to lock during hard or emergency braking. This control results in reducing braking distance as well as maintaining steerability and stability of a vehicle. The technologies of the ABS are also applied in traction control systems (TCS) and vehicle dynamic stability control (VDSC). The ABS uses longitudinal slip ratio s_i at four wheels of a vehicle, which are defined as:

$$s_i = \frac{r\omega_i - v}{v} \quad i = 1, 2, 3, 4 \tag{1}$$

where v is the vehicle speed, ω_i is the the angular speed of the wheels and r_i is the radii of the tires. The ABS looks for the large slips and high wheel accelerations associated with impending lock-up and then lower the brake pressure to prevent it.

The longitudinal slip ratio is related to longitudinal tire force through tire models like the magic formula tire model [2]. Figure 1 shows a plot of normalized longitudinal tire force versus longitudinal slip ratio for traction on several road surfaces, obtained using the magic formula. A critical slip ratio is about 15 percent, where maximum frictions are obtained. The slip curve shows that the tire force is typically an increasing function of slip ratio until the critical slip ratio. After this critical slip ratio, more slip leads to a decrease in tire force and wheel lock-up. Control algorithms of the ABS are designed to use the slip curve concept and thus, require precise calculation of the slip ratio in equation (1). For performance of the ABS, it is essential to measure the vehicle and the wheel speeds precisely.

According to some research results, it may even be possible to estimate tire/road coefficient of friction using slip information. Several researchers (Dieckmann, 1992; Gustafsson, 1997; Yi *et al.*, 1999; Hedrick and Uchanski, 2001; Uchanski, 2001; Müller *et al.*, 2002) have demonstrated that there may be potential for a slip-based estimator of the maximum tire/road coefficient of

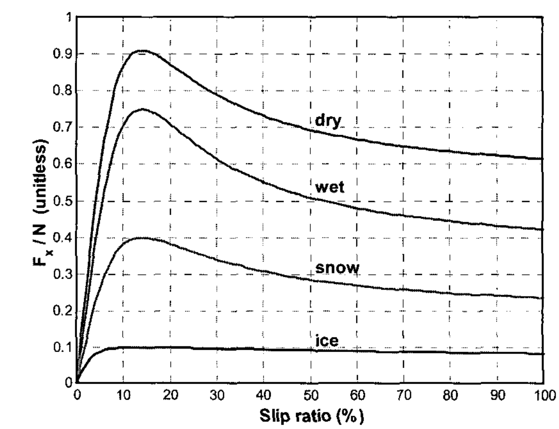

Figure 1. Simulated slip curves on several surfaces.

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Figure 2. The 5[th] wheel attached on the test vehicle.

friction, μ_{max}. A slip-based μ_{max} estimator attempts to use low slip and low tire force data from normal driving to determine the maximum amount of friction that is available to the driver. It therefore demands precise speed estimation and slip calculation. Although this is possible for two wheel drive vehicles in traction using standard wheel sensors (Dieckmann, 1992), it is not yet possible for braking vehicles without a ground reference speed sensor.

Strategies to estimate vehicle speed fall into two main categories: ground reference techniques and non-ground reference techniques. Ground reference techniques tend to be more accurate, but they also tend to be more expensive than non-ground reference techniques. Some of them include an optical cross-correlation sensor, a radar sensor, a global positioning system (GPS) (Miller *et al.*, 2001; Bevly *et al.*, 2001), and a 5[th] wheel sensor which uses a bicycle-like wheel mounted from a spring loaded arm on the back of the car (Figure 2). Because the wheel does not slip and its radius is well known, the 5[th] wheel sensor can measure the vehicle speed accurately (Oh and Song, 2002; Song, Hwang and Hedrick, 2002).

A wheel sensor is a frequently used non-ground sensor, where the vehicle speed is measured by multiplying the wheel angular speed by the effective tire radius. Recently, an accelerometer is introduced to estimate the vehicle speed more accurately. The accelerometer has some merits such as small size, low cost, and easy implementation. Several technologies of vehicle speed estimation were developed using Kalman filters (Kobayashi *et al.*, 1995) or fuzzy logic (Daiss and Kiencke, 1995; Basset *et al.*, 1997) to combine accelerometers and wheel sensors.

In this paper, a standard 50 tooth wheel sensor and a longitudinal accelerometer are used to get a high precision estimate of the longitudinal vehicle speed. We have chosen to develop the algorithms using wheel speed data only from the left front wheel. This is because it is possible to see a direct relationship between the behavior of the one wheel and the speed estimate. All results in the paper are calculated using experimental data from straight-line braking maneuvers using a rear wheel drive test vehicle. In addition to the accelerometer and wheel sensor, the vehicle is outfitted with a fifth wheel to provide a ground speed reference.

The obvious intuitive solution to the one wheel sensor estimation problem is: When tire slip is low, calculate vehicle speed using the wheel speed sensor; when tire slip is high, calculate it by integrating the accelerometer signal. Three speed estimation methods are presented on the basis of this strategy, but differ in how they implement it. The first method described in Section 2 is introduced as a basic algorithm to combined measurement data from a wheel sensor and an accelerometer. The second method, which is developed in Section 3, uses a Kalman filtering concept to reduce measurement noise. The final method, which is proposed in Section 4, uses regression to simultaneously identify the effective tire radius and the accelerometer bias, and it is found to deliver very good speed estimates, even in the presence of parameter changes.

2. BASIC ESTIMATION OF THE VEHICLE SPEED

Since the wheel speed signal is from the front wheel of a rear wheel drive car, its slip is negligible whenever the car is not braking. Therefore, when the car is not braking, which can be easily detected using the existing brake light circuit, we calculate the basic speed estimate at time step k as $v_{basic}(k) = r_{est} \cdot \omega(k)$, where r_{est} is the estimate of the effective tire radius, and $\omega(k)$ is the angular speed of the wheel. When the brakes are activated, we get the basic speed estimate at time step k by numerically integrating the accelerometer according to

$$v_{basic}(k) = v_{basic}(k_{no\_brake}) + \sum_{i=k_{no\_brake}}^{k} a_{meas}(i) \cdot \Delta t \qquad (2)$$

where k_{no_brake} is the last time index at which there is no braking, $a_{meas}(i)$ is the acceleration measurement, and Δt is the sample time interval.

Figure 3 shows the performance of this basic vehicle speed estimation algorithm. The maneuver was a straight line acceleration for approximately 5 seconds followed by braking of increasing intensity until the wheels locked at approximately 8 seconds. The true speed of the vehicle from the 5[th] wheel is shown by the thick gray line, and the speed of the braking wheel is shown by the thick line that drops abruptly to 0 m/s at approximately 8 s.

In the case when the accelerometer has no bias of 0 m/s^2, the estimate of the basic algorithm is quite good. However, the accelerometer bias of 0.5 m/s^2 makes the basic speed estimate unacceptable. Percent speed esti-

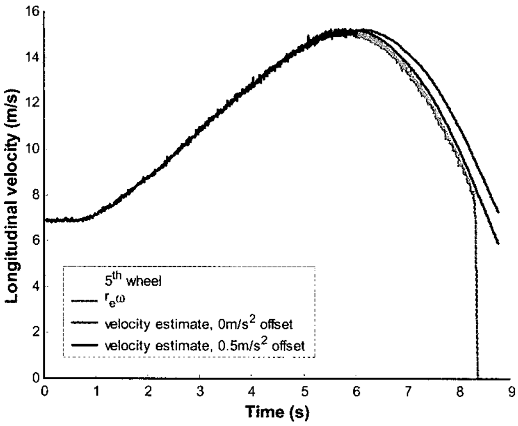

Figure 3. Speeds estimated form the basic estimator.

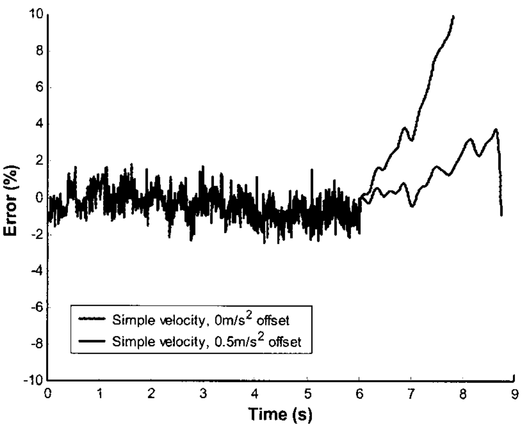

Figure 4. Errors of speeds estimated from the basic estimator.

mation errors are plotted in figure 4. The percent speed estimation error with no bias is less than 2 percent for the majority of the maneuver and only reaches a peak value of 4 percent when the wheel is locked. Note that low vehicle speed makes it easier for small absolute errors to translate to large percent errors. When the accelerometer is biased by 0.5 m/s^2, the vehicle speed estimate starts to diverge from the actual speed during braking after approximately 6 seconds. It quickly diverges to errors of 10 percent or more, rendering the basic speed estimate inutile for most uses.

Unfortunately, biases do occur quite frequently in acceleration measurements, so the basic estimator's lack of robustness to them is a serious difficulty. Some sources of longitudinal accelerometer bias include road slope, temporary pitch angle changes resulting from longitudinal accelerations, and longer-term pitch angle changes due to vehicle loading, active/semi-active suspension behavior, and suspension aging. Of these factors, road slope (Daiss and Kiencke, 1995) is typically the most important. The 0.5 m/s^2 accelerometer bias used here simulates the effect of a grade of 6 percent, which occasionally occurs on even the highest quality roads.

Similarly, this basic estimation algorithm is not robust to errors and changes in the dynamic tire radius. It is straightforward to show that during non-braking phases, the percent error in the basic speed estimate is equal to

$$\frac{r_{est} - r}{r} \times 100 \tag{3}$$

where r is the true effective tire radius, and r_{est} is the estimate of the effective tire radius used in the basic estimator. Thus, a two percent error in the effective tire radius estimate results in a constant two percent error in the speed estimate. Fortunately, the effective tire radius tends to change very little and very slowly under most circumstances.

A final difficulty with this basic estimation algorithm is that the noise of the wheel speed measurement passes directly through to the speed estimate during non-braking phases. This can be seen in Figure 3 as high frequency noise on the percent error for times less than six seconds. This is not a serious problem in these particular tests because the wheel speed noise is reasonably small. However, it warrants attention for two reasons. First, it is a problem that grows directly with the sensor noise level, so it could become a problem when the sensor noise level is high. Second, it is a problem that can be remedied by utilizing redundant sensor information and standard theoretical techniques.

3. SPEED ESTIMATION BASED ON KALMAN FILTER

A standard technique that is sometimes used for fusing imperfect measurements and system models to make less noisy is the Kalman filter (Anderson and Moore, 1979; Hayes, 1996). It is used in circumstances similar to ours in Daiss and Kiencke (1995) and Kobayashi, Cheok and Watanabe (1995). To attenuate the wheel sensor noise, we apply Kalman filtering concept to redundant information to obtain from the two different sensors of the wheel sensor and the accelerometer.

We model the vehicle as a difference equation that performs a discrete integration of the measured vehicle acceleration $a_{meas}(k)$ plus a noise term $w(k)$ to arrive at the true vehicle speed, $v(k)$. That is,

$$v(k) = v(k-1) + \Delta t \cdot a_{meas}(k) + w(k) \tag{4}$$

At each sample interval, we have a noisy measurement $\omega(k)$ of the wheel speed, which is formed from $v(k)$ and the estimated effective radius r_{est} according to the measurement equation.

$$\omega(k) = 1/r_{est} \cdot v(k) + n(k) \quad (5)$$

If the noise processes $w(k)$ and $n(k)$ were white, then the least squares optimal estimate of $v(k)$ given the measurement of $\omega(k)$ and all of its predecessors would take the form:

$$\hat{v}(k) = K \cdot r_{est} \cdot \omega(k) + (1-K)(\hat{v}(k-1) + \Delta t \cdot a_{meas}(k)) \quad (6)$$

where $\hat{v}(k)$ is the optimal estimate and K is a specially chosen gain called the Kalman gain. Of course, the noise in our situation is not white, but this same general form taken from Kalman filtering theory still provides vehicle speed estimates that tend to be better than the estimates obtained using just the accelerometer or just the wheel speed measurement.

The speed estimator of equation (6) is a more general case of the basic estimator of the previous section. For the basic estimator, K is either 0 for braking or 1 for not braking. Therefore, K can be determined by the slip ratio and plays a role as a weighting parameter. For this reason, we refer to this speed estimator as a Kalman filter-like speed estimator. Following the intuition of the basic estimation algorithm, K should not be the same under all circumstances. When the wheel slip is negligible, it should be closer to one, while it should be closer to zero when braking hard. We make K vary between 0.09 when the wheel slip is low to 0.0 when the wheel slip is high. Figure 5 shows K according to slip ratio $s(k)$, which is calculated using the current wheel speed and the previous vehicle speed estimate.

$$s(k) = \frac{r_{est}\omega(k) - \hat{v}(k-1)}{\hat{v}(k-1)} \times 100(\%) \quad (7)$$

Figure 6 shows speed estimation results of the Kalman filter-like estimator. The maneuver is the same straight line maneuver as that of Figure 3 of the previous section.

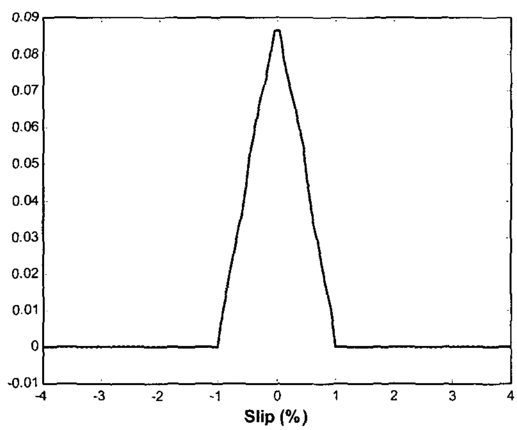

Figure 5. Weighting parameter K according to slip.

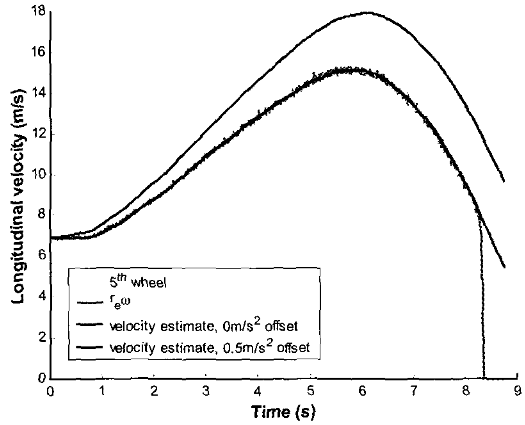

Figure 6. Speeds estimated form the Kalman filter like estimator.

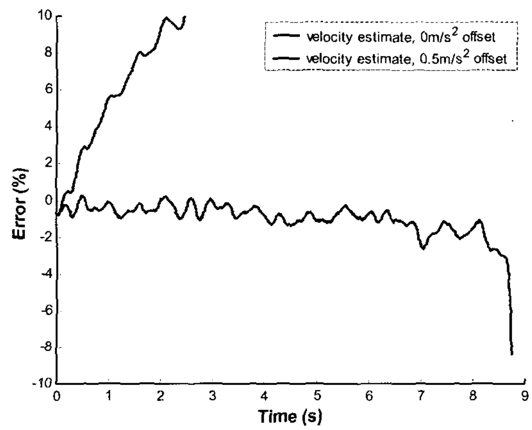

Figure 7. Errors of speeds estimated from the Kalman filter like estimator.

The 5[th] wheel signal gives the true speed reference, and the signal that drops to zero is the wheel speed signal multiplied by the effective radius estimate r_{est}. When the accelerometer has no bias, the Kalman filter-like estimation algorithm gives results with very little noise and good accuracy. As Figure 7 shows, the speed estimation error remains less than 2 percent for almost the entire maneuver. However, when an accelerometer bias of 0.5 m/s^2 is introduced, this algorithm, which relies heavily on the accelerometer measurement, gives large errors as shown in Figure 6 and Figure 7. When more weight was assigned to the wheel speed signal, the results were subjectively the same as those shown here.

4. SPEED ESTIMATION WITH BIAS AND RADIUS CORRECTOR

In the experimental results of the previous section, the Kalman filter-like estimator can successfully attenuate noise, but it heavily depends on the accelerometer bias. Furthermore, the algorithm still relies on an accurate estimate of the effective tire radius r_{est}. To correct these problems, this section adds a parameter estimator to the Kalman filter-like estimator. The parameter estimator calculates the effective wheel radius r and the accelerometer bias $\varepsilon(k)$ defined as

$$\varepsilon(k) = a_{meas}(k) - a(k)\,. \tag{8}$$

The two measurements available to aid in the parameter estimation are the wheel speed $\omega(k)$ and the measured acceleration $a_{meas}(k)$. Whenever there is no slip at the wheel, we can relate the unknown parameters to known quantities by the regression equation

$$\dot{\omega}(k) = [a_{meas}(k) \; -1]\begin{bmatrix} 1/r \\ \varepsilon/r \end{bmatrix}. \tag{9}$$

where $\dot{\omega}(k)$ is the numerical derivative of $(\omega(k) - \omega(k-1))/\Delta t$.

We can solve for the parameters ε/r and $1/r$ using either the standard least squares formula, recursive least squares, or a Kalman filter. The first two techniques have the advantage that they give least squares optimal solutions. However, the Kalman filter has the advantage that it allows one to track time varying parameters and to incorporate a prior knowledge of their relative volatility. In this particular problem, the parameters are time varying, and we understand their behavior fairly well. The radius r tends to change very little and very slowly, while the bias ε can change by much larger amounts in fairly short periods of time, which of an example is when the traveling vehicle encounters a hill. Therefore, we use a Kalman filter to solve the regression problem.

Similar to Gustafsson (1997), who discusses the use of Kalman filters to solve regression problems in more detail, we assume the parameters evolve according to the difference equation

$$\begin{bmatrix} \frac{1}{r}(k) \\ \frac{\varepsilon}{r}(k) \end{bmatrix} = \begin{bmatrix} \frac{1}{r}(k-1) \\ \frac{\varepsilon}{r}(k-1) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \tag{10}$$

where $w_1(k)$ and $w_2(k)$ are white noise processes that cause the otherwise constant parameters to change. We have a noisy measurement $\dot{\omega}(k)$ that is formed from the true parameters according to

$$\dot{\omega}(k) = \underbrace{[a_{meas}(k) \quad -1]}_{C(k)}\begin{bmatrix} 1/r \\ \varepsilon/r \end{bmatrix} + n(k) \tag{11}$$

where $n(k)$ is the measurement noise, which is substantial due to the numerical differentiation. The Kalman filter takes the form

$$\begin{bmatrix} \frac{1}{r}(k) \\ \frac{\hat{\varepsilon}}{r}(k) \end{bmatrix} = \begin{bmatrix} \frac{1}{r}(k-1) \\ \frac{\hat{\varepsilon}}{r}(k-1) \end{bmatrix} + K_{2\times 1}\left(\dot{\omega}(k) - C(k)\begin{bmatrix} \frac{1}{r}(k-1) \\ \frac{\hat{\varepsilon}}{r}(k-1) \end{bmatrix}\right) \tag{12}$$

where the hats denote estimates, and where $K_{2\times 1}$ is a time varying gain matrix gotten from the standard Kalman filter formulation (Hayes, 1996). To reflect the fact that the differentiated angular speed extremely noisy and that the accelerometer bias is far more volatile than the tire radius, the covariance of $n(k)$ associated with the measurement is chosen to be very large compared to those of $w_1(k)$ and $w_2(k)$. And the covariance of $w_1(k)$ is chosen to be still smaller than that of $w_2(k)$.

To ensure that the parameters ε/r and $1/r$ are only estimated when there is negligible slip at the wheel, their estimation is frozen when the slip surpasses a threshold value of +/–2 percent. Whenever the estimation is frozen, the parameters are set to their mean value over the past 2 seconds. The Kalman filter-like estimator of equation (6) is then used in the previous section, but wherever $a_{meas}(k)$ is used, it is replaced by $a_{meas}(k) - \hat{\varepsilon}(k)$ and wherever r_{est} is used, it is replaced by $\hat{r}(k)$.

Figure 8 and Figure 9 show the speed estimation results from combining the parameter estimator and the Kalman filter-like estimator. In case of no accelerometer bias, the error is less than 2 percent for most of the test, and the high frequency noise in the estimate is attenuated. These results are comparable with the no-bias results from the previous two methods. This indicates that parameter estimation do not degrade the speed estimation performance. Even when the accelerometer bias is 0.5 m/s^2, the speed estimate is very close to the 5th wheel speed and there is only a very minor degradation in

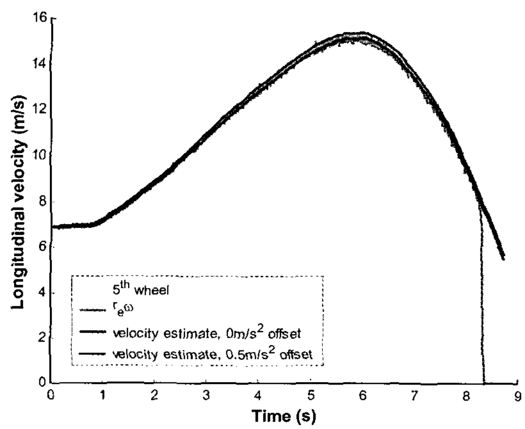

Figure 8. Speeds estimated with radius and bias correction.

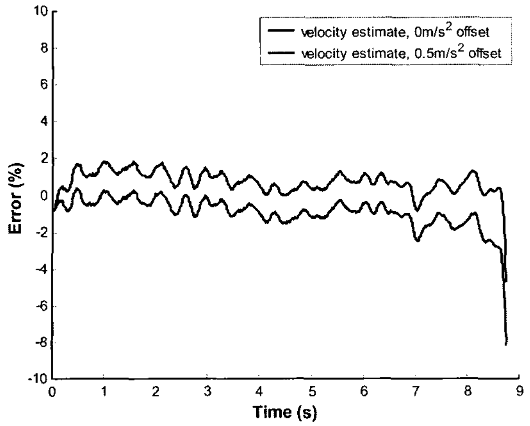

Figure 9. Error of speeds estimated with radius and bias correction.

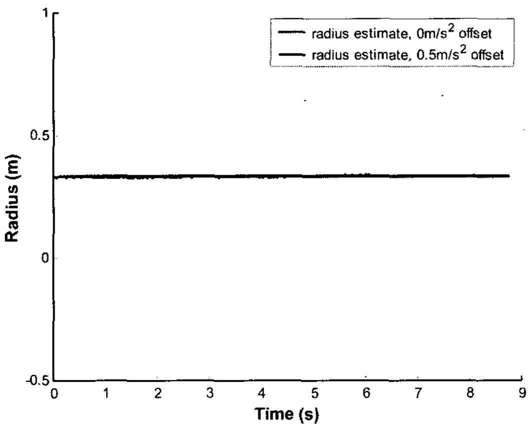

Figure 10. Radius estimates of the tire.

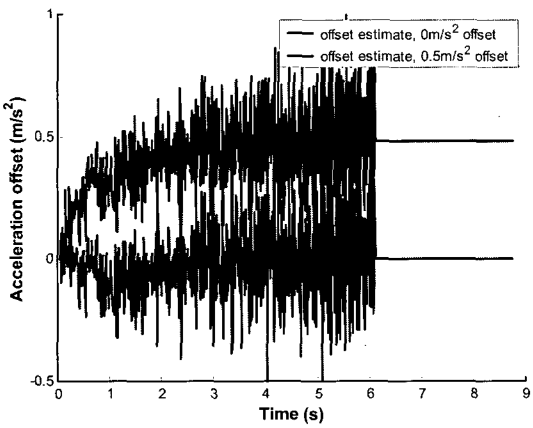

Figure 11. Bias estimates of the accelerometer.

performance, whereas there was a major breakdown in performance for the two previous algorithms. Peak error remains less than 3 percent for the duration of the test.

The estimation results of the effective radius and accelerometer bias are shown in Figure 10 and Figure 11, respectively.

The tire radius estimates are near 0.33 regardless of the two accelerometer biases and make very small scale excursions from their starting values. In both case the accelerometer bias estimate is close to the correct value, but slightly high. This is consistent either with a slight positive underlying accelerometer bias that was not adjusted out before the tests.

5. CONCLUSION

We developed the high precision speed estimator based on the Kalman filter. The developed estimator evolved out of the basic estimator, which of strategy is that during low slip the vehicle speed is calculated using the wheel speed sensor and during high slip it is calculated by integrating the accelerometer signal. We analyzed the experimental results of the basic estimator, which showed that the vehicle speed estimates were affected by the accelerometer bias, the wheel sensor noise, and error of the tire radius. The Kalman filter-like estimator was proposed for reduction of the wheel sensor noise. In addition, we applied the Kalman filter to estimation of the accelerometer bias and the tire radius. The performance of the developed estimator was evaluated trough the experiment with the severe accelerometer bias, where the peak estimation errors less than the order of 3 percent.

Most vehicles offer four wheel speeds for a speed estimation algorithm to use. In practical applications, the vehicle speed can be estimated more accurately. This is because of the flexibility to choose the wheel sensor with the least slip. However, when all four wheels are slipping, the basic difficulty of this paper may put a limit on the precision level that is available with just an accelerometer and wheel sensors.

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