Journal of Mechanical Science and Technology, Vol. 19, No. 1, ⟨Special Edition⟩ pp. 237~254, 2005 237

Multibody Dynamics of Closed, Open, and Switching Loop Mechanical Systems

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The vast mechanical systems could be classified as closed loop system, open loop system and open & closed (switching) system. In the closed loop system, the kinematics and dynamics of 3-D mechanisms will be reviewed and closed form solutions using the direction cosine matrix method and reflection transformation method will be introduced. In the open loop system, kinematic & dynamic analysis methods regarding the redundant system which has more degrees of freedom in joint space than those of task space are reviewed and discussed. Finally, switching system which changes its phase between closed and open loop motion is investigated with the principle of dynamical balance. Among switching systems, the human gait in biomechanics and humanoid in robotics are presented.

Key Words : Closed Loop System, Open Loop System, Multibody Dynamics

1. Introduction

Modern mechanical systems are often very complex and consist of many components interconnected by joints and force elements. These systems are referred to as multibody dynamic systems. The multibody dynamics has been applied in various complicated systems such as automobiles, helicopters, airplanes, aerospace vehicles, war weapons etc. In this presentation, the development of multibody dynamics will be reviewed as considering the type of dynamic systems and its applications.

In the first section, mechanical systems of the closed-loop system with complete constraints mainly 3-D mechanisms will be reviewed and some of our research activities will be presented. Later, the multibody dynamics played a key role in the open-loop mechanical system in areas of the robotics and biomechanics where the robotic system is partially constrained and biomechanical system is intermittently constrained.

In robotics area, the focus of multibody dynamics was put on the robot itself. The topics are concerned about dynamics of articulated multi-link system, open/closed-loop structure, fixed configuration and invariant topology in system structure, efficient and compact formulation of forward dynamics simulation. Inverse dynamics computation is given much attention, and flexibility in link is also dealt with.

Further more, the robot with the interaction against the external agents through a set of mechanical interactions are investigated. This includes frequently changing the configuration and topology of system structure. In this situation the number, types, and physical natures of mechanical interactions are not invariant. The number of contact points and contact mode distributions are changing and new unilateral constraints are entering into the analysis.

Next, multibody dynamics in biomechanical system applications will be presented. This includes human biped walking simulation, motion behavior of multi-pod animal, and human sports

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motion simulation and so on. In biomechanical system, the target system is modeled as a floating body system, which has no fixed body with ground. It is different from the case of mechanisms and robot manipulators that have at least one fixed base in ground. The prominent feature of this type of problem is to consider the contact constraint. The multibody dynamic applications to the humanoid robot are also demonstrated.

2. Closed Loop System

In investigations of typical closed loop mechanical system, the mechanism of multibody constraint system has a long history (Lee, 1996). Although the modern kinematics has its origin with Franz Reuleaux (1875), yet it is only in the 1950s that the motion studies of three dimensional mechanism have begun to take on analytical approach using mathematical techniques along with the extensive use of computers.

Analytical study of spatial mechanisms received great impact upon the publication of influential works of Dimentberg (1948), Denavit and Hatenberg (1955) introduced symbolic notation for the complete description of the kinematic properties of lower-pair mechanisms, and also formulated general matrix closure equations for the spatial mechanism.

The advent of large scale computers in the 1960s was crucial catalyst in the surge in research on 3-D mechanism. Among them are the 4×4 matrix iterative method (Uicker, 1964), the vector method (Chace, 1965), the dual number quarternion method (Yang, 1964), the line geometry method (Yuan, 1970) and the geometrical configuration method (Wallace, 1970) for the five bar spatial linkages.

Considerable works have been reported for derivation on the input-output displacement equations for the six bar spatial linkages as a eight degree polynomial in the half-tangent of output angular displacement (Duffy, 1971). Duffy also derived an input-output equations of degree 32 in the half-tangent angle for the 7R spatial mechanism which is the last linkage in 3-D with one degree of freedom.

2.1 Closed loop solution by DCM method and RTM methods

A method of displacement analysis of the four-link spatial mechanism is developed. The results through this analysis are exact solutions that can be obtained without resorting to numerical or iteration schemes. In this analysis, the position of a link in a mechanism can be fully defined if its direction and length are known. Therefore, this analysis involves the calculation of the unknown direction cosines and length of each link for a given configuration of the mechanism. In finding the direction cosines of the unknown unit vectors involved for each link and rotating axis, two types of coordinates, the global and the local, are generally used. Then, a direction cosine matrix between each local coordinate system and the global coordinate is established. Thus, the unknown direction cosines of the local coordinates, the links, and the rotating axes are obtained in the global coordinates. In this development, the direction cosine matrices are used throughout the analysis. As an illustration, the application of this method to the study of four-link spatial mechanisms, **RGGR**, will be presented.

2.1.1 Direction cosine matrix method for RGGR mechanism

The RGGR four-link spatial mechanism as shown in Fig. 1 is the generalization of the planar four-bar mechanism. It is one of the most versatile and practical configurations of three-dimensional mechanisms and will function as a single degree of freedom linkage with a passive degree of freedom in the connection link. A schematic diagram of an *RGGR* mechanism is shown in Fig. 2.

The known quantities of the mechanism are the lengths, l_1, l_2, l_3, l_4, the vector $\bar{l}_4$, the directions of rotations, $\hat{p}_1$, $\hat{p}_2$, the angles ξ, η, α, β, from the construction of mechanism, and the input angle θ. The unknown quantities are $\hat{l}_1$, $\hat{l}_2$, and $\hat{l}_3$.

2.1.2 Analysis of Input Link $\bar{l}_1(\theta)$

The method starts by choosing the appropriate

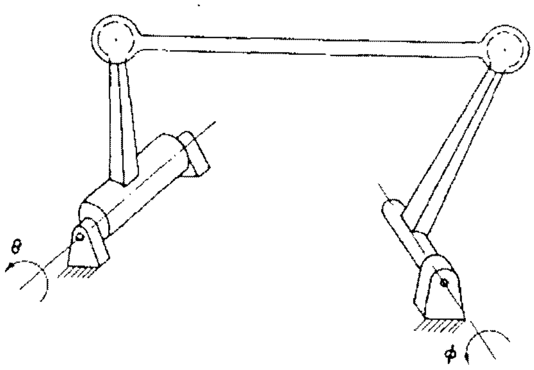

Fig. 1 A RGGR four link spatial mechanism

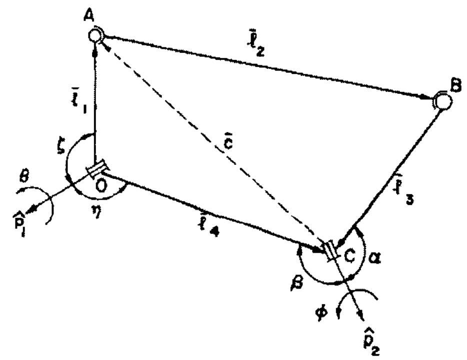

Fig. 2 The schematic diagram of a RGGR mechanism

local coordinates. Let the local coordinates x_1, y_1, z_1 associated with $\hat{p}_1$ with the origin at O be chosen as follows. The x_1-axis is set along the known rotating axis $\hat{p}_1$, the y_1-axis is set in the plane of $\hat{p}_1$ and $\bar{l}_4$ and the z_1-axis follows the right hand rule.

The direction cosines of the unit vectors $\hat{p}_1$, $\hat{l}_1$, and $\hat{l}_4$ in the local coordinates system associated with $\hat{p}_1$ are expressed in the parenthesis for each unit vector as $\hat{p}_1(1, 0, 0)$, $\hat{l}_1(\cos\xi, \sin\xi\cos\theta, \sin\xi, \sin\theta)$, and $\hat{l}_4(a_1', a_2', 0)$, respectively.

To find $\hat{l}_1$ in global coordinates, the direction cosine transformation matrix $[T_{ij}]$ should be defined.

With $\hat{p}_1$ and $\hat{l}_4$ are known in global coordinates and local coordinates, we could obtain the elements T_{11}, T_{12}, T_{21}, T_{22}, T_{31} and T_{32}. Now the rest of elements of can be found as their cofactors. This property is a key solution factor of DCM method. T_{ij} is obtained as

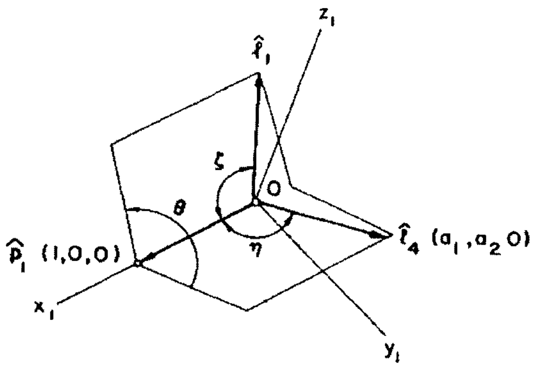

Fig. 3 A local coordinates system and input angle measurement

$$[T_{ij}]_1 = \frac{1}{\sin\eta}\begin{bmatrix} \hat{p}_{1x}\sin\eta & \hat{l}_{4x}-\hat{p}_{1x}\cos\eta & \hat{p}_{1y}\hat{l}_{4z}-\hat{p}_{1z}\hat{l}_{4y} \\ \hat{p}_{1y}\sin\eta & \hat{l}_{4y}-\hat{p}_{1y}\cos\eta & \hat{p}_{1z}\hat{l}_{4x}-\hat{p}_{1x}\hat{l}_{4z} \\ \hat{p}_{1z}\sin\eta & \hat{l}_{4z}-\hat{p}_{1z}\cos\eta & \hat{p}_{1x}\hat{l}_{4y}-\hat{p}_{1y}\hat{l}_{4x} \end{bmatrix} \tag{1}$$

Now $\hat{l}_1$ in global coordinates can be found as

$$\{\hat{l}_{1x}, \hat{l}_{1y}, \hat{l}_{1z}\}^T = [T_{ij}]\{\cos\eta, \sin\xi\cos\theta, \sin\xi\sin\theta\}^T \tag{2}$$

where

$$\begin{aligned} \hat{l}_{1x} &= (1/\sin\eta)\{(\hat{l}_{4x}-\hat{p}_{1x}\cos\eta)\sin\xi\cos\theta \\ &\quad +(\hat{p}_{1y}\hat{l}_{4z}-\hat{p}_{1z}\hat{l}_{4y})\sin\xi\sin\theta+\hat{p}_{1x}\sin\eta\cos\xi\} \\ \hat{l}_{1y} &= (1/\sin\eta)\{(\hat{l}_{4y}-\hat{p}_{1y}\cos\eta)\sin\xi\cos\theta \\ &\quad +(\hat{p}_{1z}\hat{l}_{4x}-\hat{p}_{1x}\hat{l}_{4z})\sin\xi\sin\theta+\hat{p}_{1y}\sin\eta\cos\xi\} \\ \hat{l}_{1z} &= (1/\sin\eta)\{(\hat{l}_{4z}-\hat{p}_{1z}\cos\eta)\sin\xi\cos\theta \\ &\quad +(\hat{p}_{1z}\hat{l}_{4y}-\hat{p}_{1y}\hat{l}_{4x})\sin\xi\sin\theta+\hat{p}_{1z}\sin\eta\cos\xi\} \end{aligned} \tag{3}$$

Thus, $\hat{l}_1$ is obtained as the function of the input angle θ by DCM method as $\bar{l}_1 = f(\theta)$.

2.1.3 Analysis of output link $\bar{l}_3(\phi)$

In a similar procedure as $\bar{l}_1(\theta)$, each elements of $\bar{l}_3$ can be calculated as follows.

$$\begin{aligned} \hat{l}_{3x} &= (1/\sin\beta)\{(\hat{l}_{4x}-\hat{p}_{2x}\cos\beta)\sin\alpha\cos\phi \\ &\quad +(\hat{p}_{2y}\hat{l}_{4z}-\hat{p}_{2z}\hat{l}_{4y})\sin\alpha\sin\phi+\hat{p}_{2x}\sin\beta\cos\alpha\} \\ \hat{l}_{3y} &= (1/\sin\beta)\{(\hat{l}_{4y}-\hat{p}_{2y}\cos\beta)\sin\alpha\cos\phi \\ &\quad +(\hat{p}_{2z}\hat{l}_{4x}-\hat{p}_{2x}\hat{l}_{4z})\sin\alpha\sin\phi+\hat{p}_{2y}\sin\beta\cos\alpha\} \\ \hat{l}_{3z} &= (1/\sin\beta)\{(\hat{l}_{4z}-\hat{p}_{2z}\cos\beta)\sin\alpha\cos\phi \\ &\quad +(\hat{p}_{2x}\hat{l}_{4y}-\hat{p}_{2y}\hat{l}_{4x})\sin\alpha\sin\phi+\hat{p}_{2z}\sin\beta\cos\alpha\} \end{aligned} \tag{4}$$

$\bar{l}_3 = f(\phi)$

2.1.4 Closed form solution

The following steps are proceeded in order to obtain a closed form solution.

$$\vec{l}_1+\vec{l}_2-\vec{l}_3-\vec{l}_4=0 \tag{5}$$

Taking dot product as, $\vec{l}_2\cdot\vec{l}_2=(-\vec{l}_1+\vec{l}_3+\vec{l}_4)\cdot(-\vec{l}_1+\vec{l}_3+\vec{l}_4)$.

Then, we obtain,

$$l_2^2=l_1^2 l_3^2 l_4^2-2l_1\cdot l_3-2l_1\cdot l_4+2l_3\cdot l_4 \tag{6}$$

Substituting $\vec{l}_1(\theta)$, $\vec{l}_3(\phi)$ into Eq. (6) yields the general governing equation as,

$$C_1(\theta)\cos\phi+C_2(\theta)\sin\phi+C_3(\theta)=0 \tag{7}$$

where,

$$\begin{aligned}C_1(\theta)=&2l_3l_4\sin\beta\sin\alpha+(2l_1l_3/\sin\eta\sin\beta)\\&[\{\cos\beta-\hat{p}_1\cdot\hat{p}_2)\cos\eta-\sin^2\eta\}\sin\xi\sin\alpha\cos\theta\\&-\hat{p}_1\cdot(\hat{p}_2\times\hat{p}_4)\sin\xi\sin\alpha\cos\beta\sin\theta\\&+\{(\hat{p}_1\cdot\hat{p}_2\cos\beta-\cos\eta\}\sin\eta\cos\xi\sin\alpha]\end{aligned}$$

$$\begin{aligned}C_2(\theta)=&(2l_1l_3/\sin\eta\sin\beta)\\&[\hat{p}_1\cdot(\hat{p}_2\times\hat{l}_4)\cos\eta\sin\xi\sin\alpha\cos\theta\\&+\{\cos\eta\cos\beta-(\hat{p}_1\cdot\hat{p}_2)\}\sin\xi\sin\alpha\sin\theta\\&-\hat{p}_1\cdot(\hat{p}_2\times\hat{p}_4)\sin\eta\cos\xi\sin\alpha]\end{aligned}$$

$$\begin{aligned}C_3(\theta)=&l_1^2+l_2^2+l_3^2+l_4^2\\&-2l_1l_4(\sin\xi\sin\eta\cos\theta+\cos\xi\cos\eta)\\&+2l_3l_4\cos\beta\cos\alpha-2(l_1l_3/\sin\eta\sin\beta)\\&[\{\cos\beta-(\hat{p}_1\cdot\hat{p}_2)\cos\eta\}\sin\xi\sin\beta\cos\alpha\cos\theta\\&-\hat{p}_1\cdot(\hat{p}_2\times\hat{p}_4)\sin\xi\sin\beta\cos\alpha\sin\theta\\&+(\hat{p}_1\cdot\hat{p}_2)\sin\eta\cos\xi\sin\beta\cos\alpha]\end{aligned}$$

Using Half-tangent form as

$$\cos\phi=\frac{1-\tan^2(\phi/2)}{1+\tan^2(\phi/2)} \text{ and } \quad \sin\phi=\frac{2\tan(\phi/2)}{1+\tan^2(\phi/2)} \tag{8}$$

Eq. (7) becomes $(C_1-C_3)X^2-2C_2X-(C_1+C_3)=0$, where $x=\tan(\phi/2)$. From which

$$\phi=2\tan^{-1}\left\{\frac{-C_2\pm\sqrt{C_1^2+C_2^2-C_3^2}}{C_3-C_1}\right\} \tag{9}$$

where $\pm$*signs* shows two possible solutions exist. The DCM method has successfully applied to RGCR, RCCC and RRGG.

2.1.5 Reflection transformation method (RTM)

The most difficult aspect of spatial mechanism's displacement analysis is the mathematically complicated functional relationships between input and output variables, especially for the five or more link spatial mechanism. Therefore, the most methods developed so far require numerical or trial-and-error schemes for the mobility analysis of spatial mechanisms, due to the high degree of the polynomial in the input-output relationship of the mechanism. This high-degree polynomial represents many limit points on the travel of the mechanism, which in turn makes it difficult to analysis its mobility condition. In an effort to overcome these difficulties, the RTM is developed for the derivation of the reduced-degree of polynomial as the closed form solution for the spatial RG-group five-link mechanisms. By the reflection transformation (by symmetry), the solution of the spatial mechanism is reduced order in half.

As an example of the planar 4-bar linkage, Fig. 4 is shown. We could easily obtain the functional relationship in a parabolic equation as

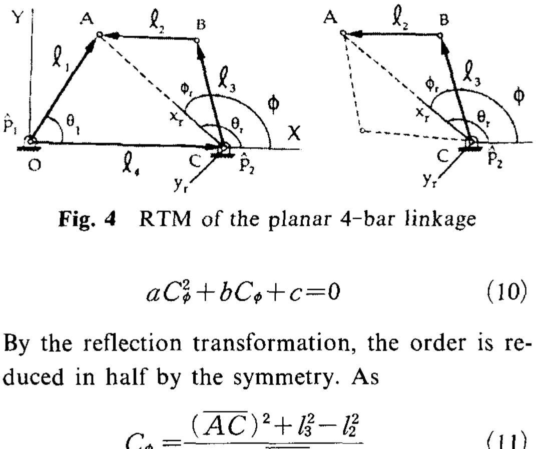

Fig. 4 RTM of the planar 4-bar linkage

$$aC_\phi^2+bC_\phi+c=0 \tag{10}$$

By the reflection transformation, the order is reduced in half by the symmetry. As

$$C_{\phi_r}=\frac{(\overline{AC})^2+l_3^2-l_2^2}{2l_3\overline{AC}} \tag{11}$$

For the spatial 4-bar mechanism shown in Fig. 5, RTM is applied as follows;

$$(\vec{l}_1)_g=(l_1S_{\alpha1}C_{\theta1},\ l_1S_{\alpha1}S_{\theta1},\ l_1C_{\alpha1},\ 1)^T \tag{12}$$

$$(\vec{l}_1)_r=([L_{\alpha_4}][Y_{\rho_4}][X_{\gamma_4}][Z_{\theta_r}])^{-1}(\vec{l}_1)_g \tag{13}$$

$$=[RT](\vec{l}_1)_g \tag{14}$$

$[RT]$: Reflection Transformation matrix

By the reflection transformation, the reduction of the order in half by the symmetry as

$$C_{\phi_r} \frac{(\overline{AC})^2 + l_3^2 S_{\alpha 3}^2 - l_2^2 + ((\vec{l_1})_{2z} + l_3 C_{\alpha 3})}{2\overline{AC}\, l_3 S_{\alpha 3}} \quad (15)$$

And output angles are

$$\phi = \theta_r \pm \phi_r \quad (16)$$

For the spatial 5-bar mechanism shown in Fig. 6, the known quantities of the mechanism are the lengths l_1, l_2, l_3, l_4, l_5, the vector $\vec{l_5}$, the directions of rotations $\hat{p}_1$, $\hat{p}_4$, the angles α_1, α_2, α_3, α_4, α_5, from the construction of mechanism. The input angle θ_1 is chosen as the angle between the two planes formed by $\hat{p}_1$ and $\hat{l}_1$, and $\hat{p}_1$ and $\hat{l}_5$ in which the $\hat{p}_1\hat{l}_5$-plane chosen as a reference. The angles θ_2, θ_3, θ_4 are the unknown output angles.

By the RTM, we had the following equations.

$$\begin{aligned} &G_1 \sin \theta_2^2 + G_2 \cos \theta_2^2 + G_3 \sin \theta_2 \cos \theta_2 \\ &+ G_4 \sin \theta_2 + G_5 \cos \theta_2 + G_6 \\ &= G_7 \cos \theta_{c4}^2 + G_8 \cos \theta_{c4} + G_9 \end{aligned} \quad (17)$$

and

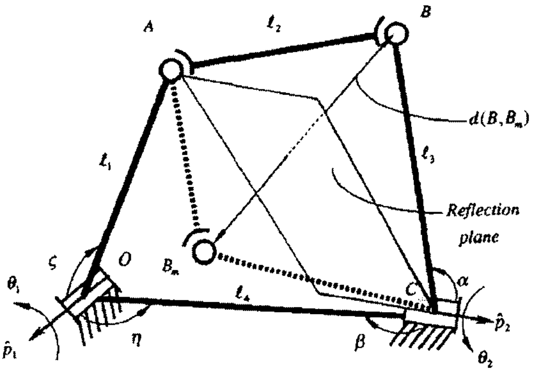

Fig. 5 The schematic diagram of RGGR mechanism

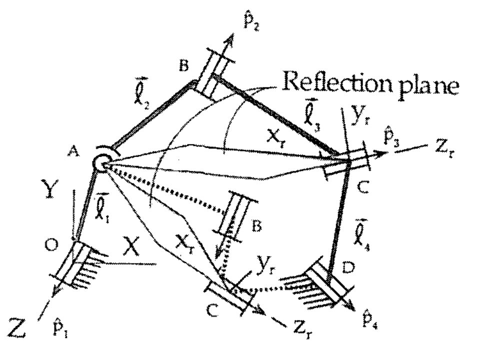

Fig. 6 The two reflection planes in the RGRRR mechanism

$$\begin{aligned} &G_{10} \sin \theta_2 + G_{11} \cos \theta_2 + G_{12} \\ &= G_{13} \cos \theta_{c4} + G_{14} \end{aligned} \quad (18)$$

where the coefficients G_is are the known values for given input angle θ_1. As arranging second equation about the angle θ_{c4} and substituting it into the first equation, we can obtain as

$$\begin{aligned} &(G_4 G_{13}^2 - 2G_7 G_{10}(G_{12} - G_{14}) - G_8 G_{10} G_{13}) \sin \theta_1 \\ &+ (G_5 G_{13}^2 - 2G_7 G_{11}(G_{12} - G_{14}) - G_8 G_{11} G_{13}) \cos \theta_2 \\ &+ G_6 G_{13}^2 - G_7 (G_{12} - G_{14})^2 - G_8 G_{13}(G_{12} - G_{14}) \\ &- G_9 G_{13}^2 + (G_1 G_{23}^2 - G_7 G_{12}^2) = 0 \end{aligned} \quad (19)$$

Above equation is the two-degree polynomial as the closed form solution of the RGRRR mechanism. After all, this method reduces the 4 degrees of polynomial equation into a quadratic one in the mechanism. Additionally, the unknown angle θ_{c4} can be calculated in second equation from the computed angle θ_2, and θ_3 can be calculated as follows,

$$\theta_3 = \theta_{r4} - \theta_{r3} \quad (20)$$

where the angles θ_{r4} and θ_{r3} can be computed from the fact that the each y component in the RTM equation. Finally, the unknown angle θ_4 can be computed.

2.2 Dynamics of mechanisms

In the 1950s, the digital computers in industry and engineering programs at university became available increasingly. In the mechanism analysis and synthesis, several programs were developed at this period by Al Hall et al. at Purdue, C. W. Mclarnan's group at Ohio State, J. E. Shigley et al. at Michigan, F. Fredenstein's group at Columbia, and J. Denavit and R. Hartenberg at Northwestern. The graphical-based techniques suggested by Burmester in 1876 were reformulated for computer solution. The computer became more available to university researchers in the early to mid 1960s. Many researchers began to utilize the power of the computer for solving equations which were too tedious by either graphical or slide rule techniques. Although there was some initial success with analog and digital computers in solving differential equations of motion, numerical methods for integration, such as Runge Kutta, caused the analog devices to be phased out. The early 1970s saw a spurt in applications on the computer. Codes such as IMP (1973), developed by P. Sheth and J. Uicker at the university of Wisconsin, and DRAM (1971) and ADAMS (1973), developed at the university of Michigan by D. Smith, N. Orlandea, and M. Chace, had early roots in this decade. Also, computer graphics applied to mechanism design received its christening in the early 1970s by G. Kaufman and KAM (1964), KIMAC (1979) and DYMAC (1979) were coded by B. Paul in this time. KINSYN (1977) was a custom-built program at M. I. T. and should be recognized as the major milestone in kinematic design. The 1980s has exhibited a burst in activity in mechanisms for several reasons, and also has seen the beginning of integration of mechanism analysis, synthesis, and dynamics with other computer-aided design areas, such as drafting, finite elements and simulation. As an example, DADS (1981) was developed by E. Haug's group, ROBSIM (1986) by J. Davidson, LINCAGES (1986) by A. Erdman. The future of the integration of the computer mechanism design looks very exciting.

Several specific areas will see increased activities. These include: use of solid modelers for the display and analysis; integration of mechanism analysis and synthesis software into other phases of computer-aided design and manufacture; many more custom applications to specific needs of industry; more computer-assisted analysis and design for machine elements: gears, cams, indexers, etc.; better techniques for analysis and simulation of more complex problem, including clearances, deflections of links, friction, damping, etc.: the development of computer-aided type synthesis techniques for inexperienced designers which will include expert systems and artificial intelligence techniques; the use of more sophisticated graphics, including vector refresh simulations; increased development of mechanism design software on micro- and desktop computer; use of the super computer that will permit large-scale design and simulation.

A new scheme of Hybrid Method for multibody displacement analysis is proposed in our research. Based on the Hybrid Method, a new software package ACUBE which deals with multibody mechanical system is developed. The Hybrid Method combines the analytical approach and the iterative one. Up to now, most of multibody systems are solved using the iterative displacement analysis schemes. However, we developed the Hybrid Method in order to use the analytical scheme in advance to depend on the iterative one.

In the analytical approach, the noble table of number synthesis, called as Lee-Youm table is developed. This table is the milestone to determine whether a given multibody mechanical system is solved using its closed form solution or not in Hybrid Method.

A large-scale kinematics and dynamics computer code, ACUBE has been developed to implement the theory presented both the kinematic and dynamic analysis of the spatial mechanical systems. The ACUBE software is computer program built in IBM-PC so that it can be used to model and predict the motion of a variety of real world mechanical systems. Using a set of data that describes the machine to be modeled, ACUBE builds a mathematical model of the real system that performs kinematic analysis, dynamic force analysis (or kinematically driven dynamic an-

Table 1 Lee-Youm table — the kinds of zero degree-of-freedom kinematic chains having the closed form solutions based on the six lower pairs

No. links	Kinds
6	3R+3P 2R+3P+1S 1R+3P+2S 3P+3S
5	2R+2P+1C 1R+2P+1S+1C 2P+2S+1C
4	1R+1P+2C 1P+1S+2C 3R+1G 2R+1P+[1G, 1F] 1R+2P+1G, 1R+1P+1S+1F 3P+1G, 1P+2S+1F
3	1R+1C+[1G,1F] 1P+1C+1G, 1S+1C+1F, 3C

alysis), and dynamic motion analysis (or initial value problem).

3. Open Loop System

In 1970s, the robot applications in industry were utilized successfully and in 1980s the robotics became one of the attractive topics for the multibody dynamic community. The earlier stage robots such as Puma, Milacron, ABB, Fanuc, P200, Adept, etc were the open loop mechanical system. However, the redundant open loop system became more popular in dynamics and control aspects in research community. A robotic manipulator which is open loop system is called (kinematically) redundant if it possesses more degrees of freedom than is necessary for performing a specified task. The extra degrees of freedom, namely redundancy, of a robotic manipulator are therefore determined relative to the particular task to be performed.

Redundancy in a manipulator structure yields increased dexterity and versatility for performing a task due to the infinite number of joint motions called self-motion or null-motion, which result in no end-effector motion. In order to take full advantage of the capabilities of redundant manipulators, effective control schemes should be developed to utilize the redundancy in some useful manner. At first the kinematic redundancy was utilized to avoid various kinematic limitations from which conventional non-redundant ones suffers, e.g. to stay within joint travel limits (Liegeois, 1977) to avoid so-called kinematic singularity (Yoshikawa, 1984), to improve connectivity of the task space (Borrel and Liegeois, 1986), for obstacle avoidance (Maciejewski and Klein, 1985; Khatib, 1986). Later there were several approaches to utilize redundancy for the control effort minimization (Hollerbach and Suh, 1987). During the past decades, redundant manipulators have been the subject of considerable research, and several methods have been suggested to resolve the redundancy.

3.1 Kinematic analysis of redundancy

In particular inverse kinematic algorithms for redundant manipulators were called the kinematic resolutions of redundancy. Literature shows that there are three main classes for kinematic resolutions of redundancy (Nenchev, 1989). Basic equations about kinematic resolutions of redundancy is as follows:

$$p=J(q)\dot{q},\ V\in R^6,\ \dot{q}\in R^N \tag{21}$$

where $J(q)$ is a Jacobian matrix of $6\times N$

$$\begin{cases} N=6 : \textit{non-redundant} \\ N>6 : \textit{redundant} \\ N<6 : \textit{deficient} \end{cases}$$

The solution about upper equation has the form as

$$\dot{q}=G(q)p+(I-G(q)J(q))z \tag{22}$$

where $G(q)$ is a generalized inverse of $J(q)$ of $n\times m$. First term in the right equation (22) is called as a particular solution and second term as a homogeneous solution.

In obtaining the homogeneous solution, the following kinematic control schemes have been introduced.

3.1.1 Resolved motion rate method

Whitney (1972) suggested first the use of pseudoinverse or Moore-Penrose inverse (Ben-Israel, 1980) of the manipulator Jacobian matrix for the kinematic control of redundant manipulators. The pseudoinverse is one of the types of generalized inverse that has a least squares property (Ben-Israel, 1980). But, in the pseudoinverse control proposed by Whitney, the pseudoinverse is one of the minimum-norm generalized inverses that minimize the sum of squares of joint velocities. Presumably any joint is prevented from moving too fast, leading to a more controllable motion. It is also presumed that squared velocities are approximately related to kinetic energy, which would then also be approximately minimized.

Liegeois (1977) proposed a modification of the pseudoinverse approach, named *resolved motion method*, by using the general solution to joint velocity which is composed of the pseudoinverse solution and the homogeneous solution corresponding to net-motion and null-motion of the end-effector, respectively. Additionally, he specified the homogeneous solution for optimizing a scalar performance function onto the null space of the joint variables by projecting the gradient vector of this function onto the null space of the Jacobian matrix, and demonstrated first the performance function for the criterion of avoidance of joint limits. Klein and Huang (1983) showed that the pseudoinverse control is not conservative, that is, repetitive motions planned with the pseudoinverse alone do not return at a given end-effector position to the same joint configuration. It was recognized that weighted pseudoinverses with constant matrices are not integrable.

3.1.2 Task space extension method

An alternative approach to the kinematic control of redundant manipulators, called the extended jacobian method, was proposed. Baillieul et al. (1984) made repetitive motions planned with the resolved motion method conservative by combining null space motions with the pseudoinverse solution via the extended jacobian method (Ballieul, 1985). This method obtains the additional equations of which number equals to the redundancy by using the orthogonality between the null space vectors of the Jacobian matrix and the gradient vector of a performance function. Then the extended Jacobian matrix is obtained by differentiating the augmented kinematic equations which are composed of the kinematic equations and the additional equations.

The extended Jacobian method, which is seemed to be velocity-based, is considered as being an application of Newton's method and thus as a position based method. A Newton-Raphson numerical procedure was developed by Oh et al. (1984), which is based on a composite Jacobian which includes rows for all members under constraint. Benhabib et al. (1985) presented a new algorithm for solving the inverse kinematics of redundant manipulators using the method of generalized inverse kinematics based on a modified Newton-Raphson iterative method. To provide the exact equilibrium state for the resolved motion method, Chang (1987) presented formulations for converting a minimization criterion into constraint function. Essentially, the constraint is that the projection of the gradient of the minimization criterion onto the null space of the manipulator Jacobian must be zero. Wampler (1987) proposed the inverse function method based on a single inverse function giving the joint coordinates for each point in some subset of the task (or operational) space. In constructing such a function, the redundancy may be used to reduce joint speeds and avoid known obstacles. For a very general class of kinematic inversion algorithm, Baker and Wampler (1988) proved that tracking algorithms which use additional constraint functions to invert the kinematics on redundant manipulators produce cyclic behavior, if the subset of the operational space in which paths can be tracked is simply connected. In order to determine a priori whether a local kinematic control strategy guarantees repeatability or not, Shamir and Yomdin (1988) deduced its global properties as well as its local properties by considering integral surfaces for a distribution in the joint space. This yields a necessary and sufficient condition, in terms of Lie brackets, for a control to be repeatable.

Seraji (1989) presented the configuration control of redundant manipulators in which the redundancy is utilized for control of the manipulator configuration directly in task space. A set of kinematic functions in joint space is chosen to reflect the desired additional task that will be performed due to the redundancy. The kinematic functions can be viewed as a parameterization of self-motion. The end-effector cartesian coordinates and the kinematic functions are combined to form a set of "Configuration variables" which describe the physical configuration of the entire manipulator in the task space.

3.1.3 Task priority based method

Maciejewski and Kiein (1985) presented a new obstacle avoidance approach by using the task-priority based on kinematic control. This approach is to identify for each period in time the point on the manipulator that is closest to an obstacle, termed the obstacle avoidance point, and assign to it a desired velocity component in a direction that is directly away from the obstacle surface. However, this obstacle avoidance algorithm requires of knowing the position on the manipulator of the obstacle avoidance point in real-time it seems that tracking the motions of this point will be fairly complicated.

Hanafusa et al.(1981) and Nakamura et al. (1987) used the homogeneous solution of the resolved motion method to impose a priority to manipulation variables. For the tasks with the order priority, if it is impossible to perform all of subtasks completely because of the degeneracy or the shortage of degrees of freedom, this task priority based on kinematic control performs the most significant sub task preferentially and the less important subtasks using the remaining degrees of freedom.

3.2 Dynamic analysis of redundancy and control

Many researchers have discussed how to resolve redundancy at the kinematic level. However, the dynamic resolution of redundancy which is taking account of manipulator dynamics shoud be discussed in comparison with the kinematic resolution of redundancy in spite of the fact that robotic manipulators are actually controlled by specifying the joint torques to track a defined end-effector trajectory. To incorporate a generalized inverse (Ben-Israel, 1980) into the manipulator dynamics, the pseudoinverse must be formulated in terms of accelerations. Khatlb (1983, 1987) was one of the first researchers to do this, in his case using the inertia- weighted pseudoinverse which truly minimizes instantaneous kinetic energy, while the pseudoinverse minimizes the sum of squares of joint velocities and thus approximately minimizes the kinetic energy. Vukobratovic and Kircanski (1984) broadened the method of Khatib to include energetic model and generated nominal joint trajectory so as to be optimal with respect to total energy consumption of the actuators.

Many dynamic control algorithms were designed based on dynamic resolution method and conventional dynamic control algorithms for non redundant manipulators (Khatib, 1987). Hirose and Ma (1989) proposed a dynamic control method, named "Redundancy Decomposition Control (RDC)," which decomposes the degrees-of-freedom of a redundant manipulator into a subset of non-redundant combinations. Hsu et al.(1989) proposed a dynamic feedback control law that guarantees the tracking of a desired end-effector trajectory and provides redundancy resolution by making the self-motion of a redundant manipulator along the projection of a given arbitrary vector field (e.g., the gradient vector of a performance function) onto the null space of the manipulator Jacobian.

Historically considered, dynamic treatment of kinematic redundancy was initiated by the motivation to minimize inverse dynamic torque to realize the task motion (Hollerbach and Suh, 1987). In order to incorporate the manipulator dynamics, Hollerbach and Suh (1987) resolved the redundancy at the acceleration level rather than at the velocity level, and then complemented the minimum-norm acceleration, i.e., the pseudoinverse solution by using a homogeneous (or null space) acceleration to locally minimize the norm of joint torque. Kazerounian and Nedungadi (1987) proposed the homogeneous acceleration term induced by Lagrange's undetermined coefficient method for the optimization of the joint driving force. A formalism for global torque optimization problem was introduced by Nakamura and Hanafusa (1987). Considering this problem as an ordinary optimal control problem, they applied the Pontryagin's Maximun Principle to the inverse kinematics which was resolved at the acceleration level. Suh and Hollerbach (1987) presented the global method of torque optimization which parameterized the redundancy of a manipulator and applied the calculus of Variations. This formulation requires an explicit inverse kinematic solutions and extra time derivative of the joint variables.

Because self-motion also evolves by its own dynamical equations and the dynamics can not be trivially specified, it will be shown that the trial to minimize joint torque using kinematic redundancy produced a puzzling phenomenon, which was named the torque instability. Nedungadi and Kazerounian (1989) pointed out that it is due to local optimization characteristics of joint torque minimization scheme. Maciejewski (1991) analyzed the kinetic effects of a homogeneous acceleration for the local torque minimization method (Hollerbach, 1987), and presented a kinematic condition to identify regions of stability and instability for this method.

The first paper dealing with self-motion dynamics explicitly appeared in Park et al.(1996). Park et al. described self-motion dynamics based on kinematically decoupled joint space decomposition and suggested the control method which puts equal emphasis on self-motion dynamic control as well as the task dynamic control and the algorithm which removes torque instability. In spite of many kinematical successes, two difficulties in dynamic control of kinematically redundant manipulator exist. One of the problems is the mentioned torque instability, and the other problem is difficulty in stabilizing self-motion (Klein and Chirco,1987; Luca, 1989; Oriolo, 1994).

4. Closed & Open Loop (Switching) Systems

In this section multibody dynamic system which has both closed and open discontinuous system is discussed. The analysis method of the multibody dynamic system has been an independent modeling about each case of system state (open/closed). This method has an advantage of simplicity in modeling but has a disadvantage that investigator must remodel the system dynamics for each varying system environment (contact/detachment with external environment). A new method, "dynamical balance" is proposed through which method the system dynamics could be modeled comprehensively according to the change in contact with situation. As an example of the switching system, human gait of biomechanics and humanoid robot which act and react with environment through contact are introduced.

4.1 Human gait study in biomechanics

Human gait is grossly divided into two phases, single support phase and double support phase. Single support phase means that human is walking with his single leg and with remaining leg swinging in the air to be an open loop system. Double support phase means that both of foots are in contact with the ground so that the gait system becomes closed loop system.

Dynamic analysis in biomechanics begins with experiment and not with the model analysis of the human system itself. As an example, in order to obtain a torque data in joints of human walking, measurements about inertia data for each body segments and calculating the mass center position/velocity during walking are performed. Finally through the free body diagrams about joint torque and forces are calculated. Also to obtain a contact force between foot and ground,

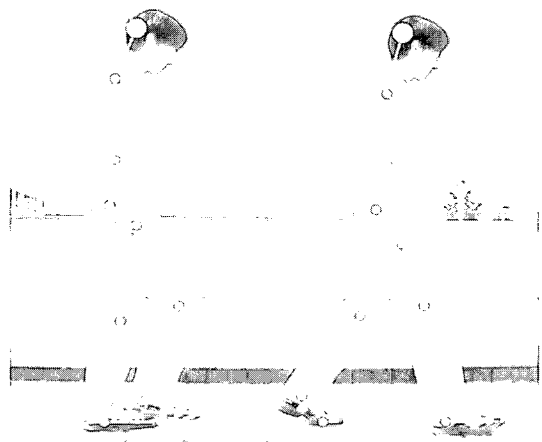

Fig. 7 Open & closed loop in human walking

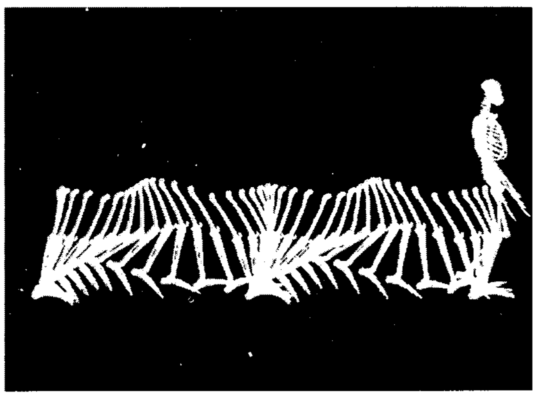

Fig. 8 The bone stereograph of human walking

force platform is used to acquire a data during human walking.

4.2 Humanoid robot

In robotics, the humanoid robot is a typical example of multibody dynamic system of closed & open phase change. The dynamics of the humanoid includes a contact, vibration, redundant system problem. Especially, humanoid system changes its kinematic loop as open and closed which is a switching dynamic system through walking and grasping an environment object with the hand.

In the following section, the method dynamical balance (Park, 2004) which could efficiently analyze a switching dynamic system as a humanoid.

4.3 Multibody system dynamics

Multibody systems have been given much research effort during the last two decades. It seems that a recent innovation was demanded by mainly three application areas : computer animation, virtual reality, and robotics. It is evident that relatively complete understanding of dynamic behavior of multibody systems and proper dynamics-theoretical approach improves the reality of virtual dynamic agents. In robotics area, greater interest lies in motion and manipulation planning consistent with multibody dynamic behaviors. With the advent of new platforms such as humanoids and dexterous robotic hands, the center-of-mass of research has seemed to move to multibody systems with continuously varying relatively fixed topology and system configuration such as open-loop manipulators and closed-loop mechanisms.

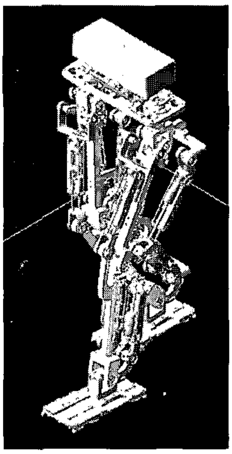

Fig. 9 Humanoid walking

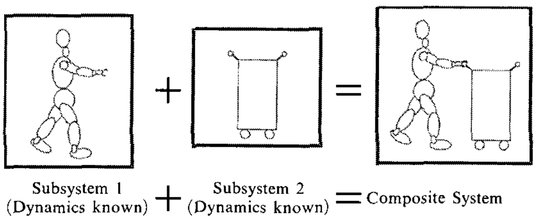

Fig. 10 Change of dynamics in humanoid task

Just with the minimalistic attitude, every multibody systems can be considered to be comprised of a number of single bodies, many instances of bilateral constraints and unilateral constraints. The dynamics of a single body is completed theoretically and is also believed so practically. It is described by Newton-Euler equations of motion, which is of the form of second-order ordinary differential equation (ODE) in generalized coordinate (GC) vector. Then, a set of independent bodies is described by a system of decoupled ODEs in aggregate of every GC vector.

Suppose that there are some active bilateral constraints. They can be represented as a system of algebraic equations in GC and/or GC velocity vector. Any attempts to violate those are negated by enforcing constraint forces. Coupled with these constraint forces, the system of single bodies under bilateral constraints are effectively described by a set of differential-algebraic equations (DAEs). Resorting to a variety of DAE, one can solve bilaterally-constrained multibody system (MBS) dynamics. Many bodies which have been independent of each other become regulated by bilateral constraints, effectively constituting a smaller number of articulated multibody systems. It is worth noting that some of them involve closed-loop in connectivity of the bodies.

Recently, this philosophy has been extended to deal with unilaterally constrained dynamic systems. One of the main tools is the linear complementarity programming (LCP). It derives from the fact that the constraint force is induced only when a unilateral constraint is active, or vice versa. It is regarded that the LCP formulation of unilateral constraints is the mathematical encapsulation of every possible cases. One should be aware that the treatment of unilateral constraints in multibody dynamics context is not completed at all. Rather, it is still evolving. In particular, if many contacts is involved between many rigid body pairs and the contact force is supported by Coulomb friction model, the solution technique is not completed in theory. Most current approaches based on complementarity formulation employ an approximation to render problem solvable by using LCP solver.

There is another sophisticated phenomenon related with impact in dealing with unilateral constraints. This behavior is quite frequent under unilateral constraint as the constraints are not constantly active. The system configuration changes intermittently as it evolves. Main difficulty of impact theory lies in the fact that the rigid body assumption is prone to fail at the impact regime. Hence, the rigid-body impact theory is only an approximation of complex physical processes, involving wave propagation and infinitesimal deformations, which actually occurs during impact. Further, the approximation is much cruder. Nevertheless, the net effect of impact process could be summarized as an abrupt velocity jump. Hence, once the post-impact velocities have been obtained, one can continue without impact for the time being. A greater difficulty is encountered in analysis when an articulated multibody system (AMBS), already subject to many active unilateral and bilateral constraints, experiences multiple, possibly simultaneous, impacts. If frictional effect is not negligible, which is indeed the case for almost every problems, the difficulty becomes more exaggerated. Within our knowledge, it does not seem that there exists a complete mathematical model or tool to deal with such a general problem, which is favorable from the rigid-body dynamics-theoretic viewpoint. Every available method relies on its own approximation and can solve only a class of problems.

4.4 Principle of dynamical balance

We have recently proposed a new formalism for multibody dynamics within the context of composition of subsystem dynamics. Composition of subsystem dynamics refers to a sort of mathematical operations which generate valid equations of motion for a composite system, consisting of two interacting multibody subsystems. It is assumed that the subsystem dynamics are already known. Then the question is how to compose two subsystem dynamics to derive the composite system dynamics ?

To this end, first we develop the Lie group theoretic formulation of rigid body kinematics and dynamics. Within the formulation, two most beneficial geometric entities are the body twists and wrenches, which are velocity- and force-equivalents in particle kinematics and dynamics. The notion of twists and wrenches provides us with a succinct and consistent mean to express spatial rigid body motion. In addition, it facilitates systematical joint modeling at the twist (or velocity) level, not the homogeneous transform (or position) level.

We concretize the notion of dynamical balance between two subsystems. To do this, the concept of d'Alembertian wrenches and torques are devised to encapsulate, or abstract, subsystem dynamics. We propose the principle of dynamical balance that the d'Alembertian wrenches and torques of two subsystems should be balanced for a composite system. In other words, composite system dynamics is obtained by taking dynamical balance between d'Alembertian wrenches and torques of two subsystems. The principle is quite unique in that the dynamical balance equation is derived just from the dual expression of kinematical constraints due to mechanical interaction. We will prove the principle using d'Alembert's principle later in this article.

It is worth noting that the dynamical balance equation is a sort of encapsulated version of the

(a) Fixed loading (b) Jointed loading

Fig. 11 Loading

equations of motion for the system. The d'Alembertian wrench encapsulates the details of dynamics of every individual system. In other words, d'Alembertian wrench is the abstraction of the system dynamics.

As long as the system dynamics is known, one can obtain the concrete expression of d'Alembertian wrench by rearranging the dynamics. Then, the dynamical balance yields the true equations of motion for the system, once the d'Alembertian wrench for the system is substituted.

4.4.1 Relative twist formulation of geometric constraints

There are many kinds of geometric constraints on motion of multibody systems. For example, a body of an AMBS can be joined to another body of a different AMBS, composing a larger AMBS (called *joining*); a body of an AMBS can be connected to the ground (called *earthing*); when a body of an AMBS is joined to another body of the same AMBS, a kinematic loop is created (called *looping*); or the like.

All of the exemplifying constraints share one common aspect that relative motion between two bodies is restrained effectively. Hence, the aforementioned geometrical constraints can be formulated in terms of the homogeneous transform and the associated relative body twist. Specifically, for any two bodies α and β, the constraint can be expressed as a parameterized homogeneous transform ${}^{\alpha L\beta}T_{\beta L\alpha}$, where $\{\alpha L\beta\}$ and $\{\beta L\alpha\}$, called the loading frames, are body-fixed frames attached to body α and β, respectively.

The relative body twist between two loading frames $\{\alpha L\beta\}$ and $\{\beta L\alpha\}$ provides the generalized notion of the relative velocity of two points in point kinematics. It is related with the time-derivative of the homogeneous transform between two frames, but not directly. Let us denote the relative twist by ${}^{\alpha L\beta}V_{\beta L\alpha}=({}^{\alpha L\beta}\nu^T{}_{\beta L\alpha},\ {}^{\alpha L\beta}\omega^T{}_{\beta L\alpha})$. As mentioned, constraints restrain the relative body twist of two bodies. Basically, the relative twist is the difference between the body twists of the two bodies. This is expressed succinctly as

$$ {}^{\alpha L\beta}V_{\beta L\alpha}={}^{\beta}\mathrm{Ad}_{\beta L\alpha}^{-1}V_\beta-{}^{\alpha L\beta}\mathrm{Ad}_{\beta L\alpha}^{-1}{}^{\alpha}\mathrm{Ad}_{\alpha L\beta}^{-1}V_\alpha \quad (23) $$

where the coordinate transformation of body twists is done by the adjoint transformation defined as follows.

General geometric constraints on relative motion of two bodies are expressed as

$$ {}^{\alpha L\beta}T_{\beta L\alpha}=T_j(q_j,\ t) \quad (24) $$

Hence, feasible relative body twists are parameterized by the joint velocity $\dot{q}_j$ as

$$ {}^{\alpha L\beta}V_{\beta L\alpha}=E_j(q_j,\ t)\,\dot{q}_j+E_t(q_j,\ t) \quad (25) $$

When the motion constraint (25) is made use of, the relative body twist (23) yields

$$ V_\beta={}^{\alpha}\mathrm{Ad}_\beta^{-1}V_\alpha+{}^{\beta}\mathrm{Ad}_{\beta J\alpha}E_j\dot{q}_j+{}^{\beta}\mathrm{Ad}_{\beta J\alpha}E_t \quad (26) $$

This is called the loading constraint between two bodies α and β.

4.4.2 Joining two subsystems

One effect of geometric constraints is to kinematically restrain the relative motion. The other effect of dynamical significance is the so-called dynamical balance of system dynamics. Consider the jointing operation on two subsystem dynamics. Provided that each subsystem dynamics is known, joining operation formulates the system dynamics of a larger system composed of two subsystems. Fig. 12 shows one such example. Let us assume jointed loading between body α and β. Consequently, we are dealing with the composite system shown in Fig. 13. The loading imposes the loading constraint with $E_t=0$, rewritten below

$$ V_\beta={}^{\alpha}\mathrm{Ad}_\beta^{-1}V_\alpha+{}^{\beta}\mathrm{Ad}_{\beta J\alpha}E\dot{q} \quad (27) $$

Recognize that the loading constraint is written with the body twists of intermediate bodies, not directly with the GC vectors themselves. Using the joint index partition, the jointed-loading (27) can be regarded as a constraint between V_1, $\dot{q}_1$, V_2, $\dot{q}_2$ and $\dot{q}$. In particular, it is written in the following form

$$ \begin{aligned} V_2={}&{}^{2}\mathrm{Ad}_{\beta\alpha}\mathrm{Ad}_\beta^{-1}V_1+{}^{2}\mathrm{Ad}_\beta{}^{\alpha}\mathrm{Ad}_\beta^{-1}G_{1:\alpha}\dot{q}_{1:\alpha}\\ &+{}^{2}\mathrm{Ad}_\beta{}^{\beta}\mathrm{Ad}_{\beta J\alpha}E\dot{q}-{}^{2}\mathrm{Ad}_\beta G_{2:\beta}\dot{q}_{2:\alpha} \end{aligned} \quad (28) $$

which is the canonical form of the loading constraint in composition.

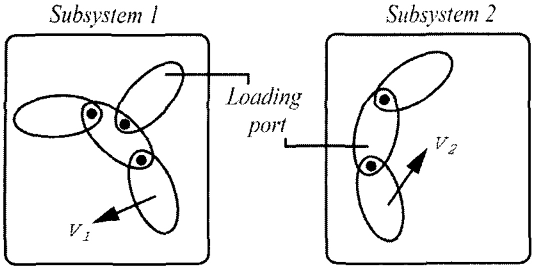

Fig. 12 Two decoupled subsystems

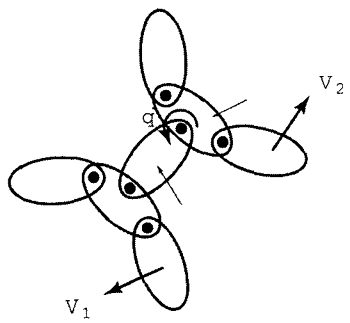

Fig. 13 Extended systems

Now, we can state the basic version of the principle of dynamical balance for joining two subsystem dynamics.

Theorem 1 (Principle of Dynamical Balance) Under the loading constraint, e.g. (28), between two bodies of two subsystems, the composite system should satisfy the balance condition, between d'Alembertian wrenches and torques of two subsystems, defined by the dual expression of the constraint

$$F_1^* = -{}^1\mathrm{Ad}_\alpha^{-T\,\alpha}\mathrm{Ad}_\beta^{-T\,2}\mathrm{Ad}_\beta^T F_2^* \tag{29a}$$

$$\tau_{1:\alpha}^* = -G_{1:\alpha}^T{}^{\alpha}\mathrm{Ad}_\beta^{-T\,2}\mathrm{Ad}_\beta^T F_2^* \tag{29b}$$

$$\tau_{1:\alpha}^* = 0 \tag{29c}$$

$$\tau = -E^{T\beta}\mathrm{Ad}_{\beta J\alpha}^T{}^2\mathrm{Ad}_\beta^T F_2^* \tag{29d}$$

$$\tau_{2:\beta}^* = G_{2:\beta}^T{}^2\mathrm{Ad}_\beta^T F_2^* \tag{29e}$$

$$\tau_{\beta:}^* = 0 \tag{29f}$$

Dynamical balance is described as the dual expression of the loading constraint in terms of d'Alembertian wrenches, associated with the body twists, and torque, associated with the joint velocity.

4.4.3 Notion of d'Alembertian wrenches and torques

Consider a single rigid body B which can move by the body twist V_B under the influence of the body wrench F_B. Assume that the body takes static equilibrium. Then it is easy to see that the net body wrench F_B should be zero. The null body wrench is the necessary and sufficient condition to static equilibrium for a single rigid body. This condition is called the *statical balance*

$$F_B = 0 \tag{30}$$

The term 'balance' reflects the idea that possibly many wrenches derived from different sources are canceling out each other, which leads to zero net wrench. Now assume that the net wrench F_B is not zero. Obviously statical equilibrium cannot be sustained, and the body begins to move. It is the equations of motion that describes the ensuing motion of the body exactly. For a single rigid body, it is written as

$$F_B = A_B\dot{V}_B + B_B V_B \tag{31}$$

From a different perspective, the equations of motion are just the statement of balance between the statically unbalanced, hence nonzero, wrench F_B and the wrench-equivalent induced by dynamical motion of the body. The right-hand side expression $A_B\dot{V} + B_B V_B$ corresponds to the induced dynamical wrench. Just as statical balance is written in terms of the net wrench, the so-called d'Alembertian wrench enables us to express *dynamical balance* succinctly as

$$F_B^* = 0 \tag{32}$$

The d'Alembertian wrench is denoted by the same notation as the body wrench except the superscript '*'. The notational similarity is intentionally devised in order to develop systematically the principle of dynamical balance. Roughly speaking, the principle asserts that any statical balance equation expresses dynamical balance by replacing the involved wrenches with their respective d'Alembertian wrenches. By the notational convention, this is done by simply attaching '*' to each body wrench. For the system in consideration, statical balance is expressed by

Eq. (30). When the involved body wrench F_B is replaced with its d'Alembertian wrench F_B^*, this yields the equation, i.e. Eq. (32), describing dynamical balance condition.

It is worth noting that the dynamical balance equation is a sort of encapsulated version of the equations of motion for the system. The d'Alembertian wrench encapsulates the details of dynamics of every individual system. In other words, d'Alembertian wrench is the *abstraction* of the system dynamics. As long as the system dynamics is known, one can obtain the concrete expression of d'Alembertian wrench by rearranging the dynamics. For instance, as the equations of motion of the single body is Eq. (31), the d'Alembertian wrench of the system is written as

$$F_B^* = F_B - A_B \dot{V}_B - B_B V_B \tag{33}$$

Then, the dynamical balance equation Eq. (32) yields the true equations of motion for the system, once the d'Alembertian wrench for the system is substituted.

The notion of d'Alembertian wrench might be regarded as a restatement of the celebrated d'Alembert's principle, which states that the inertial acceleration is equivalent to the force. The principle is expressed for this simple example as

$$F_B + \widetilde{F}_B = 0 \tag{34}$$

where,

$$\widetilde{F}_B = -A_B \dot{V}_B - B_B V_B \tag{35}$$

However, the notion of dynamical balance is distinctive in expressing the equations of motion of a system in the following sense. It can be said that the d'Alembert's principle originates from the system dynamics. We may say, at least for this simple system, that the d'Alembert's principle Eq. (34) is a restatement of the Newton-Euler equations of motion Eq. (31). The concept of statical balance has nothing to do with the principle. On the other hand, the notion of dynamical balance is deeply related with that of statical balance. Once the statical balance equation is established, the expression of dynamical balance is also obtained by the same equation by replacing the involved wrenches with the d'Alembertian wrenches. The latter operation is called *promoting* the wrenches (to d'Alembertian). One of the reasons why we cling to the notion of statical balance is that it is much easier to formulate than the equations of motion itself because it is just a dual statement of available kinematical constraints. This idea is of great significance within the context of composing two subsystem dynamics into the composite dynamics. Once we formulate kinematical constraints for the system, we can express the dynamical balance, and the equation of motion is derived in a straightforward manner.

Consider an AMBS consisting of K bodies interconnected by N joints. Denote the body wrench applied to the body k by $F_k \in R^6$, and the joint torque at the joint j by $\tau_j \in R^{n_j}$. Suppose that the system is described using the independent GC velocity vector and hence unconstrained, consisting of $V \in R^6$, the body twist of a base body, and the joint velocity vector $\dot{q} = (\dot{q}_1^T, \dot{q}_1^T, \cdots, \dot{q}_N^T)$. The closed-form equations of motion is formally written as

$$WF + B\tau = M\begin{bmatrix} \dot{V} \\ \ddot{q} \end{bmatrix} + C\begin{bmatrix} V \\ \dot{q} \end{bmatrix} \tag{36}$$

where $F = (F_1^T, F_2^T, \cdots, F_K^T)^T \in R^{6K}$ and $\tau = (\tau_1^T, \tau_2^T, \cdots, \tau_N^T)^T \in R^{\Sigma n_j}$; the matrices W and B are the influence coefficient matrices; the matrices M and C correspond to the generalized inertia matrix and the generalized coriolis' and centrifugal matrix. Gravitational forces are all incorporated in the term F, that is, each F_k is the sum of the gravitational forces applying to the body k and the other wrenches.

The d'Alembertian wrenches and torques are obtained from the equations of motion of the system Eq. (36). For the sake of effective description, we will make advantage of the structural properties of W and B in Eq. (36). Then, one can define the d'Alembertian wrench and torque of the subsystem by

$$\begin{bmatrix} F_1^* \\ \tau^* \end{bmatrix} = WF + B\tau - M\begin{bmatrix} \dot{V} \\ \ddot{q} \end{bmatrix} - C\begin{bmatrix} V \\ \dot{q} \end{bmatrix} \tag{37}$$

or

$$
\begin{aligned}
F_1^* &= F_1 + \sum_{a=2}^{K} W_{0a} F_a - M_{00} \dot{V} - M_{01} \ddot{q}_1 \cdots \\
&\quad - C_{00} V - C_{01} \dot{q}_1 \cdots \\
\tau_1^* &= \tau_j + \sum_{j=a} W_{ja} F_a - M_{0j}^T \dot{V} - M_{1j} \ddot{q}_1 \cdots \\
&\quad - C_{j0} V - C_{j1} \dot{q}_1 \cdots
\end{aligned}
\tag{38}
$$

5. Conclusions

In this paper, the development of multibody dynamics are reviewed as considering the type of dynamic systems and its applications. The dynamic systems are classified as the closed, open loop system, and switching system. In case of closed loop system, we discuss mechanical systems with completely constrained 3-D mechanism and some of the research activities. Also, the multibody dynamics of the open loop mechanical system in the areas of the robotics partially constrained are reviewed. Lastly, the switching system are presented as focusing the dynamics of articulated multi-link system, invariant topology in system structure, and the multi-body dynamics in biomechanical system applications. In an effort to simulate the gait of the humanoid robot, the formulation of the dynamical balance is investigated.

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