Name, Quilt and Transformation Geometry[1]

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The author has been teaching with an instructional module consisting of many mathematical concepts, based on designs formed by personal names or words to arouse students' interesting in learning mathematics. This module has been growing since it was first used as a supplementary lesson for calculus students. Now it consists of concepts that connect with mathematical topics such as number sense, algebraic thinking, geometry, and statistical reasoning, as well as other subjects such as art and quilt design. With its content we can provide our students the basic mathematical knowledge needed for further study in their own fields.

In this article, we will demonstrate the latest development of this instructional module, which makes connections between mathematical knowledge and the design of personal quilt patterns. We will exhibit a "Quilt of Nations" which consists of the designed quilt blocks of different countries, such as USA, Japan, Taiwan, Korea and others, as well as a quilt design using the abbreviation of this seminar. Then we will talk about how the connections are built, and how to design these mathematically rich, uniquely created, beautifully designed, and personalized quilt block patterns.

Keywords: transformation geometry, instructional module, art and quilt design, quilt patterns
ZDM Classification: U60
MSC2000 Classification: 97C80, 97U60

1. BACKGROUND

1.1. The school

The author teaches mathematics in a technological and business-oriented institute in the southern part of Taiwan. This institute has about 9000 students. The levels of

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schooling this institute represented, are a) five-year junior college; b) two-year college; c) two-year institute of technology and d) four-year university of technology. This institute recruits middle school graduates for its five-year junior college and high school graduates for its two-year College and four-years University of technology. This institute also recruits five-year junior and two-year junior college graduates for its two-year institute of technology. These four branches of levels of schooling also divide into three different class schedules, based on the students' class time; this institute divides into regular daytime school, the night school, and the weekend school.

1.2. The Mathematical curriculum

Mathematics is one subject required for all the departments, but the content, the course name and the credits required are different. For example, the business department requires a general mathematics course of two credits, while the engineering departments require not only two credits general mathematics combined with four credits calculus course but also two credits of differential equations as well.

1.3. The students

Although the institute requires the study of mathematics in the curriculum, most of these students are not enthusiastic about learning mathematics. In many cases the students have forgotten the mathematics they learned and some of them have mathematics anxiety and refuse to learn. The author's challenge has been to come up with pedagogical devices that will attract the students' interest and motivation in order to help them overcome their mathematics anxiety and become willing to learn mathematics.

This instructional module we discuss in this paper, which we design based on the students' names (Lee 2002, 2003, 2004), is mathematically rich, providing students with opportunities to learn important mathematical concepts and procedures with understanding. With the access to technologies this module can broaden and deepen students' understanding and appreciation of mathematics.

1.4. The design of this instructional module

The development of this module has been through several stages. That is because the author gained in pedagogical content knowledge along the way of teaching. At each teaching stage, the mathematical content became more abundant. In the first stage, the author has used this activity to design each student's personal logo. There consists very little mathematics in it, just an activity to make students enjoy mathematics. And based on the design of students' logo, the author asks her students to identify whether the design matches their personal characteristics for an amusement.

In the second stage, the author has been using this activity to do some algebraic mathematics, especially, the mathematics knowledge (Lee 2002) for finding the distance between two points, the slope of a line segment, or the equations of given lines. Based on what we learned from studying all the examples produced by the class, students come up with their own definitions and conjectures concerning parallel and perpendicular lines. For example, what do zeroes in the denominators or in the numerators of the slopes have to do with the vertical lines or the horizontal lines? What is the characteristic when two lines have the same slope? Or what is the relationship of the slopes of two vertical lines?

In the third stage, the author hopes the students can identify one of the geometric shapes in our name design as a square (Lee 2003).How does one prove a quadrilateral is a square? The slopes and lengths of line segments of two points become a must. But if the end points of the segments are not on the grid, what are we supposed to do? We hope the students have the initiative to pose questions to find the coordinates of the intersection points of two line segments. The students have to learn to solve systems of linear equations. In traditional courses students usually have no idea why we need to solve systems of linear equations. In this activity, they have a sense of why.

At the fourth stage, we collect data to learn statistics. For instance, how many points, how many lines, how many parallel lines and how many perpendicular lines; how many lines have the same distance; how many geometric shapes; and so on. Hopefully, the students can conjecture that the number of letters in their first name has something to do with the number of points and lines and so on. Not only data can help students to learn statistics, but geometry as well.

At the fifth stage, similarity knowledge is added to our growing content of instructional module. We make connection between the geometrical shapes and linear algebraic expressions. Since these designs are polygonal, many lines appear, thus we must talk about slopes and linear equations. We do not use the traditional method to find the slopes. Instead, we use the rectangle with the line segment as the diagonal. The ratio of height and width is the slope of the diagonal line segment. The main diagonal and sub-diagonal distinguish the sign of the slope. We also do not use the traditional method to find the equation of the line containing the diagonal segment. We know by now that we have the slope of the diagonal segment. What we need to do is to find a similar rectangle with the diagonal crossing the y-axis, in order to find the y-intercept. The concept of ratio comes into the picture. The slope of the line segment gives the ratio of y-coordinate to x-coordinate.. For example, if we know the slope of a given line segment is one half, then whenever we move every one unit of y-coordinate we have to move two units of x-coordinate.

At the sixth stage, the transformation geometry content (Lee 2004) is included in our lecture. Instead of providing students the table resulting from three consecutive 90

degrees rotations around a point (5, 5), we provide them a program created by my colleague. This program will produce the basic shape, three consecutive 90 degrees rotated figures and one overlapping final figure. We then asked our students to actually cut out the basic shape and then move the piece to find the final result. This activity helps them to understand the isometrics of transformational geometry.

At the seventh stage, based on two articles involving the concept of mathematics as applied to quilting (Ernie 1995; Westegaard 1998), the author helps students to make connections of transformation geometry knowledge to a new art creation other than a single logo design. This leads to multiple combinations of beautiful quilt block pattern designs.

In this article, we will demonstrate the latest development of this instructional module which makes connections between mathematical knowledge and the design of personal quilt patterns. We will exhibit a "Quilt of Nations" which consists of the designed quilt blocks of different countries, such as USA, Japan, Taiwan, Korea and others, as well as a quilt design using the abbreviation letters of this seminar ISME a quilt named "Cooperation of Friendship".

2. RELATED STUDIES

2.1. Meaningful learning

The National Council of Teachers of Mathematics (NCTM 1989, 2000) suggests that the emphasis of the mathematics curriculum should move away from rote memorization of facts and procedures to the development of mathematical concepts, and that students should investigate through problem solving not only how to make connections among various representations of those concepts, but also how to make these concepts meaningful to themselves.

2.2. Good problems

House (2001) concludes that a good problem is a problem that just keeps giving and giving (Lee 2003 & 2004). What are the characteristics of investigations that can lead to good mathematics problems for students? Good problems (adapted from Russell, Magdalene & Rubin 1989; Wheatley 1991; Clements 2000):
1. Are meaningful to the students;
2. Stimulate curiosity about a mathematical or non-mathematical domain, not just ask for an answer;
3. Engage knowledge that students already have, about mathematics or about the

world, but challenge them to think harder or differently about what they know;

4. Encourage students to devise solutions;

5. Invite students to make decisions;

6. Lead to mathematical theories about a) how the real world works or b) how mathematical relationships work;

7. Open discussion to multiple ideas and participants; there need not be a single correct response or only one thing to say;

8. Are amenable to continuing investigation, and generation of new problems and questions.

2.3. Creativity and quilting

In the paper written by Ernie (1995), the mathematical content was explored by the class using quilt blocks which include the design, construction, and measurement of the geometric shapes. Each child selected for his or her own patchwork. The actual project created by each student may be sewn or glued using fabric, or it could be created as an art-design poster using other materials: paper, paint, makers, stickers, or wood. The well-known Amish quilt, often referred to as "Sunshine and Shadow" or "Trip around the World" is used as a creative inspiration for students to design their own quilts. With Amish quilt as an example of learning symmetry students can find other transformational geometry on their own patchworks.

Westegaard (1998) provides an activity, which includes five sheets of quilt block designs, asks students to explore coordinate-geometry concepts–coordinate, positive and negative slopes, intercepts, and equations for horizontal and vertical lines. This activity can be used with small groups, individuals, or large groups. This article also includes questions and promptings that will lead students to notions of probability, area, and geometry.

3. THE INSTRUCTIONAL MODULE

The instructional module starts with translations of student's first names into sequences numbers from one to ten. With the numbers we get, we can do easy arithmetic operations or we can do more complicated mathematics. In order for students to become interested in this activity and then learn to do mathematics, we ask them to design their own logos based on their own names. In order to eliminate the mathematics anxiety, we design a computer program which will draw the graphs for students. They can use it to produce the basic shape, each individual transformed shape, and the final overlapping figure.

3.1 The assignment rule

In the following table we assign each alphabet letter a number from one to ten. We did try to assign letters to digit numbers from 0 to 9, but it turns out the center (4.5, 4.5) of the rotation is not on the grid which adds complication to further transformation analysis. By using the following letter-to-number assignment, every word can be transferred into number.

Table 1. Chart showing correspondence between alphabets and numbers

1	2	3	4	5	6	7	8	9	10
a	b	c	d	e	f	g	h	i	j
k	l	m	n	o	p	q	r	s	t
u	v	w	x	y	z				

3.2. Designing the quilt blocks

Step 1: Finding the set of numbers

The letters ISME (the acronym of "International Seminar of Mathematics Education") according to Table 1, correspond to numbers 9, 9, 3, and 5. With these numbers, if students like they can do fundamental arithmetic operations to create their own other related numbers, such as sum number, digit number, negative numbers, unit fractions and common fractions, decimals, square or cubic numbers and square root numbers.

Step 2: The set of points

Next, as an illustration, we find a point set corresponding to the letters I, S, M, and E. So let the set of numbers {9, 9, 3, 5} be the x-coordinates of our points. We then pair the consecutive numbers to create the set of coordinate points. After we connect the set of points, we form a closed polygon. We call this closed polygon our basic shape. Start from the first x -coordinates 9 pairing the next consecutive x -coordinates 9 to form the y-coordinates, we have our starting point A (9, 9). The last number in the number set has to pairing with the first number.

Step 3: The basic shape

After we form all the points, we connect all points based on the order of appearance of the points, and make sure the last point is connected to the first point to form a closed polygon. Therefore the points of A (9, 9), B (9, 3), C (3, 5), D (5, 9) and A (9, 9) will form the basic quilt block from our name (See Figure 1). The students not only can learn to draw the point on the coordinate plane and connect the points, they can also discover

some mathematics concepts here. With the college students, we hope they will be able to pose some mathematical questions based on their own basic shapes. Such as, what are the lengths of these lines? What are the slopes of these lines? Are there vertical lines? Are there horizontal lines? Are there parallel lines? Are there perpendicular lines? What are the equations of these lines? Can we find the intersection point of any two lines? What kinds of geometric shapes do we have? Do we have squares? Do we have parallelograms?
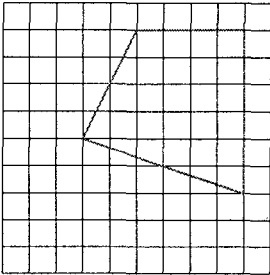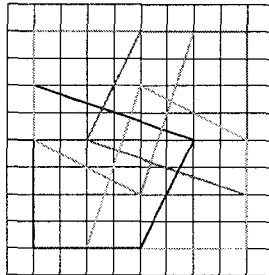

Figure 1. Basic Block   *Figure 2.* Transformed Block   *Figure 3.* Basic Block
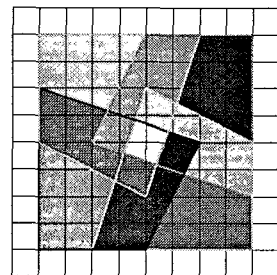
Step 4 : The transformed quilt block

In order to make the basic shape more complicated and interesting we add on three transformed images to our basic quilt block pre-image. We subject the basic block to three 90-degree rotations around the point (5, 5) to form three new congruent blocks. We then overlay each geometric shape on the same coordinate plane. This overlapping new quilt block is called the transformed quilt block (See Figure 2). Now what can we tell about this quilt block? What do you see? We can ask the same questions as for the basic quilt block and ask further new questions. For example, are there any squares in this transformed quilt block? We can now color the block and design our final quilt block (See Figure 3) to make some beautiful pictures. In the Appendix 1, we use this designed quilt block to create a quilt which we have named as "Cooperation of Friendship." We also provide a beautiful quilt designed by using quilt blocks we have named as "Quilt of Nations" (See Appendix 2).

4. CLOSING REMARKS

Mathematics and quilt block art offer a rich environment for students to explore. The personal and creative quilt block design presents a natural focus for studying geometric shapes and transformations. The patterns used in quilts were designed and named by students, thus providing meaningful mathematical learning to students. They not only

have the sense of ownership of mathematics knowledge, but also appropriate the mathematical learning behind their own works. Most important students' original creations of their own quilt designs help them to take away their anxiety about learning mathematics.

5. REFERENCES

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APPENDIX 1: The Cooperation of Friendship (Single quilt block design)
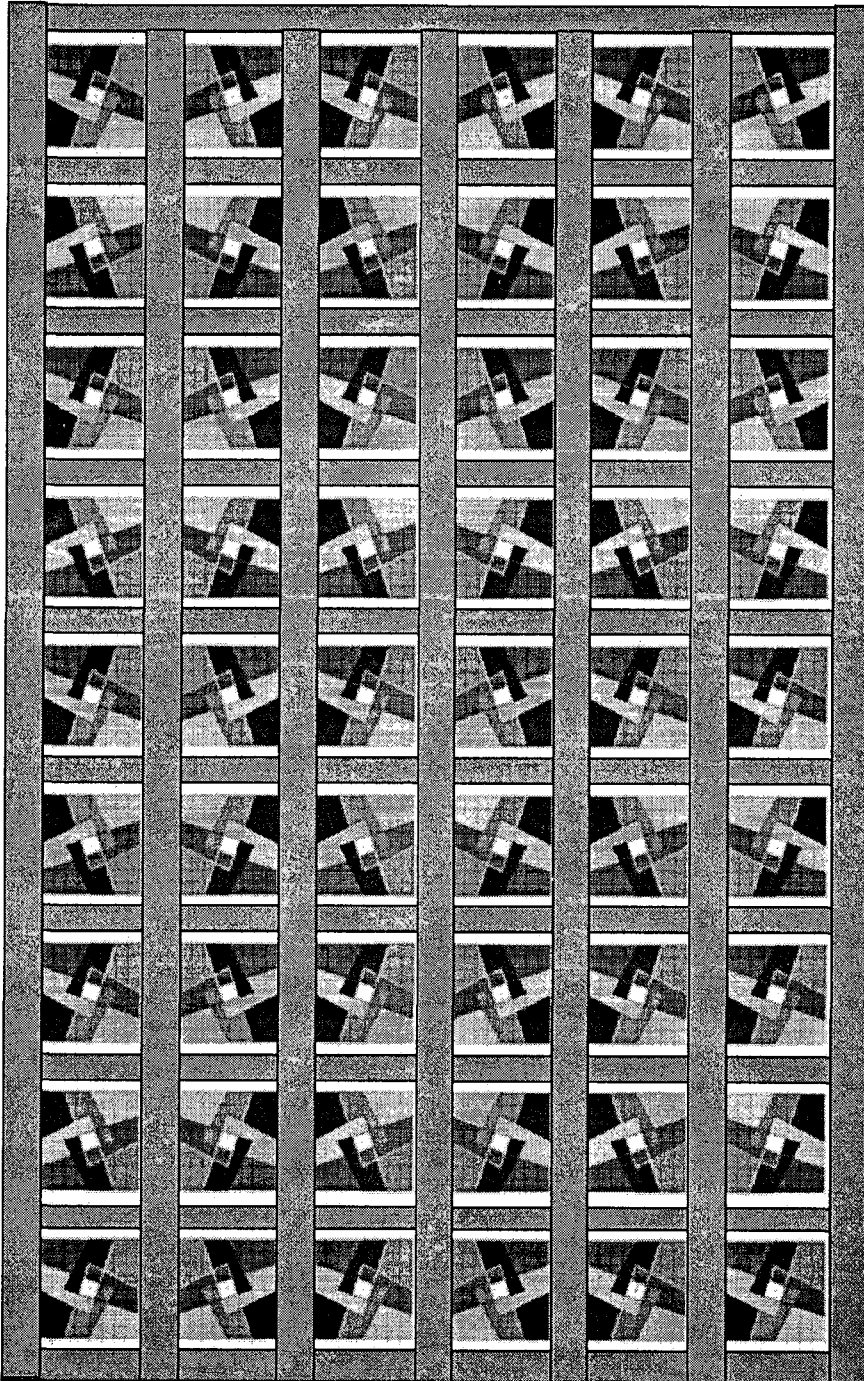
APPENDIX 2: The Quilt of Nations (Multiple quilt blocks design)