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Is Mathematics Teaching in East Asia Conducive to Creativity Development? – Results from the TIMSS 1999 Video Study and the Learners' Perspective Study[1]

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Students in East Asia have consistently out-performed their counterparts in the West in recent international studies of mathematics achievement. But some studies also show that East Asian students are more rigid in thought, and lack originality and creativity. While different theories have been proposed to account for these student performances, relatively few research studies have been done on classroom practices, potentially a major variable for explaining student performances. This paper will report on the results of two classroom studies: the TIMSS 1999 Video Study and the Learners' Perspective Study (LPS).

Results the quantitative analysis of the TIMSS 1999 Video Study data show that the East Asian classrooms were dominated by teacher talk, and the mathematics content learned was abstract and unrelated to the real life. On the other hand, the characteristics of the instructional practices in Hong Kong as judged by an expert panel are that student learned relatively advanced mathematics content; the components of the lessons were more coherent, and the presentation of the lessons was more fully developed. Hong Kong students seemed to be more engaged in the mathematics lessons, and the overall quality of the lessons was judged to be high. Results of the analysis of the LPS data also show that the classrooms in the East Asian city of Seoul were in general teacher dominated, but students were usually actively engaged in the mathematics learning. Emphasis on exploration of mathematics and practicing exercises with variation was common.

It is argued that the quality teaching in the East Asian classrooms laid a firm foundation in mathematics for students, and that constitutes a necessary condition for the development of students' creativity. In order to fully develop the creativity of East

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Asian students, they need to be given the right environment and encouragement.

Keywords: TIMSS 1999 Video Study, Learners' Perspective Study (LPS)
ZDM Classification: C10, D10
MSC2000 Classification: 97U70

INTRODUCTION

Creativity in education can be defined as elements that help develop flexible thought, motivate the construction of problems and situations, promote the solution of problems in real context and improve imagination (Guerra & Servat 2005). Amabile (1996) claimed that "a social environment that encourages autonomy or self-directed learning, or that can provide optimal challenge, competence, or task involvement, should contribute positively to creative performance" (cited in Niu & Sternberg 2003). In particular, classroom teaching should be a major factor in nurturing the development of creativity in children. In this paper, we will consider the results of two classroom studies in order to see whether the classroom teaching in East Asia is conducive to creativity development

BACKGROUND

Students in East Asia have consistently out-performed their counterparts in the West in many international studies of mathematics achievement (Robitaille & Garden 1989; Stevenson et al. 1990, 1993; Lapointe et al. 1992). In particular, they performed extremely well in two recent large-scale studies, TIMSS (Beaton et al. 1996; Mullis et al. 1997, 2000, 2004) and PISA (OECD 2001, 2003, 2004). However, in some smaller-scale studies, conflicting results were obtained. For example, Mayer et al. (1991, p. 69) found that "students in the USA performed relatively better on problem solving than on computation, whereas students in Japan showed the reverse trend", and Cai (1995) found that Chinese students were superior to the US students in simple routine problems but not in more complex tasks. Yoshida et al. (1997) also found that Belgian students performed slightly better than Japanese students on non-routine, realistic mathematics problems.

For performance related to creativity, some earlier studies showed that East Asian students were more rigid in thought, and lacked originality and creativity. For example, Liu & Hsu (1974) found that Taiwanese university students performed poorly in a Chinese version of the Torrance test (Torrance, 1966) that measured creative thinking. They were also found to be more rigid in thought and less original. In another study by Douglas & Wong (1977), Hong Kong pupils, especially female, were found to perform worse than their counter-parts in America on three Piagetian formal-operations tasks (Combination of Colours, Invisible Magnet and Projection of Shadows). Douglas and Wong attributed these results to the Hong Kong culture which did not encourage an "active, *searching mode of behaviour*" (Douglas & Wong 1977, p. 692).

More recently, in a study by Niu & Sternberg (2001), it was found that Chinese students' artwork was generally perceived as less creative by both Chinese and American judges. Niu and Sternberg suggested that Chinese students' demonstration of a lower level of creativity might be due to their exposure to a school system that predominantly emphasized the learning of basic knowledge and analytical skills.

Decision makers and educators in Asian countries are beginning to realize the importance of developing the creativity of their students. For example, Sinlarat (2002) opined that since the Colonial Period, a "consumer culture" has developed in Asia, and Asian countries are becoming more and more dependent on Western countries. Sinlarat urged that education in Asia must nurture students to become creative and productive. In Japan, the government realizes that the past way of learning which focused on memorizing patterns of solution and formulas is becoming insufficient in preparing students for the large scaled and more complicated problems they face nowadays. So in the revised course of study in 2002, the objective was to improve students' creativity and to develop their own individuality and basic scholastic abilities (Yanagimoto 2005).

Different theories may be proposed to account for East Asian students' seemingly lack of creativity (and superior mathematics achievement), and an important source of factors in accounting for such performances is the teachers and their teaching. In an analysis of the TIMSS teacher questionnaire, Philippou & Christou (1999, pp. 394-395) found that East Asian teachers "adopt an algorithmic interpretation of mathematics, view mathematics as an abstract subject, teach mathematics as a set of rules and algorithms, emphasize drill and practice, follow activities from the textbook, and pay limited attention to understanding and creative thinking." Philippou & Christou related their findings to the high achievement of East Asian students in TIMSS, but these teacher conceptions and the resulting teaching styles may have contributed to East Asian students' performance in creativity as well.

Philippou & Christou's study relied on the TIMSS questionnaire, which is a self-reporting by the teachers in the study. Such methodology has obvious limitations (see Stigler & Hiebert 1999). Videotaping, on the other hand, offers a form of cross-cultural documentation which is both true to the original classroom and amenable to analysis by independent researchers.

In this paper, we will examine the results of two video studies of the mathematics classroom: the TIMSS 1999 Video Study and the Learners' Perspective Study (LPS). Characteristics of mathematics teaching in the two East Asian cities of Hong Kong and

Seoul will be identified respectively to see whether they are conducive or restrictive to creativity development.

HONG KONG: TIMSS 1999 VIDEO STUDY

The goal of the TIMSS 1999 Video Study was to examine instructional practices in eighth-grade mathematics across seven countries: Australia, Czech Republic, Hong Kong SAR, Japan[2], Netherlands, Switzerland, and United States. The study utilized a national probability sample of a target of 100 schools for each country. One lesson per teacher per school was videotaped, and the videotaped lessons were sampled across the school year. Altogether, videotapes of 638 lessons were collected for analysis, and the achieved sample for Hong Kong was 100 lessons.

Data collection followed standardized camera procedures, with one camera focusing on the teacher and another camera focusing on the whole class. A fluently bilingual international team of coders recruited from each of the participating countries applied a set of 45 codes in 7 coding passes to each of the videotaped lessons. The "in-point, out-point, and category" of each code were evaluated and the reliability measured. If reliability fell below a certain minimum acceptable standard, the code was dropped from the study.

The research team was aware that a fine grained analysis of the video data may have the danger of breaking the observed lessons down into minute constituent parts, but the parts may not fit with each other to form back a meaningful picture of the lesson. For this reason, a number of qualitative analyses were performed on the data, one of which was a judgment of the quality of the content of the lessons by a specialist group of mathematicians and mathematics educators known as the Mathematics Quality Analysis Group. The group reviewed a randomly selected subset of 120 lessons (20 lessons from each country except Japan[3]) based on expanded lesson tables including details about classroom interaction, nature of mathematics problems worked on, mathematical generalizations etc. created by the international coding team. The descriptions were "country-blind," with all indicators that might reveal the country removed.

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2 The 1995 Japanese data were re-analyzed using the 1999 methodology in some of the analysis.

3 The Japanese data was not included in this analysis because the same group of experts performed a similar analysis on the data in the 1995 study, which included the Japanese data.

RESULTS IN HONG KONG

Instructional practices in Hong Kong as portrayed by the analysis of the codes

Whole-class interaction dominated

The TIMSS 1999 Video Study classified lesson time into public interaction, private interaction and "optional student presents information." As far as Hong Kong lessons are concerned, it was found that "comparing across countries, eighth-grade mathematics lessons in Hong Kong SAR spent a greater percentage of lesson time in public interaction (75%) than those in the other countries, except the United States." (Hiebert et al. 2003, pp. 54-55).

Table 1. Average percentage of lesson time devoted to public interaction, private interaction, and optional student presents information

Country	Public interaction	Private interaction	Others
Australia	52	48	0
Czech Republic	61	21	18
Hong Kong	75	20	5
Japan	63	34	3
Netherlands	44	55	1
Switzerland	54	44	1
United States	67	32	1

Teacher talked most of the time

The coding team counted the number of words spoken by the teachers and the students during the lessons. From the count, it was found that Hong Kong teachers talked a lot in class. Along with American teachers, they were the most talkative among the teachers in the study. And "Hong Kong SAR eighth-grade mathematics teachers spoke significantly more words relative to their students (16:1) than did teachers in Australia (9:1), the Czech Republic (9:1), and the United States (8:1)" (Hiebert et al. 2003, p.109). When we factor in the large class size in Hong Kong (the average class size for the Hong Kong lessons in this study was 37), the reticence[4] of the Hong Kong students was even more striking.

[4] A ratio of 16:1 is in effect a ratio of nearly 600:1 as far as an individual student is concerned.

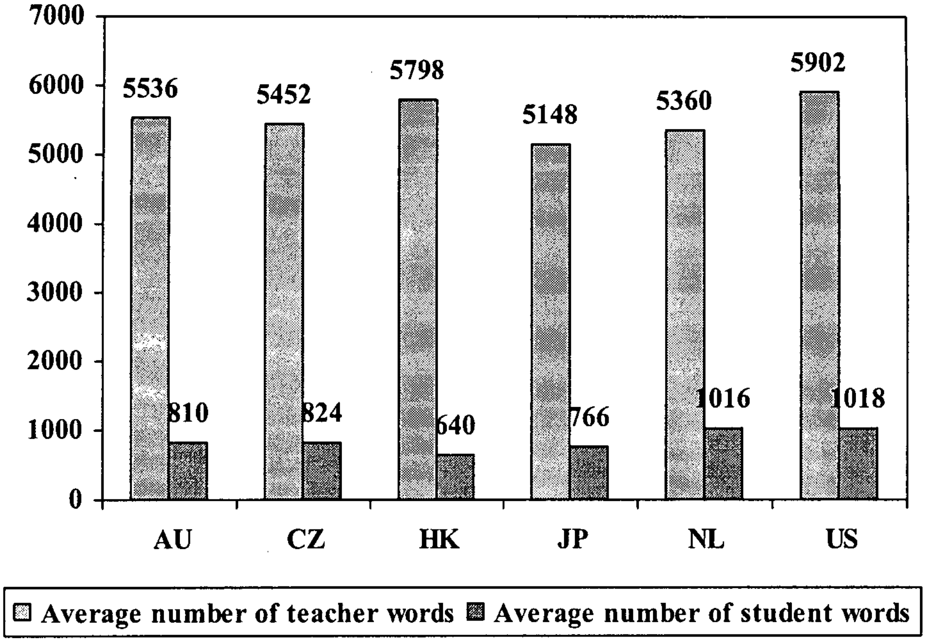

Figure 1. Average number of teacher and student words per lesson

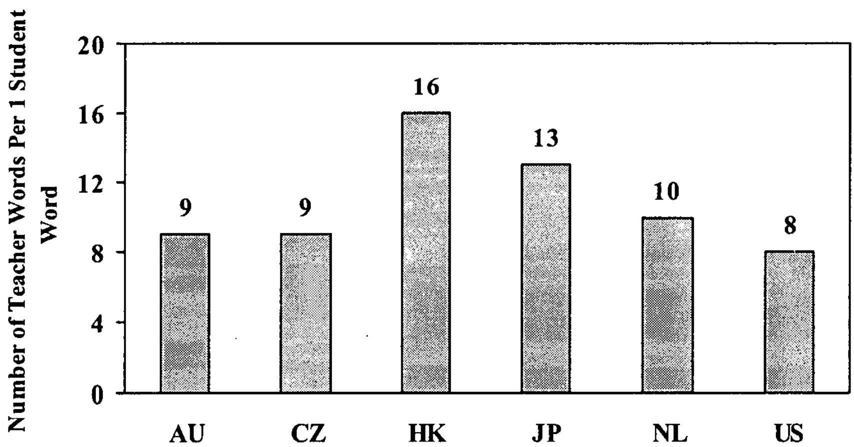

Figure 2. Average number of teacher words to every one student word per lesson

Students solved procedural problems unrelated to real-life following prescribed methods

1. The nature of the problems

The TIMSS 1999 Video Study classified problems into three types according to the nature of the problem statements: using procedures, stating concepts and making connections. It was found that "Hong Kong SAR lessons contained a larger percentage of problem statements classified as using procedures (84%) than all the other countries except the Czech Republic (77%)" (Hiebert et al. 2003, p.98).

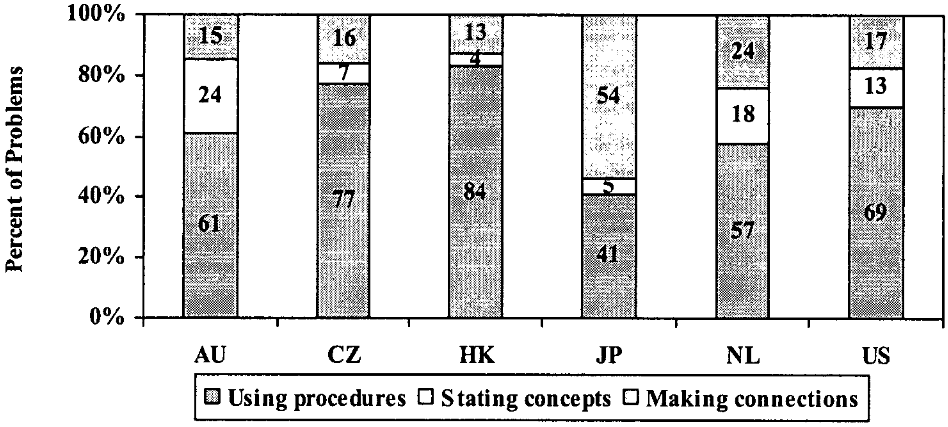

Figure 3. Average percentage of problems per lesson of each problem statement type

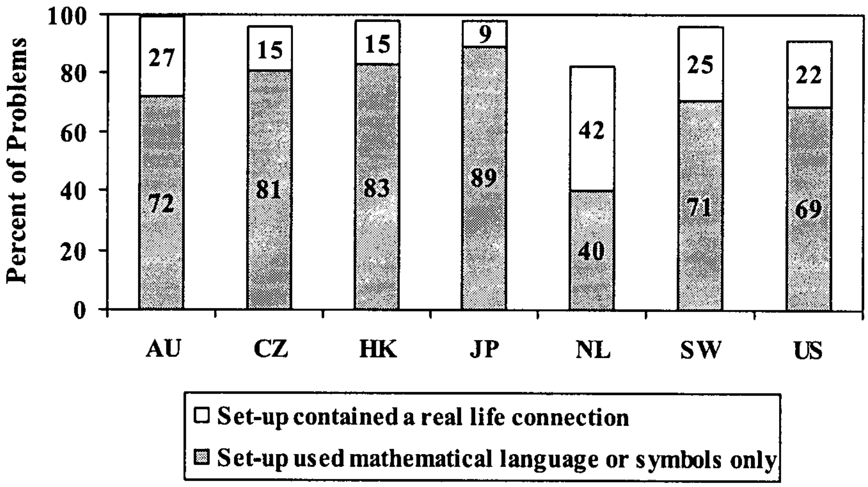

Figure 4. Average percentage of problems per lesson set up with a real life connection or with mathematical language or symbols only

2. Contexts of the problems

Another aspect of the problems that the research team looked at is whether the problems were set in real-life contexts. Many mathematics educators argue that mathematics problems presented within real-life contexts make mathematics more meaningful and interesting for students, and are thus more conducive to developing students' creativity. From the results presented in Figure 4, it can be seen that most of the problems dealt with in the Hong Kong lessons (and the Japanese and Czech lessons as well) were set up using mathematical language or symbols only. Only 15% of the problems were set up with a real life connection (15% and 9% for Czech Republic and Japan respectively).

3. Choice of solution methods

To promote the development of creativity, it is important for students to be given "space" to deal with the content they learn, especially in the way they solve problems. To enhance students' creativity, they should be encouraged to solve the same problem with different methods. In the TIMSS 1999 Video Study, the problems were analyzed to see whether students were given a choice of solution methods. It was found that the Hong Kong classrooms had the lowest percentage of problems (measured in terms of either the percentage of problems or the percentage of lessons where there was at least one problem in which students had a choice of solution methods) for which students were given a choice of solution methods.

Table 2. Average percentage of problems per lesson and percentage of lessons with at least one problem in which students had a choice of solution methods

Country	Average percent of problems with a choice of solution methods	Percent of lessons with at least one problem with a choice of solution methods
Australia	8	25
Czech Republic	4	20
Hong Kong	3	17
Japan	15	31
Switzerland	7	24
United States	9	45

Summary

From the discussions above, it can be seen that the characteristics of mathematics teaching in Hong Kong as portrayed by the analysis of the codes in the TIMSS 1999 Video are as follows:

- Whole-class interaction dominated
- Teacher talked most of the time
- Problems solved by students:
    - procedural problems
    - unrelated to real-life
    - following prescribed methods

All these are not very conducive to developing students' creativity!

Quality of content as judged by the Mathematics Quality Analysis Group

More advanced content

Based on country blind descriptions of a subset of the videotaped lessons, the Mathematics Quality Analysis Group made judgment on the how advanced the mathematics content in the lesson was. As can be seen from Figure 5, lessons in Hong Kong (and the Czech Republic) were found to deal with more advanced content. As the international report pointed out, "the ratings for countries with the most advanced (5) to the most elementary (1) content in the sub-sample of lessons, were the Czech Republic and Hong Kong SAR (3.7), Switzerland (3.0), the Netherlands (2.9), the United States (2.7), and Australia (2.5)" (Hiebert et al. 2003, p. 191).

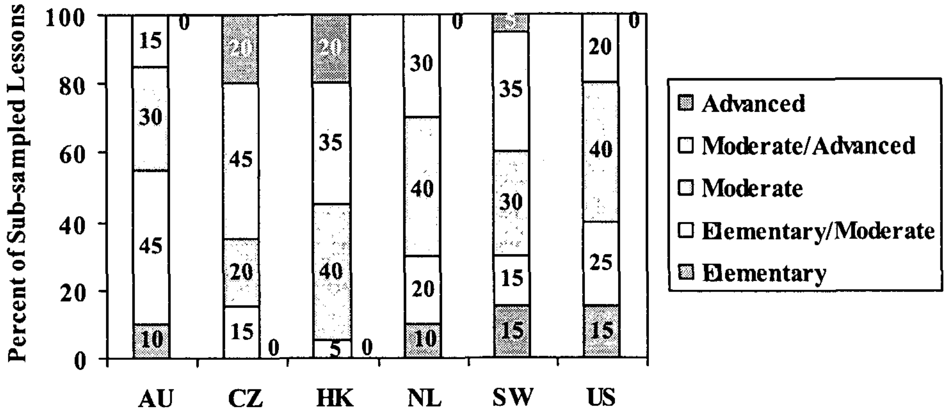

Figure 5: Percentage of lessons in sub-sample at each content level

Overall quality

In addition to judging how advanced the content of the lessons was, the Mathematics Quality Analysis Group also made judgment on the quality of the lessons in the sub-sample along four dimensions: coherence, presentation, student engagement and overall quality.

Coherence was defined by the group as "the (implicit and explicit) interrelation of all mathematical components of the lesson," and 90% of the Hong Kong lessons were judged to be thematically coherent, the remaining 10% being moderately coherent.

The second dimension the group looked at was how developed the presentation of the lessons was. Presentation was defined by the group as "the extent to which the lesson included some development of the mathematical concepts or procedures." The presentation was more "developed" if mathematical reasons or justifications were given for mathematical results. Presentation ratings took into account the quality of mathematical arguments. Higher ratings mean sound mathematical reasons were provided by the teacher or the students for concepts and procedures. Mathematical errors made by the teacher reduced the ratings. As can be seen from Figure 6 below, most of the Hong Kong lessons were judged to be more fully developed.

The group also made judgments on how likely students were engaged in the lessons given the detailed descriptions of the lessons (remember the group did not actually watch the videotapes). Student engagement was defined as "the likelihood that students would be actively engaged in meaningful mathematics during the lesson." Rating of very unlikely (1) means students were asked to work on few of the problems, and problems did not stimulate reflection on mathematics concepts or procedures. Rating of very likely (5) means students were expected to work actively on, and make progress solving, problems that raise interesting mathematical questions for them, and then to discuss their solutions with the class. The Hong Kong lessons received very high rating in this dimension from the Mathematics Quality Analysis Group. Hong Kong students were judged to be most likely to be engaged in the lessons compared to all other countries in the study.

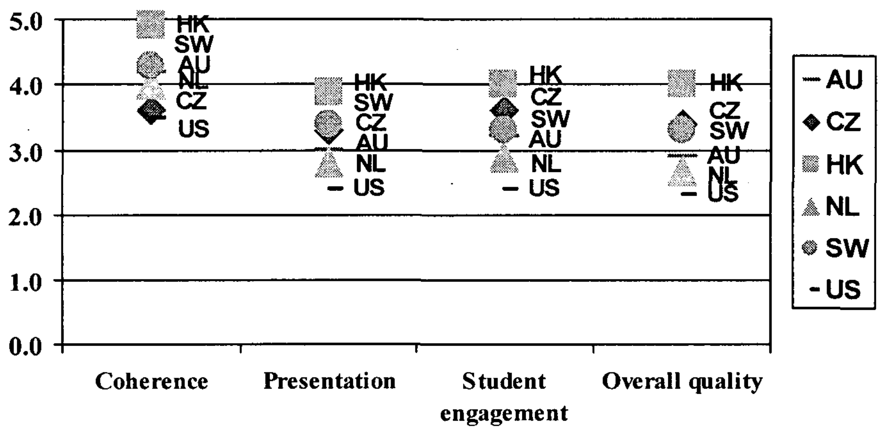

Figure 6. General ratings for each overall dimension of content quality of lessons

Finally, the group made an overall judgment on the quality of the lessons in the sub-sample. The quality the group look for was "the opportunities that the lesson provided for students to construct important mathematical understandings". It was found that "the relative standing of Hong Kong SAR (in terms of the quality of lessons) was consistently high" (Hiebert et al. 2003, p. 200).

The ratings in these four dimensions are shown in Figure 6 below.

Summary

From the discussions above, it can be seen that the qualities of content of the lessons in Hong Kong as judged by the mathematics quality analysis group are:

- Relatively advanced content
- More coherent
- More fully developed presentation
- Students are more engaged, and
- Overall quality is high

These are qualities that are thought to be conducive to quality teaching and learning, and it may be argued that such qualities constitute the necessary condition for creativity development. For without a thorough understanding of mathematics, there is nothing based on which we may be creative about. So we can conclude from the TIMSS 1999 Video Study data that the teaching in Hong Kong provided a sound foundation in mathematics based on which students' creativity may be developed. But the teaching strategies used in the Hong Kong lessons do not seem to have encouraged the development of creativity.

SEOUL: LEARNER'S PERSPECTIVE STUDY (LPS)

The Learner's Perspective Study, led by David Clarke at the University of Melbourne, is another international video study of the mathematics classroom which intends to supplement the TIMSS Video Study data by in-depth documentation of the student perspective over several lessons in the same classroom. The methodology of the Learner's Perspective Study (LPS) combines two traditions – the macro-study of culturally specific scripts and practices, and microanalysis of the construction of mathematical and social meanings in classroom settings. One teacher from each of three schools in each participating country was sampled for study, and a series of 10 to 15 consecutive lessons taught by the teacher were videotaped. The teachers chosen were judged to be competent teachers in their respective countries.

The study combines videotape data with participants' re-constructions of classroom events. Three cameras were employed in the videotaping – a "Teacher Camera," a "Student Camera" and a "Whole Class Camera." An audio-video mixer was used for on-site mixing of the images from the Teacher camera and the Student camera to provide a split-screen record of both teacher and student actions. The integrated images were used for stimulated recall in interviews conducted immediately after the lessons to get students' reconstructive account of the teaching and learning.

The sample for the Korean component of the study is as follows:

Table 3. The sample for the Korean component of the study

	School H	School K	School W
Type of schools	Girls' school	Co-educational	Co-educational
SES of parents	Mostly middle class		
Teacher gender(age)	Male (47)	Female (32)	Female (33)
Teaching Experience	18 years	6 years	7 years
Class size	40	38	37

Data Analysis: Theory of Variation (by Marton)

A learning theory is utilized in the data analysis. According to Marton et al. (2003), learning is a process in which learners develop certain capability through discerning some features of the learning object. Experiencing variation in the features i s essential for discernment. It is also important to attend to what varies and what is invariant in a learning situation. An "enacted space of learning" is constructed through creating certain dimensions of variation for the experience of students

This is consistent with the "Teaching with Variation" espoused by Gu (1994). According to Gu, meaningful learning enables learners to establish substantial and non-arbitrary connections between new knowledge and previous knowledge. Two types of variation are helpful for meaningful learning: **conceptual variation** and **procedural variation**.

RESULTS IN KOREA

A preliminary analysis of the Korean LPS data echoes the findings of the TIMSS 1999 Video Study that the lessons in this East Asian city were dominated by teacher talk and reticence of students. Mathematics content was delivered efficiently in the lessons, with mathematics concepts often stated directly. The focus of the lessons seemed to be on the final product rather than the process of arriving at the product. There was much use of formal mathematical language rather than everyday life language such as metaphors in the teaching (Leung & Park 2004).

A more fine-grained analysis of the Korean LPS data shows that there were a lot of variations in concepts and practicing exercise in the Korean lessons. In this analysis, one lesson typical of the teaching of the teacher from each school was chosen for in-depth analysis. The details of the three lessons can be found in the Appendix. Conceptual and procedural variations found in the three lessons include:

1. Introducing a new topic based on a review of the content covered in previous lessons (HP1, WP1)[5]
2. Consolidation through summary (HC4)
3. Learning concepts through comparison and contrast (HC1)
4. Linkage between mathematics and concrete examples (KC1)
5. Multiple representation of a concept (HC2, KC2)
6. Generalization through abstraction (WP2)

Systematic and continuous variation

One kind of variation warrants further discussion. It is a kind of systematic and continuous variation that is very prevalent in the videotaped Korean lessons:

HC3

Basic equation, $x + y = 5$: coefficients and domains for the variables are natural numbers

Domain being natural numbers unchanged, coefficients changed systematically, first to another set of natural numbers ($3x + y = 15$), and then to include a negative whole numbers ($-3x + y = 12$), i.e., the coefficients are now integers

Next, domains for the variables extended to include negative whole numbers (i.e., integers)

Then the domains are further extended to real numbers

At this point, the concept that the graph of a linear equation with two unknowns is a straight line in the coordinate plane is well expounded

Finally, the general form of a linear equation with two unknowns ($ax + by + c = 0$) where the domains for the unknowns (x and y) are real numbers is introduced

In the analysis above, we can see that there are systematic variations starting with the basic equation $x + y = 5$ and moving step by step to the general form of $ax + by + c = 0$. In each variation, all but one of the components of the equation concerned are kept constant, so that the effect of the varied component is elucidated

[5] WP1 stands for the first procedural variation in school W. Similarly, HC4 stands for the fourth conceptual variation in school H, etc.

KC3 and KC4

Basic diagram: pair of rectangles with ratio of similarity 1 : 2

Then ratio of similarity kept constant (1 : 2), and basic diagram changes to a pair of triangles

Next, geometric figures (pair of rectangles) unchanged, and ratio of similarity changes to 1 : 3

Then both figures (from rectangles to triangles) and ratio of similarity (from 1 : 2 to 2 : 3) change

Ratio of similarity generalized to m : n

Finally, keeping ratio of similarity (2 : 3) constant, figure further changed to a pair of pentagons

With these systematic variations, students are guided to understand the concept that for a pair of **any** similar polygons, if the ratio of similarity is m : n, then the ratio of areas is m2 : n2

WC1

Basic situation: pouch with three red stones

Situation varies systematically: each time a red stone being replaced by a blue one until eventually there are three blue stones

These systematic variations bring out the facts that:

1. probability of an impossible event is 0
2. probability of a certain event is 1
3. probability is a value lying between 0 and 1

In the three examples above, the teaching all started with a certain simple basic situation. Then only one of the different aspects of the basic situation was varied at a time, and the variations followed a systematic pattern until the situation reached a target form. These systematic variations constitute a kind of exploration on the part of the students. It seems that the variations were carefully designed by the teacher, leading students to discern attributes of the object of learning or the concepts involving the final situation. According to the theory of variation, such systematic variations will create the necessary condition for different features or critical attributes of the object of learning to be experienced by the students.

In addition, in all the lessons analyzed above, there were also systematic variations in the exercise given to students. So students after being exposed to systemic variations in the presentation of the concepts would now have an opportunity to practice the application of the concepts systematically in class and/or at home.

DISCUSSION

Results of the quantitative analysis of the TIMSS 1999 Video data show that the East Asian classrooms were dominated by teacher talk, and the mathematics content learned was abstract and unrelated to real life. On the other hand, the characteristics of the instructional practices in Hong Kong as judged by an expert panel are that students learned relatively advanced mathematics content; the components of the lessons were more coherent, and the presentation of the lessons was more fully developed. Hong Kong students seemed to be more engaged in the mathematics lessons, and the overall quality of the lessons was judged to be high. Results of the analysis of the LPS data also show that the East Asian classrooms were in general teacher dominated, but students were usually actively engaged in the mathematics learning. Emphasis on exploration of mathematics and practicing exercises with variation was common.

Literature on creativity development often stressed the importance of exploration in enhancing students' creativity. Exploration in the Western context often means students were given open-ended tasks and engaged in free exploration activities usually conducted in a small group or individualized setting. This is in sharp contrast to the teacher directed East Asian classroom as illustrated by the video data of the TIMSS 1999 Video Study and the Korean LPS project reported above. However, the fine-grained analysis of the two studies above also shows that in the seemingly teacher-directed learning in the East Asian classroom, students were actually actively involved in learning. They explored mathematics concepts under the close guidance of the teacher, and such exploration was possible since students had a good grasp of the mathematics through learning more advanced and fully developed content that was presented by the teacher in a coherent manner. Hence, in the words of the variation theorists, a necessary condition of learning has been created (Marton & Booth 1997). Under such a condition, students may develop their mathematics competence and excel in traditional paper and pencil tests (such as TIMSS and PISA), but it may be argued that the competence in mathematics also provides the necessary condition for the development of creativity.

However, competence in mathematics is only a necessary condition for creativity development; it is not a sufficient condition. To fully develop students' creativity, they need a teaching and learning environment which "helps develop flexible thought, motivate the construction of problems and situations, promote the solution of problems in real context and improve imagination" (Guerra & Servat 2005). And mathematics teaching characterized by abstract, procedural content unrelated to real life and in a classroom dominated by teacher talk as presented above do not seem to be the right environment for the development of students' creativity.

Niu & Sternberg (2003) found that the Chinese students in their study were able to increase their creative performance by simply being told to be creative. It seems that the instruction to Chinese students (to be creative) gave them the "permission to break through the norms and restrictions of the environment" (Niu & Sternberg 2003, p.112). It was also found that giving the Chinese students detailed coaching on how to be creative led to their increased creative performance. So Niu & Sternberg's findings seem to support the notion that the necessary condition for the development of creativity already exists among East Asian students. They already have the basic mathematics competence and thus the potential of developing their creativity, and once given the right environment and encouragement, they can excel. What they need is an environment that allows them to "break through the norms and restrictions of the environment."

CONCLUSIONS

The instructional practices in the East Asian mathematics classroom identified through the analysis of the TIMSS 1999 Video Study and the Learner's Perspective Study results as presented in this paper do not only provide explanation for the high achievement of East Asian students in mathematics. They also point to the possibility that the potential already exists for East Asian students to develop their creativity.

Roots and wings are common metaphors used in education. Our children need to be rooted in our cultures and traditions, and yet have the wings to get out of the box. Applying the metaphor to what has been presented in this paper, East Asian students have already the roots of solid mathematics competence. To develop their creativity, what they need perhaps is to be given a pair of wings through an unbounded and favorable learning environment so that they can fly freely in the world of mathematics with their imaginations!

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APPENDIX

1. School H (Lesson 1)

Topic of the lesson

The topic of the lesson was 'the graph of linear equations with two unknowns'. It is the first of a series of lessons on this topic.

Description of the different stages of the lesson

Stage 1: Review and induction

At the beginning of the lesson, the teacher presented 2x+1=0 as an example of a linear equation which was addressed in the 7th grade. The teacher confirmed that this linear equation had only one unknown x. Then the teacher wrote down the equation x+y-5=0, and let the students recognize that there were two unknowns. Here, the teacher naturally introduced the linear equation with two unknowns using the linear equation with one unknown as scaffolding. The teacher started to investigate the roots of the linear equation with two unknowns by mentioning that the roots of the new equation should satisfy the equation x+y=5, just as the root x=-1/2 satisfies the equation 2x+1=0.

Stage 2: Exploring new concepts

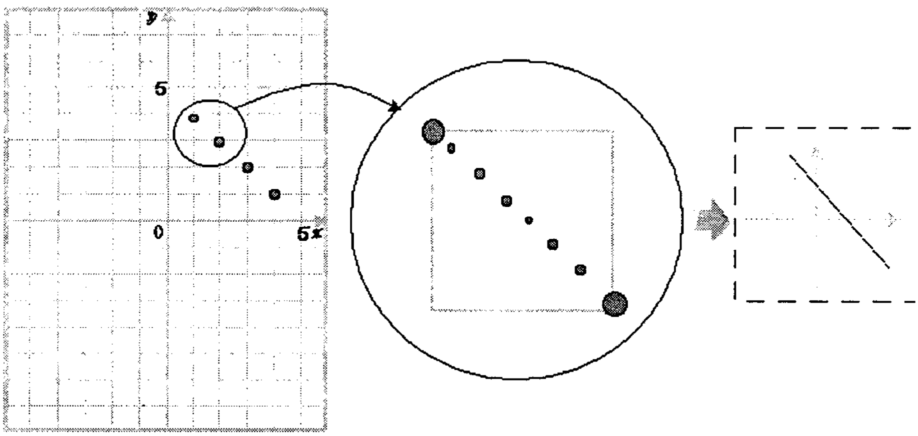


Figure 1(a) *Figure* 1(b) *Figure* 1(c)

The teacher asked the students what the values of y would be when x was substituted by the natural numbers 1, 2, 3 in the equation x+y=5. From the answers students gave, the teacher mentioned that there were infinitely many roots of the equation x+y=5, and then marked the ordered pairs of x and y (1,4), (2,3), (3,2), (4,1) as points in a coordinate plane (Figure 1(a)). Then the teacher asked again what the values of y would be when x was substituted by 1.1 and 1.2, and let the students understand that there was an infinite number of ordered pairs between two points (1, 4) and (2, 3) (Figure 1(b)). Finally, the teacher explained that if all these points were connected, then they resulted in a straight line (Figure 1(c)).

Stage 3: Examples and exercise

After the explanation of the root and graph of linear equations with two unknowns, the following four tasks were handled one by one.

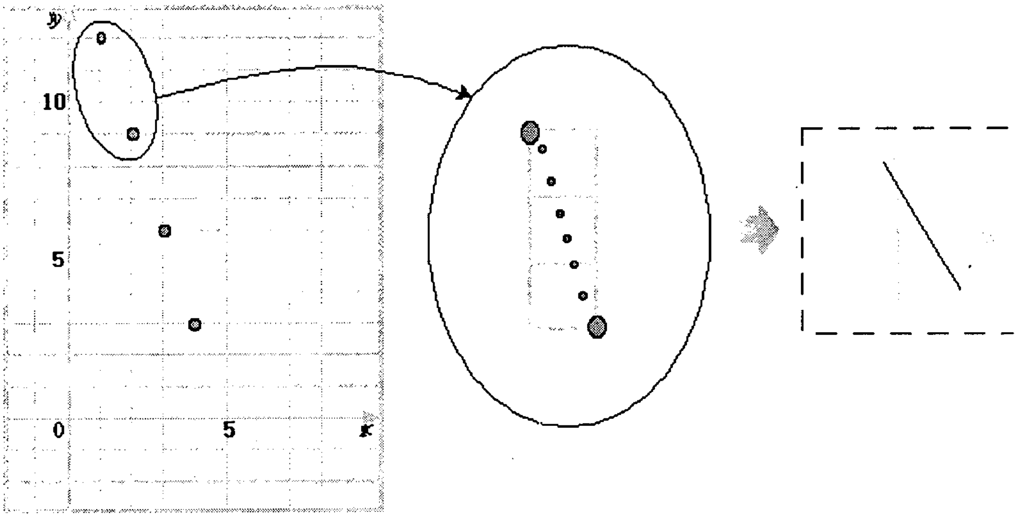


Figure 2(a) *Figure* 2(b) *Figure* 2(c)

Task 1: Solve the linear equation 3x+y=15 when x and y are natural numbers.

(1) Finding the ordered pairs satisfying the equation within the scope of natural number
Mark the ordered pairs (1, 12), (2, 9), (3, 6), (4, 3) as points in coordinate plane which satisfy the linear equation 3x+y=15 (Figure 2(a)).

(2) Expansion of the scope of x and y
If there is no condition saying that x and y are natural numbers, there would be an infinite number of points (ordered pairs) between two points (Figure 2(b)).

(3) Equation of a straight line
The linear equation 3x+y=15 is an equation for a straight line since many points (ordered pairs) in the coordinate plane which satisfy the equation 3x+y=15 form a

continuous straight line (Figure 2(c).

Task 2: Solve the linear equation -3x+y=12 when x and y are natural numbers.

The ordered pairs of x and y which satisfy the equation -3x+y=12 within the scope of natural number are infinite; (1, 15), (2, 18), (3, 21), (4, 24), ... Without the condition that x and y are natural numbers, this equation becomes the equation of a straight line.

Task 3: With the given values of x {-2, -1, 0, 1, 2, 3, 4, 5, 6, 7}, draw the graph of the equation x+2y=6.

With the given values of x, find the ordered pairs, and mark them on the coordinate plane as dots.

X	-2	-1	0	1	2	3	4	5	6	7
Y	4	7/2	3	5/2	2	3/2	1	1/2	0	-1/2

Like the previous tasks, this graph becomes a straight line without the condition that x is an integer.

Task 4: Draw the graph of x+y=3.

Find the four ordered pairs (0, 3), (1, 2), (2, 1), (3, 0) which satisfy the equation x+y=3, and mark them as points in the coordinate plane. There are many ordered pairs such as (2.1, 0.9) which satisfy the equation x+y=3, but it is not necessary to find all these ordered pairs. Two points are enough to determine a straight line, thus a straight line can be drawn with two ordered pairs (1, 2) and (2, 1).

Stage 4: Summary and assignment

Through the series of tasks, the teacher summarized that when x and y are real numbers, there are an infinite number of solutions that satisfy the linear equation ax+by+c=0 and therefore ax+by+c=0 is the equation of a straight line. The teacher also mentioned that conversely the points on the straight line, which is the visual presentation of the linear equation, become the root of the equation. Lastly, the teacher explained that only two ordered pairs are needed to draw the graph of a linear equation ax+by+c=0 and concluded by assigning the students the self-assessment and the basic or enrichment problems in the textbook as homework.

Analysis of the lesson: enacted objects of learning

Procedural Variation 1: *Review previous knowledge (linear equation with one unknown)* *and introduce the new topic (linear equation with two unknowns)*

Conceptual Variation 1: *Familiarizing students with the concept of linear equations in two unknowns through comparison with linear equations in one unknown*

Conceptual Variation 2: *Multiple representations of the root of an equation*

Procedural Variation 2: *Progressively unfolding the mathematical concept*

Conceptual Variation 3: *Variation of the 'basic equation'*

Conceptual Variation 4: *Consolidation and enhancing the formation of concepts*

2. School K (Lesson 7)

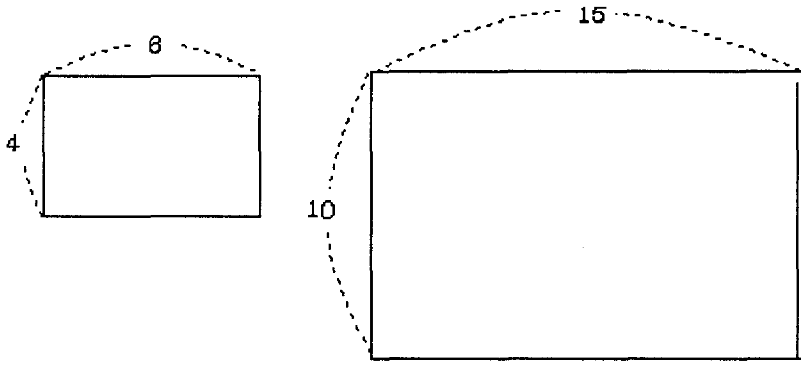
Topic of the lesson

The topic of the lesson was 'the area of similar figures'. It is the 7th in a series of 10 lessons on the theme of similar figures. Topics under this theme include: centre of similarity, similarity of triangles, and volume of similar figures.

Description of the different stages of the lesson

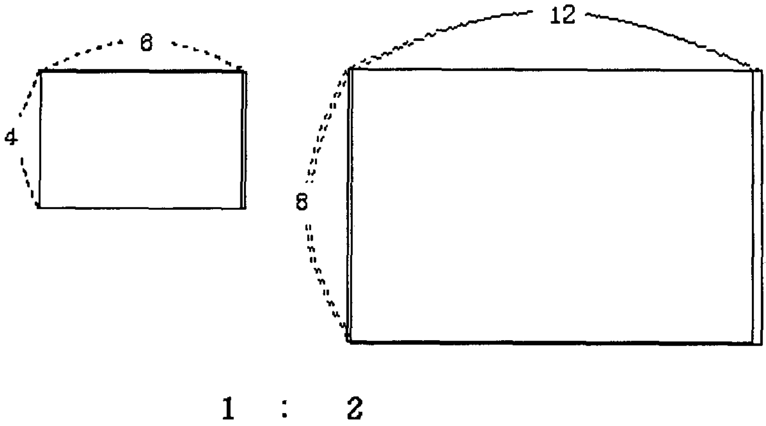
Stage 1: Review and induction

The teacher started with a realistic situation to introduce the concept of the ratio of areas between similar figures: "I want to take a picture with my digital camera and have it printed. If I were to enlarge the original 4×6 photo to a size 10×15, how many times more ink would it take?" Through this example, the teacher enabled the students to explore the relation between the content of the previous lesson (the ratio of similarity between similar figures) and that of the present one (the ratio of areas).



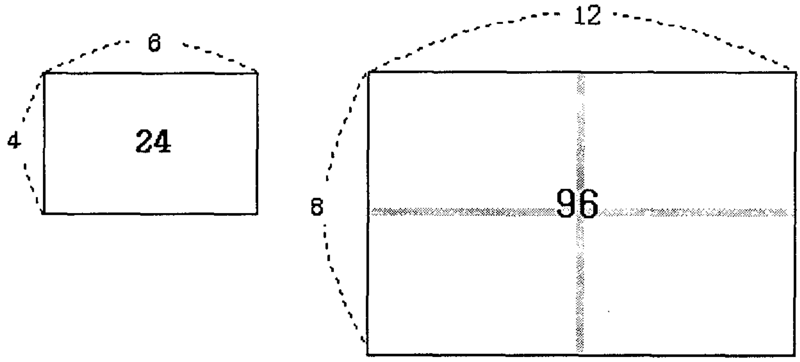
Stage 2: Exploring new concepts

Starting from the problem of finding out the amount of ink needed according to the size of the photo, the students explored the relation between the ratio of similarity and the ratio of areas. However, the teacher realized that the initial ratio of similarity of 1:2.5 was rather complex for students to compute. Accepting the suggestion of a student, the teacher changed the size of the photos to a ratio of similarity 1:2 to make the calculation easier. But the teacher made it clear that the students also had to solve the first problem afterwards.



To visually show the ratio of areas when the ratio of similarity of the rectangles is 1:2, the teacher modified the rectangle so that the ratio of similarity became 1:2. Through this visual presentation, the students were able to intuitively infer that the ratio of areas between the two figures is 1: 4 because the large rectangle is made up of four smaller ones.

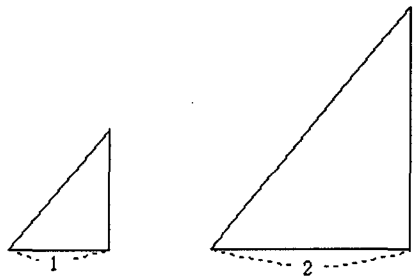
The teacher mentioned that in calculating the ratio of areas, other than dividing the larger rectangle into smaller ones, one could also multiply the height and base to get the area of each rectangle and then calculate the ratio.



Stage 3: Examples and exercise

After the explanation of the ratio of areas based on the ratio of similarity, the following four tasks were handled one by one.

Task 1: Find the ratio of similarity and the ratio of areas of the given figures.



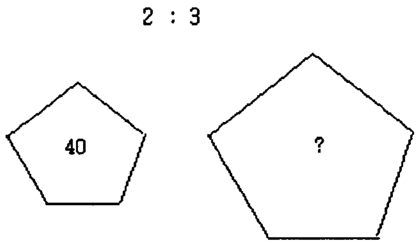
Task 2: Find the ratio of the areas of rectangles when the ratio of the sides is 1:3

Task 3: Find the ratio of the areas of triangles when the ratio of the sides is 2:3

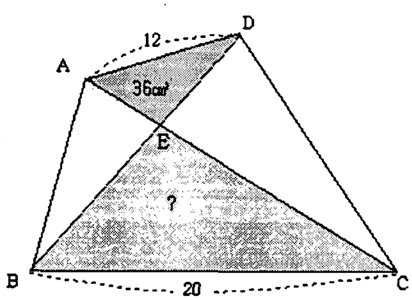
Task 4: Fill in the blanks:

The ratio of areas between similar figures is the _______ of the ratio of the sides. The ratio of areas between similar figures is _______ when the ratio of the sides is m:n.

Task 5: Compute the area of a large pentagon when the ratio of similarity between two pentagons is 2:3 and the area of a small pentagon is 40.



Stage 4: Assignment



One student was called up to the front to solve the following problem included in the handout given to the students: "△AED and △BEC are similar figures and the area of △AED is 36 ㎠. Calculate the area of △BEC." The teacher decided that it was difficult to give sufficient explanation on this advanced problem in the short time left and assigned the problem as homework before concluding the lesson.

Analysis of the lesson: enacted objects of learning

Conceptual Variation 1: *Introducing a new concept (ratio of areas) through concrete examples in everyday life*

Procedural Variation 1: *Improvement of given problem situation*

Conceptual Variation 2: *Perceiving the ratio of areas in a variety of ways*

Conceptual Variation 3: *Variations of the 'basic diagram'*

Procedural Variation 2: *Changing the modified problem back to the original*

Conceptual Variation 4: *More variations of the 'basic diagram'*

3. School W (Lesson 6)

Topic of the lesson

The topic of the lesson was 'the properties of probability'. It is the 6th in a series of 10lessons on the theme of probability. Topics under this theme include: the number of cases,the definition of event and complementary event, the definition of probability, and computations of probability.

Description of the different stages of the lesson

Stage 1: Review and induction

At the beginning of the lesson, the teacher presented a problem on probability which could be solved based on the definition of probability which was addressed in the previous lesson. The problems was to find the probability of making a number over 40 when you pick two cards out of five numbered from 1 to 5 to make a two digit number. Then the teacher presented the following simple problem to inquire into the properties of probability: starting from a pouch with three red stones and then changing a red stone for a blue one each time, find the probability of picking out a blue stone from a pouch in each case. This sequence of situations became the 'basic problem' for students to explore 'the properties of probability'.

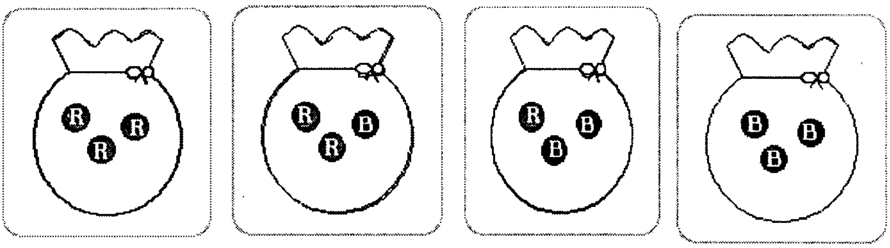


Figure 3(a) *Figure* 3(b) *Figure* 3(c) *Figure* 3(d)

Stage 2: Exploring new concepts with the guidance of the teacher

From Figure 3, students observed that the probability changed from 0, and then reached 1. Through this, the students inferred that the probability of an event not taking place is 0 and an event which is certain to take place is 1, and that probability is a value between 0 and 1. While the probability for each individual event had been found till this point, now the range of the values of the probability has been formally expressed.

Stage 3: Examples and exercise

Task 1 is a problem whose results echo with those of the basic problem.

Task 1: Find the probability when we throw a dice.
(1) Probability of getting an odd number
(2) Probability of getting 7
(3) Probability of getting 6 or less.

Task 2: There are two red, one green and three blue stones in the pouch Figure 4.
(1) When we pull out one, what is the probability of getting a red one?
(2) When we pull out one, what is the probability that it will not be red?

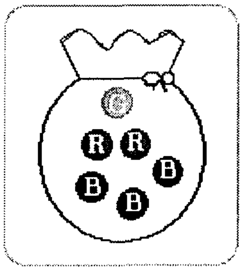


Figure 4

From the results of task 2, the teacher elicited the following fact: When the probability of event A is p, the probability of not being event A is 1 minus p.

The problems were not solved through a top-down approach by simply telling students this fact, but the probability of the complementary event was elicited through a bottom-up by first guiding students to think of the probability asked for in the problems. After that, task 3 was given as an opportunity for students to put this newly acquired concept into practice.

Task 3: There are 20 marbles numbered from 1 to 20. Find the probability of getting a number that is not a multiple of 3.

A series of tasks within various contexts were then presented.

Stage 4: Summary and assignment

As not a lot of new content was covered in the lesson, an informal formative evaluation was conducted on the number of cases, the definition of probability, and the properties of probability. A short quiz of 15 questions was conducted and was marked by peers during the lesson.

Analysis of the lesson: enacted objects of learning

Procedural Variation 1: *Reviewing previous knowledge (the definition of probability) and introducing the new topic (the properties of probability)*

Conceptual Variation 1: *Continuous change in situation to show the range of probability*

Procedural Variation 2: *Abstraction into general properties*