Topology Optimization of Linear Motor for Rope-less Elevator by Using Density Method and ON/OFF Method

Yoshifumi Okamoto* and Norio Takahashi*

Abstract - The reduction of the ripple of driving force is especially required for the practical utilization of linear synchronous motor for rope-less elevator. In this paper, the magnetic region of the linear motor is optimized by using topology optimization techniques (density method and ON/OFF method) in order to reduce the ripple of driving force. The optimal results of both methods are compared, and useful information for the optimal design of linear motor is obtained.

Keywords: density method, ON/OFF method, topology optimization, core-less linear synchronous motor, adjoint variable method

1. Introduction

In the conventional optimization problem, such as size and shape optimization [1] the outline of magnetic circuit should be given beforehand. The topology optimization based on the density method [2], [3] does not need such an initial shape of magnetic circuit, and also the knowledge of experience. This method provides useful information for engineers in order to start the design of electromagnetic machines.

In the conventional density method [2], [3], the material density is the design variable which changes continuously from zero to unity. There occurs some gray scale elements (the medium value of material density) in the optimal topology obtained by the above-mentioned method. Then, an efficient method called as ON/OFF method, by which the continuous shape of magnetic material can be successfully obtained, is proposed [4], [5] by utilizing the adjoint variable method [6].

A basic demand for an elevator system is the smoothness of motion of the permanent magnet synchronous motor. The controlling performance extremely drops because of the feedback of the oscillation of acceleration, velocity, and position by the ripple of driving force. Therefore, the design of lower ripple linear motor is needed for the practical utilization of rope-less elevator system.

In this paper, in order to reduce the ripple of the driving force, the optimal topology of magnet region is obtained by using the ON/OFF method [4], [5], in which the existence of magnetic material in each element is decided by using the design sensitivity. The ON/OFF method is compared with the conventional density method [2], [3] in order to illustrate the effectiveness of these methods from the viewpoint of convergence characteristic and practical utilization of obtained topology.

* Department of Electrical and Electronics Engineering, Okayama University, Okayama 700-8530, Japan (okamoto, norio@eplab.elec.okayama-u.ac.jp)
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2. Topology Optimization Technique

2.1 Sensitivity Analysis Method

The sensitivity is accurately calculated by using the adjoint variable method [6]. The equation for finite element method (FEM) is given as:

$$\boldsymbol{HA} = \boldsymbol{G} \tag{1}$$

where $\boldsymbol{H}$ is the coefficient matrix, $\boldsymbol{A}$ is the magnetic vector potential, and $\boldsymbol{G}$ is the right-hand vector. Taking the derivative of (1) with respect to the remanence B_r in an element k:

$$\frac{\partial \mathbf{A}}{\partial B_{rk}} = \mathbf{H}^{-1}\left(\frac{\partial \mathbf{G}}{\partial B_{rk}} - \frac{\partial \mathbf{H}}{\partial B_{rk}}\tilde{\mathbf{A}}\right) \tag{2}$$

where $\tilde{A}$ is obtained by solving (1). If the objective function is expressed as the function $W(B_{rk}, A)$ of the permeability in design domain and the magnetic vector potential, the sensitivity with respect to the remanence B_{rk} is given by:

$$\frac{\mathrm{d}W}{\mathrm{d}B_{rk}} = \frac{\partial W}{\partial B_{rk}} + \frac{\partial W}{\partial A}^{T} \frac{\partial A}{\partial B_{rk}} \tag{3}$$

substituting (2) into (3):

$$\frac{\mathrm{d}W}{\mathrm{d}B_{rk}}=\frac{\partial W}{\partial B_{rk}}+\frac{\partial W}{\partial A}^{T}\boldsymbol{H}^{-1}\left(\frac{\partial \boldsymbol{G}}{\partial B_{rk}}-\frac{\partial \boldsymbol{H}}{\partial B_{rk}}\tilde{\boldsymbol{A}}\right) \quad (4)$$

In order to avoid the calculation of the inverse of $\boldsymbol{H}$, an adjoint vector λ is introduced. The adjoint equation is given by:

$$\boldsymbol{H}^{T}\lambda=\frac{\partial W}{\partial \boldsymbol{A}} \quad (5)$$

$\mathrm{d}\tilde{W}\ \mathrm{d}B_{rk}$ is calculated by substituting λ into (6).

$$\frac{\mathrm{d}W}{\mathrm{d}B_{rk}}=\frac{\partial W}{\partial B_{rk}}+\lambda^{T}\left(\frac{\partial \boldsymbol{G}}{\partial B_{rk}}-\frac{\partial \boldsymbol{H}}{\partial B_{rk}}\tilde{\boldsymbol{A}}\right) \quad (6)$$

(6) means that only one extra solution for the adjoint vector is needed in order to determine the sensitivity to all parameters, rather than obtaining each value per parameter, providing a computationally fast method for deriving the gradients.

2.2 Density Method

In the case of the calculation of the magnet topology, the relation of the material density ρ and remanence B_r in an element i is given by:

$$B_{ri}=B_{r0}\rho_i^{n} \qquad (0\le\rho\le1) \quad (7)$$

where B_{r0} is the standard value of remanence, which is chosen as 1.2 T, and n, which is set as 2, is the exponent for the relationship. The normalized density ρ takes the value between 0 and 1. The design sensitivity of W with respect to ρ_i can be written as:

$$\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}\rho_i}&=\frac{\partial W}{\partial B_{ri}}\frac{\partial B_{ri}}{\partial \rho_i}\\&=nB_{r0}\rho^{n-1}\frac{\partial W}{\partial B_{ri}}\end{aligned} \quad (8)$$

where $\partial\tilde{W}\ \partial B_{ri}$ is calculated by (6). The calculated sensitivity is used to search the optimal distribution of the material density in magnet region. The steepest descent method is adopted as an optimization algorithm because there is no constraint on the volume of the magnet region. In the steepest descent method, ρ in the (k+1)-th iteration is updated by using the design sensitivity following (9):

$$\rho^{(k+1)}=\rho^{(k)}+\Delta\rho^{(k)} \quad (9)$$

where $\Delta\rho^{(k)}$ is the change vector. $\Delta\rho^{(k)}$ is given as:

$$\Delta\rho^{(k)}=-\alpha^{(k)}\frac{\partial W^{(k)}}{\partial\rho^{(k)}} \quad (10)$$

where $\alpha^{(k)}$ is the step size. $^{(k)}$ is given as:

$$\alpha^{(k)}=W^{(k)}\Big/\left\|\partial W^{(k)}/\partial\rho^{(k)}\right\|^{2} \quad (11)$$

Then, $\rho^{(k+1)}$ is given by

$$\rho^{(k+1)}=\rho^{(k)}-\frac{W^{(k)}}{\left\|\partial W^{(k)}/\partial\rho^{(k)}\right\|^{2}}\frac{\partial W^{(k)}}{\partial\rho^{(k)}} \quad (12)$$

When the change of the material density $|\Delta\rho|$ is less than 10^{-3} in each element, the topology optimization is terminated.

2.3 ON/OFF Method

The gray scale element is generated by using the above density method. Then, the ON/OFF method [4] [5], in which the existence of magnetic material in each element is decided by using the design sensitivity, is adopted because of no existence of gray scale element. The algorithm of the ON/OFF method is as follows:

- **Step 1:(Initialization)** The initial material location is decided. The initial material of the design domain is air.
- **Step 2:(forward analysis)** The calculation of initial topology by using the finite element method (FEM).
- **Step 3:(sensitivity analysis)** The calculation of design sensitivity by using the adjoint variable method.
- **Step 4:(modification of topology)** The topology is modified using the design sensitivity. If $\partial\tilde{W}\ \partial B_{ri}$ is negative, the remanence in an element i is should be increased. Therefore, the magnet is allocated in the element i. On the other hand, if the sensitivity $\partial\tilde{W}\ \partial B_{ri}$ is positive, the remanence in the element i is should be decreased. Then, the air is allocated in the element i.
- **Step 5:(forward analysis)** The magnetic field of the modified topology is calculated using FEM. If the objective function is improved, return to Step 3. Otherwise, go to Step 6.
- **Step 6:(annealing)** If the objective function is not improved, the changeable element number of material N_m is relaxed by using the following equation (13):

$$N_m = \gamma \cdot N_m \tag{13}$$

where γ is the annealing factor and chosen as 0.9. The obtained topology in this step is calculated by FEM. This step is continued until some improvement of the objective function is detected. If the objective function is improved, go to Step 3. Otherwise, the optimization is terminated.

3. Linear Synchronous Motor Model

3.1 Analyzed Model

The analyzed model of core-less type linear synchronous motor model is shown Fig. 1. The analyzed region is the upper half region in consideration of x-axis symmetry. The hybrid-type infinite element [7] is adopted in the outer region of air in order to reduce the calculation cost. The analyzed region is subdivided by the first order rectangle element. The amplitude of current density J_0 in 3-phase system is 3.125×10^6 A/m^2.

The dimension of the analyzed model is shown in Fig. 2. The gap length between magnet and coil is 4 mm, the thickness of magnet is 8 mm. The number of elements nd in design domain is 6720, and the total number of elements is 45600. The waveform of 3-phase alternative current is shown in Fig. 3. One period is equal to the distance of movement of 120 mm. The FEM calculation is performed at every 5 mm displacement of magnet region in 2-dimensional magnetostatic field. Therefore, the total number of calculation is 25 steps.

3.2 Objective Function

The design goal in this problem is to reduce the ripple of driving force generated on the coil region. The objective function is set as follows:

$$W = \sum_{l=1}^{np} \left(\frac{F_{xl} - F_0}{F_0} \right)^2 \tag{14}$$

where np is the number of magnet position (= 25), F_{xl} is the x-component of the electromagnetic force generated on the coil at the magnet position l, and F_0 is the target value (= 370 N/m). F_{xl} is calculated by the BIL law as follows:

$$F_{xl} = \sum_{ie=1}^{nc} \left(-B_y^{(ie)} J_0 S^{(ie)} \right) \tag{15}$$

where nc is the total number of elements in the coil region, $B_y^{(ie)}$,and $S^{(ie)}$ is the y-component of flux density, and the area in the element ie. Substituting (15) into (14), the adjoint equation at the magnet position l is given as (17).

$$\boldsymbol{H}^T \lambda = 2 \frac{\left(F_{xl} - F_0\right)}{F_0} \sum_{ie=1}^{nc} \left(-J_0 S^{(ie)} \frac{\partial B_y^{(ie)}}{\partial A} \right) \tag{16}$$

Then, the design sensitivity of W with respect to B_{ri} is expressed as follows:

$$\frac{\partial W}{\partial B_{ri}} = \sum_{l=1}^{np} \frac{\partial W_l}{\partial B_{ri}} \tag{17}$$

where W_l is the component of objective function at the magnet position l. The design sensitivity is calculated as a linear combination of the sensitivity at each magnet position.

4. Numerical Result

4.1 Optimization Using Density Method

The topology optimization is performed by using density method. The initial value of material density ρ is set up 0.5. The ripple r_d of driving force is given by

$$r_d = \frac{F_{max} - F_{min}}{F_{average}} \times 100 \tag{18}$$

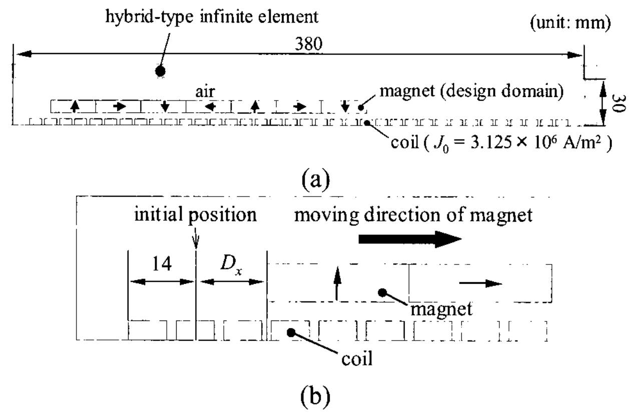

Fig. 1 Linear motor model. (a) Whole region. (b) Magnet displacement.

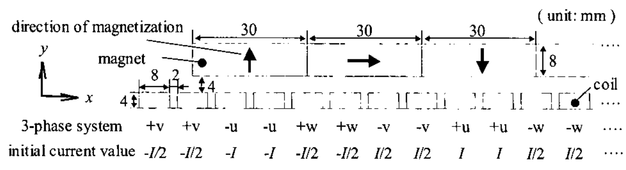

Fig. 2 Dimension of model.

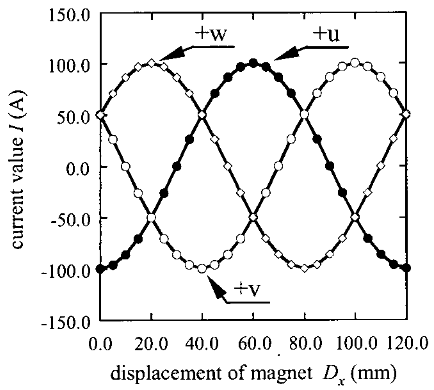

Fig. 3 Waveform of 3-phase alternative current.

where F_{max} is the maximum force, F_{min} is the minimum force $F_{average}$ is the average value of all magnet position D_x defined in Fig. 1.

The convergence process of magnet topology by using the density method is shown in Fig. 4. The approximate solution is obtained at 100th iteration, and the optimal solution is obtained at 1033rd iteration. Many gray scale elements are generated. In order to get a practical design, the optimal solution is converted into an ON/OFF data in which the material of the element (ρ> 0.69, optimal criterion value in this problem) is ON (magnet), otherwise the material is OFF (air). This is called as an extraction model.

A good convergence is observed from the viewpoint of the improvement of objective function W and r_d using the density method. However, the extraction model has a lower performance than the gray scale model. Fig. 5 shows the flux distribution of the optimal solution obtained using the density method. The flux in the coil region is uniformly distributed.

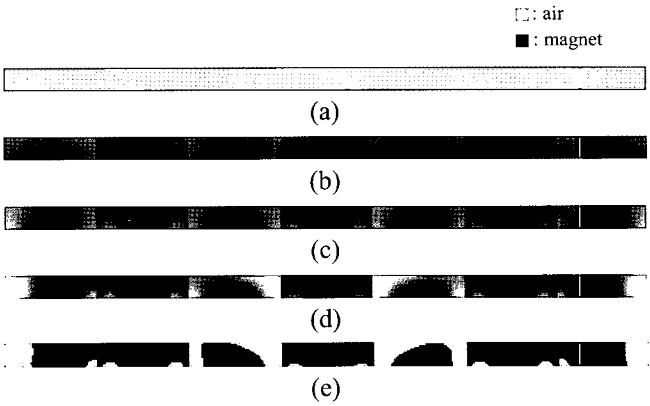

Fig. 4 Convergence process of density method. (a) Initial state (W = 11.09, r_d = 6.97 %). (b) 10th iteration (W = 2.63x10^{-3}, r_d = 3.89 %). (c) 100th iteration (W = 2.59x10^{-3}, r_d = 1.34 %). (d) 1040th iteration (optimal solution, W = 7.73x10^{-5}, r_d = 0.55 %). (e) Extraction model (W = 3.91x10^{-4}, r_d = 1.04 %).

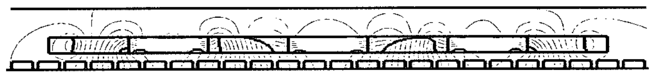

Fig. 5 Flux distribution of extraction model obtained by density method (D_x = 60 mm).

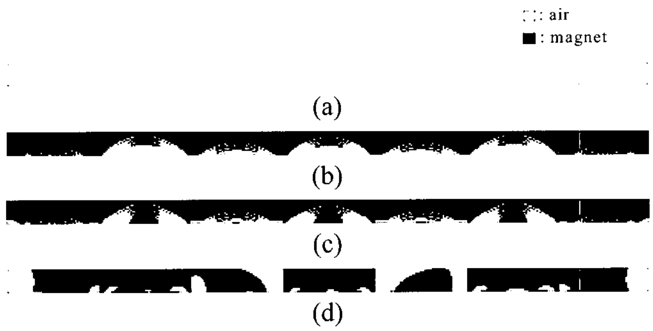

Fig. 6 Convergence process by using ON/OFF method. (a) Initial state (only air, W = 25.01). (b) 20th iteration (W = 0.15, r_d = 2.47 %). (c) 50th iteration (W = 1.58x10^{-3}, r_d = 2.72 %). (d) 101st iteration (optimal solution, W = 7.28x10^{-5}, r_d = 0.68 %).

4.2 Optimization Using ON/OFF Method

The convergence process by using the ON/OFF method is shown in Fig. 6. The optimal topology is obtained at 101st calculation. In the optimal topology, the magnet region has holes. This result is almost same as the extraction model obtained by the density method shown in Fig. 4 (e). Fig. 7 shows the flux distribution of the optimal topology obtained using the ON/OFF method.

Fig. 7 Flux distribution of optimal solution obtained by ON/ OFF method (D_x = 60 mm).

4.3 Comparison between Density Method and ON/ OFF Method

The convergence process of driving force at every magnet position is shown in Fig. 8. The driving force at every step approaches to the target value, when both methods are used. The number of iterations using the ON/OFF method is less than that using the density method.

Table 1 Optimization Results

	W×10^{-5}	r_d[%]	iterations	CPU time [h]
density method	7.73	0.55	1040	8.0
extraction model	39.1	1.04		
ON/OFF method	7.28	0.68	41	15.5

CPU: Intel Pentium 4 Processor 3.2 GHz, RAM: 2.0Gbyte

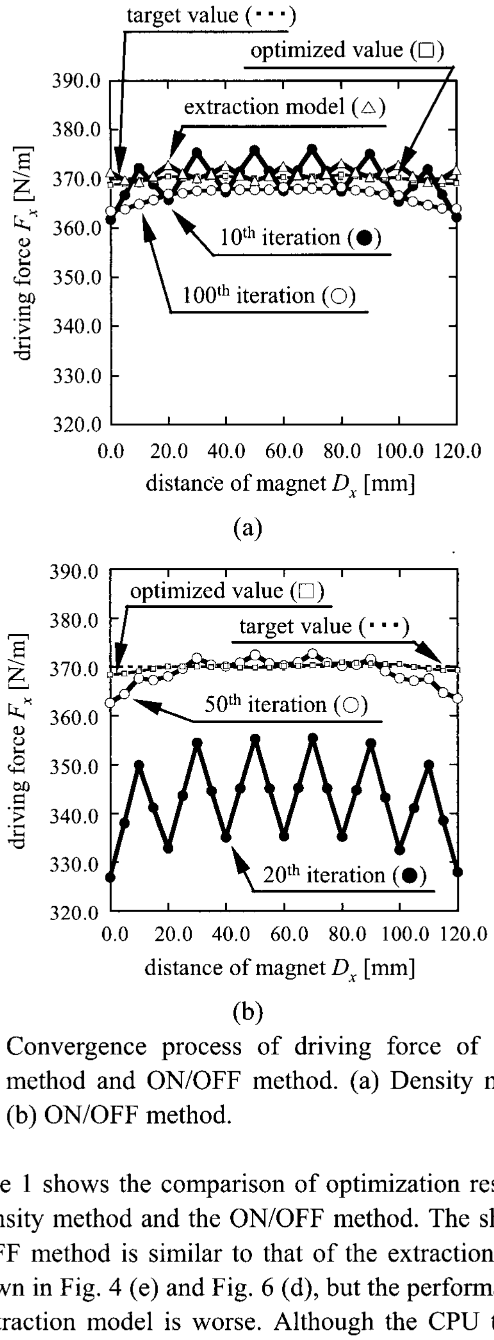

Fig. 8 Convergence process of driving force of density method and ON/OFF method. (a) Density method. (b) ON/OFF method.

Table 1 shows the comparison of optimization results of the density method and the ON/OFF method. The shape of ON/OFF method is similar to that of the extraction model as shown in Fig. 4 (e) and Fig. 6 (d), but the performance of the extraction model is worse. Although the CPU time of the ON/OFF method is longer than that of the density method, the actual magnetic circuit with a lower ripple r_d can be obtained using the ON/OFF method. The reason for the longer CPU time in spite of the smaller number of iterations of the ON/OFF method is the annealing process denoted at Step 6 in Section II. C.

5. Conclusions

In this paper, the core-less type linear synchronous motor is optimized by using the density method and the ON/OFF method in order to reduce the ripple of driving force. The obtained results can be summarized as follows:

(a) The convergence characteristics of the density method is equal to that of the ON/OFF method.

(b) The actual magnetic circuit with a lower ripple can be obtained using the ON/OFF method than the extraction model obtained by the density method.

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