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Teaching Mathematics as an Alternative Approach to School Mathematics[1]

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Mathematics has developed dramatically in today's world and come to be increasingly put into practical use in various fields in society. However, many Japanese students dislike mathematics.
We have to study mathematics education with this situation in our mind. When we consider a better educational material, we don't have to consider only within the framework of the current school mathematics. We can expect to find good mathematical materials in fields beyond the school mathematics.
In this paper, we study how the inclusion of idea such as "fuzzy theory" and "graph theory" influences pupils' approaches to learning mathematics.

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MSC2000 Classification: 97U70

1. THE PRESENT SITUATION-ISSUES FACING TODAY'S SCHOOL MATHEMATICS IN JAPAN

According to a survey covering 38 countries and areas of IEA (TIMSS), Japanese students get the third highest grades (1995), and the fifth highest grades (1999) in the world. Comparing the findings for Japanese students with those for students from other surveyed countries, their consciousness of math displays the following features: There are many students (i) who hate math, (ii) who don't want to have a job connected with math, (iii) who find math important in life, and (iv) who enjoy studying math.

Many students are quite convinced that math problems only ever have one solution,

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and that to study math is mainly to memorize the method of arriving at the solution. This belief probably arises from the fact that math education in Japan has long been focused on the school entrance examinations, and has trained students merely to memorize.

In modern society, however, in contrast to the attitudes of these students, with the widespread use of computers, math is being applied in various fields, not only naturally in science and engineering, but also in social science (Kondo 1973). In the future, children will no doubt encounter math more and more frequently in other fields, and will unconsciously reap its benefits. Given this situation, it is surely necessary to review any existing school mathematics teaching that has nothing to do with real life and the functioning of society (Yanagimoto 1999).

Furthermore, in the current highly advanced information society, more mathematical thinking in problem solving comes to be needed. Setting problems complicated by a variety of factors on a larger scale than before requires grasping the structure of a problem and solving it more logically than ever.

Mathematics education in Japan must find out what ability is needed for Japanese students to cope with and live through this situation vigorously. Then not only can we improve upon a teaching material in the current teaching content, but also create a better material through putting mathematics out of the school mathematics.

In this paper, I will introduce two case studies in which we have fuzzy theory and graph theory as a teaching material. Then I will examine the educational significance of alternative approach to school mathematics.

We can find that Mathematics has developed dramatically in today's world. Fuzzy theory and graph theory are also both the leanings rapidly developing for the past-half century. According to this approach, students have the opportunity to experience the fresh and vivid mathematics of today, which exists beyond the boundaries of that which is taught in schools. The traditional mathematics that is taught in schools now does not contain any areas of the subject that are under development (Everything is already complete and finished). There is no room for students to either doubt it or to improve it. However, the kind of mathematics that has been developed recently, such as graph theory and fuzzy theory, is still under development, and many problems still remain to be solved. This newer mathematics is easy enough for those who are not specialized researchers to understand its basic elements and the meanings of unsolved problems. In future, it is expected that it will become more important to work with undeveloped problems that have no proper answers-that is, in which it is more important to have a creative scientific mind than to understand a given body of knowledge. Under this more modern approach, the aim is to show the students a zigzag thought process in the making, as well as models of refined perfection, in order to remind them of the importance of flexibility in ideas. Such kinds of thinking can easily be conducted within the framework of the new mathematical theories that are still under development.

The main objections in the following experiments are to teach modeling through fuzzy theory and to challenge students to help them grasp structurally and explain mathematiccal phenomena reasonably. And observing how they are to be influenced under our guidance of these theories is what this paper is all about.

2. A CASE STUDY-1: TEACHING MODELING BY USING FUZZY THEORY

(1) The significance of fuzzy modeling within education

Fuzzy theory recognizes the presence of human subjectivity and ambiguity, and reformulates these according to fuzzy concepts. Until now, the old science has been aiming to account for things strictly and in detail without accepting any vagueness of this sort. This is where its limitations have come to be felt, and as a result, fuzzy theory has come into being. The following significant meanings are conceivable in fuzzy theory from the viewpoint of mathematics education.

Firstly, we can help students developing a more up-to-date view of science through "fuzzy theory." It is only because of the changing approach to science in general in recent ages that fuzzy theory has been able to develop. As mentioned above, this is a way of addressing problems without involving rigid ways of thinking. Looking around in our daily lives, we notice that our attention has been turned away from "easy-to-make electric appliances" towards "easy-to-use electric appliances"; away from "effective housing" towards "healthy housing"; away from "wholehearted regional development" towards "regional environmental protection." Science has shifted its focus away from "enhancing industrial development" towards "making it possible for human beings to enjoy a more comfortable and richer life." It could safely be said that fuzzy theory is a human-friendly type of modeling (Tanaka 1990; Tanaka, Nakayama & Yanagimoto 1995).

Secondly, by coming into contact with a new theory, we can really follow and experience the process of its development. The study of fuzzy theory is still fresh. It started in Zadeh (1965). From the outset, Zadeh faced some criticism, but even today, the theory he created is still being structured. This does not only apply to only fuzzy theory, of course, but to any area of science that is in the making. It has a long way to go before it becomes systematically organized, and at first it encounters criticism. In time, it is modified, added to, and gradually completed.

Today, we should point out the important of modeling training in mathematics education (Yanagimioto 2005). When we examine students' awareness of mathematics as mentioned in section 1, a large proportion of them say that mathematics is not effectively used in society or that it is not necessary in real life. We wonder if this is because they have had fewer experiences of grasping various kinds of real problems as mathematical modeling. Modeling does not just end when we gain understanding through applying the established learning contents to problems in real life.

Effectively using the mathematics studied so far, students in the researched classroom went on to make new rules, modeling by themselves with real problems based on a new mathematical concept of "Fuzzy Sets." They also evaluated by themselves whether or not their modeling was appropriate. The model thought out by students sometimes differs from the formulation established in Fuzzy theory. However, in this case, the learning process of modeling is important for the students.

(2) Using fuzzy modeling for teaching in class

A 6-hour experiment was conducted with 15 first-and second-year students from Seifu Gakuen Senior High School.

Throughout the experiment, the aim was to establish how the students were thinking and how they were making their models. In particular, the following points had to be clearly established:

- How the students perceived various notions, including that of ambiguity.
  How the students made models using fuzzy concepts, what points they felt were difficult, and what aspects they found interesting.
- In what ways the students found this approach different in its thinking from that of traditional mathematics learning.

The following guidelines were developed.

Theme I. Human-Friendly 5-grade Evaluation

1) The vague - quantifying and graphing the vague
2) The Sets "Average" and "Not Average" - The Fuzzy Set and the Complement Set
3) The Sets "Average" and "somewhat too fat" - the union and product of fuzzy sets
4) Calculation of degrees - formulation of the membership function
5) Human-Friendly 5-grade Evaluation of the Laurel Index

Theme II. Forecasting data-forecasting data while thinking of possibility

1) Forecasting winning records in the Olympics
2) Estimated expression and real calculation

(3) Consideration

When making models based on real-life problems, there is no one correct model. Various models intended to express real-life problems more suitably may be made. The students themselves have to examine which is the most suitable. It is both important and a delight for students not only to reach a conclusion, but also to think out the process of coming to that conclusion. They also seem to have felt a sense of achievement in the guided result because they brought it to completion themselves. In the case of traditional mathematics, complete answers exist beforehand, and the students feel that they could come to a correct answer by a previous-taught path to the solution. In this respect, the old school mathematics is quite different from the approach of modeling. Below are some of the students' impressions.

"Unlike the mathematics I've been studying in class, I thought and created something like a formula by myself and could proceed with my work in my own way, so in the end, I really felt I made it in my own right. When we learned mathematics at school, we could only work out a problem in the way our mathematics teacher taught, and in explaining it to others; we could tell them how to do it by applying this formula. In this lesson, though, I found it difficult to convey exactly to others what I thought."

"In my school, I was just calculating by applying a difficult formula, and mathematics did not feel familiar to me. After this lesson, however, I came to realize that mathematics could be quite helpful to us in our real lives."

In this case study, I have attempted to show that fuzzy ideas may be seen as a theory that is useful in real life. I have also tried to illustrate the role of mathematical modeling within the general field of education. I believe that modeling can have the following educational benefits:

- It can help students solve complicated problems involving actual phenomena using mathematics.
- It can enable students to study ways of approaching problems that they have not used before. It is also useful in training students to approach real problems using creative thinking.

3. A CASE STUDY-2: TEACHING GRAPH THEORY

(1) The significance of graph theory within education

Negami (1998) has proposed that teaching discrete mathematics that includes graph theory helps students understand the structure of problems. And, Negami & Nakamoto (2003) have illustrated mathematical problems for understanding the structure of problems and logical thinking. Furthermore, Formin, Genkin & Itenberg (1996) have introduced a Russian experience that helps learners' mathematical thinking by using discrete mathematics.

However, in Japan, many students in mathematics class concentrate not on understanding the structure or thinking logically but on calculating or memorizing formulas and patterns of solutions.

The first significance of teaching graph theory in our understanding is that we can improve students' ability of explaining mathematical phenomena reasonably, which is one of the main purposes of mathematics education. In ordinary mathematics classes, students who are good at calculations, memorizing formulas and solving problems with the same patterns of solutions sometimes are the ones considered to be good in mathematics.

In fact, many students even those who are in mathematics teachers courses wasted time on calculating and memorizing formulas and patterns of solutions in their study of mathematics before they enter university. In junior and senior high school mathematics, there are some parts of mathematics that students have to memorize formulas to be able to solve technically and fast, in preparation for university admission exam, without thinking deeply of the meaning of formulas. Therefore, it seems that students are deprived of the opportunity to appreciate mathematical and logical thinking as well as excellent solutions in schools. On the other hand, graph theory is a mathematics on which one can appreciate mathematical and logical thinking using more natural language than formul as. In this area, one can solve problems by using mathematical logic and basic operations without necessarily using a lot of tools such as formulas and theorems. One has to think without depending on calculation, find relevant points and use logical methods such as mathematical induction.

(2)-1: Using graph theory for teaching in class in elementary school

A 4-hour experiment was conducted with 33 pupils in the 6th grade of elementary school. We helped them grasp the structure of the graph and think and explain reasonably by using Euler circuit. We set the following 2 themes in this teaching.

Theme 1: Is the figure that consists of a group of circuits regarded as the Euler circuit?
Theme 2: If all points of the graph have an even degree, does "Euler circuit" exist?

We helped the pupils understand the following mathematical fact and explain the reason by using daily terms.

- The figure that consists of a group of circuits is drawn without lifting the pencil from the paper (Figures 1 and 2).

- If the figure can be drawn without lifting the pencil from the paper, all points of it have an even degree.
- If all points of the figure have an even degree, it can be drawn without lifting the pencil from the paper.
- If all points of the figure have an even degree, it consists of a group of circuits.

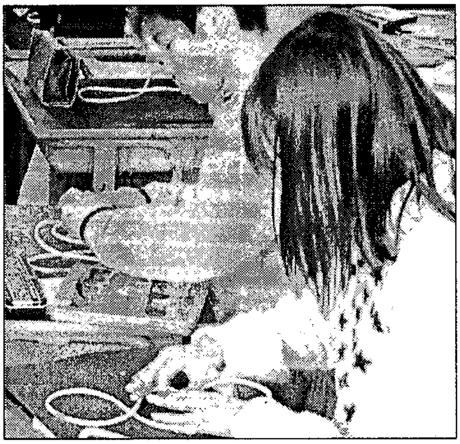

Figure 1. Graph theory in elementary school

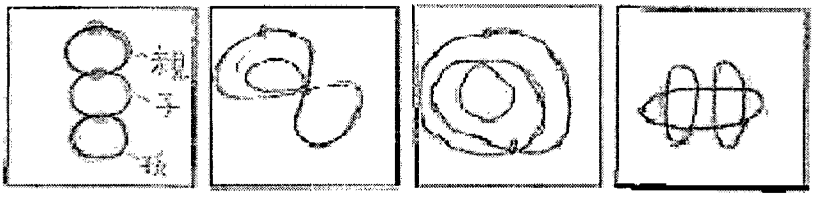

Figure 2. All points have an even degree

(2)-2: Using graph theory for teaching in class in junior high school

A 2-hour experiment was carried out for 44 students in the 2nd grade of junior high school. We helped them grasp the structure of the graph and think and explain logically by Matching Theory. In Matching Theory, the necessary and sufficient conditions for a bipartite graph and general graph to have a complete matching are clearly shown.

With these in mind, it is expected that the Junior High School Students will be able to grasp this theorem and the succeeding theorems without difficulty. We set the following 2 themes in this teaching.

Theme 1: To fully cover the floor area of a room

Theme 2: Matching

Example of the question is as follows:

Q1: Suppose that there are three groups of tennis players as shown on the figure below. Each point represents each player. If two players know each other, there is a line that connects them. In each group, is it possible for players who know each other to form a team for tennis' doubles?

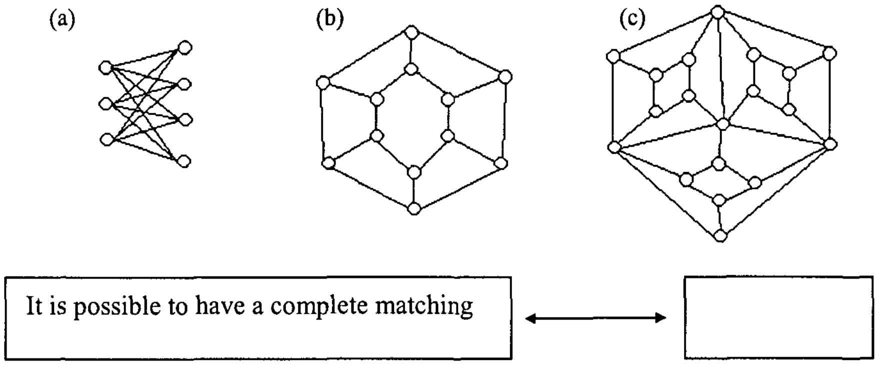

Figure 3. Matching

(3) Consideration

The following points have been found through the experimental teaching:

1) The second grade students in the junior high school are became to investigate the logic "$p \rightarrow q$" more cautiously through considering questions about matching. Furthermore they could point out the structure for solving the given problem of a graph.
2) Pupils in the primary school could understand the sufficient condition of the Euler theorem and explain the meaning of it by are also capable of understanding and explaining logically in their own simple words.
3) It can be said that graph theory is an appropriate material to develop logical thinking because of the above facts.

4. CONCLUSION

The case studies above led us to do following. Through the guidance fuzzy theory, students could have the opportunity to be part of creative activities, different from the traditional mathematics, by making mathematical modeling in their own ways.

The guidance of graph theory enabled students to have the chance to grasp the structure of a graph and explain the reason for mathematical phenomena in their own daily word and clearly with logic.

In creating the coming mathematics education, we find it necessary to consider and look flexibly into proper teaching contents, materials to really develop children's capabilities from a broad perspective.

REFERENCES

Yanagimoto, T. (2005): Teaching modeling as an alternative approach to school mathematics. *Teaching Mathematics and its Applications* **24(1)**, 1–15.

Yanagimoto, T.; Nakamoto, A. & Masuda, N. (2004): *A Study on Teaching Graph Theory-For Primary & Junior High School Students,* TSG13 at ICME-10 in Copenhagen, Denmark.

Negami, S. & Nakamoto, A. (2003): *Kiso-Sugakuryoku training.* Tokyo: Nihon-Hyoronsha.

Yanagimoto, T.; Nakamoto, A. & Fukuyama, S. (2002): A Study on Teaching Graph Theory–For Pupils in Elementary School. *Memoirs of Osaka Kyoiku Uni. Ser. V* **51(1)**, 201–212.

Yanagimoto, T. (1999): For the Development of Mathematics Education from the viewpoint of computer science. In: D. Tinsley & D. C. Jhonson (Eds.), *Information and Communication Technologies in School Math. IFIP TC3/WG3.1 Conference* (pp. 65–67). London: Chapman & Hall.

Negami, S. (1998): Discrete Mathematics Changes Mathematics Education. In: Japan Society of Mathematics Education (Ed.), *Mathematical Education in Japan 1998, Towards the Reform of Curriculum in Mathematics* (pp. 129–146). Tokyo: Sangyo-Tosho-Press.

Formin, D.; Genkin, S. & Itenberg, I. (1996): *Mathematical Circles (Russian Experience).* American Mathematical Society.

Ttanaka, H.; Nakayama, H. & Yanagimoto, T. (1995): *Possibility Portfolio Selection, Proceedings on Fuzzy Systems and the Second International Fuzzy Engineering Symposium* (pp. 813–818).

Tanaka, T. (1990): Fuzzy modeling and its application. In: Control and Information Engineers (Ed.), *The Institute of Systems*. Tokyo: Asakura-Press.

Kondo, J. (1973): *Operations Research.* Tokyo: Nikkagiren.

Zadeh, L. A. (1965): Fuzzy Sets. *Information and Control* **8(3)**, 338–353.