Hyper Redundant Manipulator Using Compound Three-Bar Linkages

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A new mechanism for hyper redundant manipulator (HRM) is presented, which comprises of serially assembled compound three-bar linkages (CTL). The CTL mechanism has some unique properties. This paper presents the forward and inverse kinematics of this mechanism and shows the simulation of the HRM havig 9 CTL units. The recursive algorithm of the inverse kinematics that the author originally developed is employed. It is fast and stable; moreover, it enables us to obtain a solution in which the end-point of the HRM is controlled by a portion of joints. It also presents the method of the dynamical analysis. There exist kinematical constraints in the proposed closed linkage mechanism. In the dynamic analysis constraints are sufficiently sustained by the constraint stabilization method that the author developed. The mechanical structure of the HRM having some CTL units that is under construction is shown.

Key Words: Hyper-Redundant Manipulator, Inverse Kinematics, Constraint Stabilization

1. Introduction

Manipulator having many number of DOF exceeding far from the number of end-point's Cartesian coordinate variables (Hyper Redundant Manipulator, HRM, hereafter) has been numerously studied so far, since it has many potential properties, wide movability of the end-point, intrusion of narrow area, obstacle avoidance, and so on. HRM having many parallel rotary axes (Pettinato and Stephanort, 1989), truss type structure (Naccarato, and Hughes, 1989; Chirikjian and Burdick, 1993), tendon driven type (Ma and Konno, 1997), those of having active universal joints (Mochiyama et al., 1999) have been proposed. This paper proposes an alternative approach of HRM, which has compound three-bar linkages mechanism (CTL, hereafter). The CTL has particular properties; the most prominent one is that the motion of the end-point is more sensible to the joint coordinate variable locating at more proximal part. It implies the end-point is able to control by some actuators located at the proximal parts of the manipulator. This mechanism also allows us to design the lightweight and high rigid mechanical structure due to the closed link mechanism. This property is very important for HRM because it has a long serially connected links sustained at the base that is located at the most proximal part so that it may bring about the vibration of the end-point due to insufficient rigidity.

This paper is organized as follows.

After introducing the mechanism of the proposed HRM, one presents the formula for the forward and the inverse kinematics in the following section. The algorithm of inverse kinematics for HRM is originally developed and applied into this system. It has one prominent feature that is suitable for applying for the mechanism having a lot of DOF. Its recursive structure allows us to obtain the minimum DOF that is enough for the end-effecter to reach the specified position. The

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third section is devoted to describe the dynamical analysis. Since the CTL has a closed linkage structure it presents the problem of solving the dynamical equation while sustaining some holonomic constraints. In this paper one introduces a originally developed method for stabilizing the constraints during numerical integration for dynamical analysis. The conclusive remarks are shown in the last section.

2. HRM Having CTL Structure and its Kinematics

Fig. 1 shows the structure of a planar hyper redundant manipulator composed of serially arranged three-bar linkages. In Fig. 1 only three CTL units compose a manipulator. One unit consists of a three-bar linkage with one actively rotating joint, one passive joint and a slider. In the base, exceptionally two links are rotated actively by individual actuators. This mechanism will enable us to design lightweight and highly rigid structure due to the closed linkage structure. The particular property is that the rotation of one joint gives rise to rotations of all of the joints that are distally located. It induces that a few active joints that are proximally located may control the end-point. The simulation described later will verify this insight.

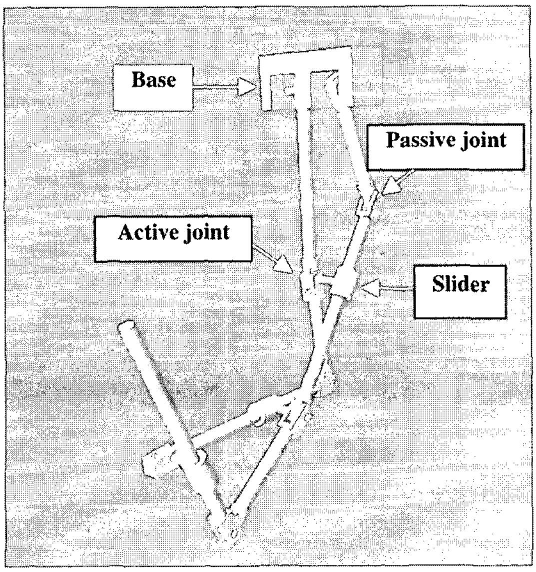
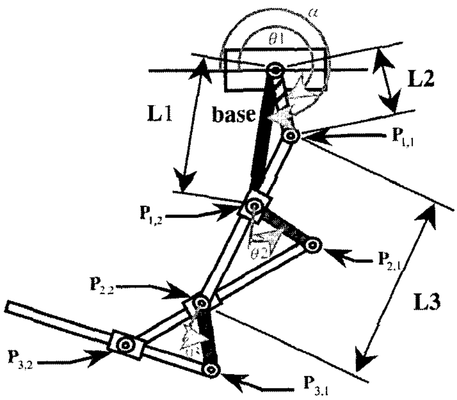


Fig. 1 CTL structure

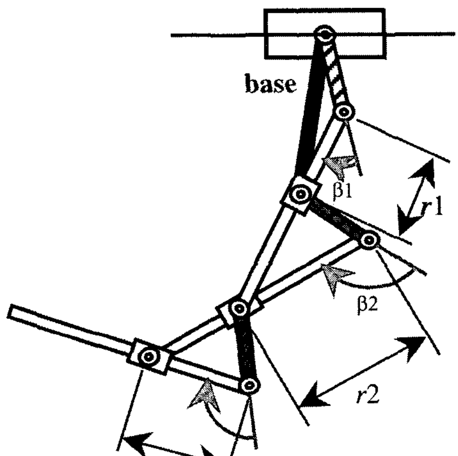
2.1 Forward kinematics

Fig. 2 describes the geometric parameters of the CTL mechanism. The i th CTL has two angular variables θ_i and β_i, and one linear variable r_i. But β_i and r_i are dependently determined once θ_i is specified due to the closed linkage structure. Exceptionally the base has additional angular parameter α.

Hence one CTL unit has one actuator to specify the angle θ_i and the HRM has $n+1$ actuators if it has n CTL units since one more actuator is joined for controlling the angle α. The planar



(a) Links and joints with active joint angles



(b) Passive joint angles and passive lengths

Fig. 2 Geometric parameters of the proposed mechanism joint coordinates are recurrently calculated by the following equations.

$$\begin{cases} \mathbf{P}_{2k-1,1}=\mathrm{P}_{2k-2,2}+\mathrm{L}_2 e^{j\left(\alpha+\sum_{i=1}^{k-1}(\theta_{2i-1}+\beta_{2i-1})+\theta_{2k-1}\right)} \\ \mathbf{P}_{2k-1,2}=\mathbf{P}_{2k-2,1}+\langle \mathrm{L}_3\rangle e^{j\left(\alpha+\sum_{i=1}^{k-1}(\theta_{2i}+\beta_{2i})\right)} \\ \qquad =\mathbf{P}_{2k-1,1}+\gamma_{2k-1} e^{j\left(\sum_{i=1}^{k}(\theta_{2i-1}+\beta_{2i-1})\right)} \end{cases} \tag{1}$$

where,

$$\langle \mathrm{L}_3\rangle=\begin{cases} \mathrm{L}_1 \text{ if } k=1 \\ \mathrm{L}_3 \text{ if } k>1 \end{cases}$$

Even CTL unit $[2k]$, $(k=1, 2, \cdots)$

$$\begin{cases} \mathbf{P}_{2k,1}=\mathrm{P}_{2k-1,2}+\mathrm{L}_2 e^{j\left(\alpha+\sum_{i=1}^{k-1}(\theta_{2i}+\beta_{2i})+\theta_{2k}\right)} \\ \mathbf{P}_{2k,2}=\mathbf{P}_{2k-1,1}+\mathrm{L}_3 e^{j\left(\sum_{i=1}^{k}(\theta_{2i-1}+\beta_{2i-1})\right)} \\ \qquad =\mathbf{P}_{2k,1}+\gamma_{2k} e^{j\left(\alpha+\sum_{i=1}^{k}(\theta_{2i}+\beta_{2i})\right)} \end{cases} \tag{2}$$

with initial positions, $\mathbf{P}_{0,1}=\mathbf{P}_{0,2}=0$.

Dependent variables β_i and $\gamma_i (i=1, 2, \cdots)$ are obtained from the above simultaneous equations.

2.2 Jacobian

If the HRM has n CTL unit, the planar coordinates of its end-point $\mathbf{P}_n=\mathrm{P}_{n,x}+j\mathrm{P}_{n,y}$ are functions of 3n variable α, θ_1, β_1, γ_1, θ_2, β_2, γ_2, $\cdots$, θ_n, β_n, i.e.,

$$\begin{aligned} \mathrm{P}_{n,x}&\equiv\mathrm{P}_{n,x}(\alpha, \theta_1, \beta_1, \gamma_1, \theta_2, \beta_2, \gamma_2, \cdots, \theta_n, \beta_n) \\ \mathrm{P}_{n,y}&\equiv\mathrm{P}_{n,y}(\alpha, \theta_1, \beta_1, \gamma_1, \theta_2, \beta_2, \gamma_2, \cdots, \theta_n, \beta_n) \end{aligned} \tag{3}$$

Moreover the direction of the end-point is simply calculated by

$$\phi_n=\sum_{i=1}^{n}(\theta_i+\beta_i) \tag{4}$$

Equations (3) and (4) lead the linear relationship between the infinitesimal displacement vector of the end-point ; $[\Delta \mathrm{P}_{n,x}\ \Delta \mathrm{P}_{n,y}\ \Delta\phi_n]^T$ and infinitesimal variables, $[\Delta\alpha\ \Delta\theta_1\ \Delta\beta_1\ \Delta\gamma_1 \cdots \Delta\theta_n\ \Delta\beta_n]$ as follows,

$$\begin{pmatrix} \Delta\mathbf{P}_{n,x} \\ \Delta\mathbf{P}_{n,y} \\ \Delta\phi_n \end{pmatrix}=\mathbf{J}_n(\Delta\alpha\ \Delta\theta_1\ \Delta\beta_1\ \Delta\gamma_1 \cdots \Delta\theta_n\ \Delta\beta_n)^T \tag{5}$$

where, $\mathbf{J}_n=\Re^{3\times 3n}$ is Jacobian defined by

$$\mathbf{J}_n=\begin{pmatrix} \frac{\partial\mathbf{P}_{n,x}}{\partial\alpha} & \frac{\partial\mathbf{P}_{n,x}}{\partial\theta_1} & \frac{\partial\mathbf{P}_{n,x}}{\partial\beta_1} & \frac{\partial\mathbf{P}_{n,x}}{\partial\gamma_1} & \cdots & \frac{\partial\mathbf{P}_{n,x}}{\partial\theta_n} & \frac{\partial\mathbf{P}_{n,x}}{\partial\beta_n} \\ \frac{\partial\mathbf{P}_{n,y}}{\partial\alpha} & \frac{\partial\mathbf{P}_{n,y}}{\partial\theta_1} & \frac{\partial\mathbf{P}_{n,y}}{\partial\beta_1} & \frac{\partial\mathbf{P}_{n,y}}{\partial\gamma_1} & \cdots & \frac{\partial\mathbf{P}_{n,y}}{\partial\theta_n} & \frac{\partial\mathbf{P}_{n,y}}{\partial\beta_n} \\ \frac{\partial\phi_n}{\partial\alpha} & \frac{\partial\phi_n}{\partial\theta_1} & \frac{\partial\phi_n}{\partial\beta_1} & \frac{\partial\phi_n}{\partial\gamma_1} & \cdots & \frac{\partial\phi_n}{\partial\theta_n} & \frac{\partial\phi_n}{\partial\beta_n} \end{pmatrix}\in\Re^{3\times 3n}$$

The relationship of Eq. (5) includes dependent variables β_i and $\gamma_i (i=1, 2, \cdots)$. In order to omit them, the following constraint equations derived from the eqs. (1) (2) are used,

$$C_{i,x}+jC_{i,y}=\mathbf{P}_{i,2}-\mathbf{P}_{i,1}-\gamma_i e^{j\gamma_i}=0+j0 \tag{6}$$

where,

$$\gamma_i=\begin{cases} \sum_{j=1}^{(i+1)/2}(\theta_{2j-1}+\beta_{2j-1}) & \text{if } i \text{ is odd} \\ \alpha+\sum_{j=1}^{i/2}(\theta_{2j}+\beta_{2j}) & \text{if } i \text{ is even} \end{cases}$$

Total differentiation of Eq. (6) leads the following relations,

$$\mathbf{A}_i\begin{pmatrix}\Delta\gamma_i \\ \Delta\beta_i\end{pmatrix}+\mathbf{B}_i(\Delta\alpha\ \Delta\theta_1\ \Delta\theta_2\ \Delta\theta_3 \cdots \Delta\theta_i)^T=\begin{pmatrix}0 \\ 0\end{pmatrix} \tag{7}$$

where, $\mathrm{A}_i\in\Re^{2\times 2}$, $\mathbf{B}_i\in\Re^{2\times(i+1)}$.

Eq. (7) can be solved with respect to $(\Delta\gamma_i\ \Delta\beta_i)^T$ if the matrix A_i is non-singular,

$$\begin{pmatrix}\Delta\gamma_i \\ \Delta\beta_i\end{pmatrix}=-\mathbf{A}_i^{-1}\mathbf{B}_i(\Delta\alpha\ \Delta\theta_1\ \Delta\theta_2\ \Delta\theta_3 \cdots \Delta\theta_i)^T \tag{8}$$

Using Eq. (8), following relations are derived,

$$\begin{pmatrix} \Delta\alpha \\ \Delta\theta_1 \\ \Delta\beta_1 \\ \Delta\gamma_1 \\ \Delta\theta_2 \\ \vdots \\ \Delta\theta_n \\ \Delta\beta_n \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ \multicolumn{2}{c}{-\mathrm{A}_1^{-1}\mathrm{B}_1\in\Re^{2\times 2}} & 0 & 0 & \cdots & 0 \\ & & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \multicolumn{3}{c}{-\mathrm{A}_2^{-1}\mathrm{B}_2\in\Re^{2\times 3}} & 0 & \cdots & 0 \\ & & & 0 & & 0 \\ & \vdots & & & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \multicolumn{4}{c}{-\mathrm{A}_n^{-1}\mathrm{B}_n\in\Re^{1\times n}} & & \end{pmatrix}\begin{bmatrix} \Delta\alpha \\ \Delta\theta_1 \\ \Delta\theta_2 \\ \Delta\theta_3 \\ \vdots \\ \Delta\theta_n \end{bmatrix} \tag{9}$$

Eq. (5) and (9) give us the desired relation between the infinitesimal displacement vector of the end-point and the independent angle vector as follows,

$$\begin{pmatrix} \Delta\mathbf{P}_{n,x} \\ \Delta\mathbf{P}_{n,y} \\ \Delta\phi_n \end{pmatrix}=\mathbf{J}_n\mathbf{J}_c(\Delta\alpha\ \Delta\theta_1 \cdots \Delta\theta_n)^T \tag{10}$$

2.3 Inverse kinematics

Eq. (10) is inversely solved by using pseudoinverse of Jacobian. Denoting $\mathbf{J} \equiv \mathbf{J}_n \mathbf{J}_c \in \Re^{3\times(n+1)}$ we have,

$$\begin{pmatrix} \Delta\alpha \\ \Delta\theta_1 \\ \vdots \\ \Delta\theta_n \end{pmatrix} = \mathbf{J}_D^{\#} \begin{pmatrix} \Delta\mathbf{P}_{n,x} \\ \Delta\mathbf{P}_{n,y} \\ \Delta\phi_n \end{pmatrix} + (\mathbf{I} - \mathbf{J}_D \mathbf{J}) \Delta\psi \qquad (11)$$

$$\Rightarrow \Delta\theta = \mathbf{J}_D^{\#} \Delta\mathbf{x} + (\mathbf{I} - \mathbf{J}_D \mathbf{J}) \Delta\psi$$

where, $\mathbf{J}_D^{\dagger}$ is the weighted pseudoinverse of the Jacobian $\mathbf{J}$ defined by,

$$\mathbf{J}_D^{\dagger} \equiv \mathbf{D}\mathbf{J}^T (\mathbf{J}\mathbf{D}\mathbf{J}^T)^{-1}$$

with a weight matrix $\mathbf{D} \in \Re^{(n+1)\times(n+1)}$. $\Delta\psi \in \Re^{n+1}$ is a vector that can be arbitrarily determined. It will be useful to regulate the posture of HRM on the occasion of avoiding obstacles, passing through a narrow gate, and so on. For simplicity the weight matrix $\mathbf{D}$ is assumed to be diagonal, i.e.,

$$\mathbf{D} = diag\{d_1, d_2, \cdots, d_{n+1}\} \in \Re^{(n+1)\times(n+1)}$$

In order to calculate the right hand side of Eq. (11), we employ the recursive algorithm (Koganezawa, 1997) as follows,

[Recursive algorithm of inverse kinematics]

Initial setting : $\mathbf{H}_0^{-1} = c\mathbf{I} \in \Re^{3\times 3}$

where, c is a scalar value sufficiently large.

$\Delta\mathbf{x} = (\Delta P_{n,x}\ \Delta P_{n,y}\ \Delta\phi_n)^T$ and $\Delta\psi = (\Delta\psi_1, \Delta\psi_2, \cdots, \Delta\psi_n)^T$ and are specified.

For $\{\nu = 1\ ;\ \nu \leq n+1\ ;\ \nu++\ ;$

$\mathbf{j}_\nu$; ν th vector of Jacobian $\mathbf{J}$

$\mathbf{J}_\nu = (\mathbf{J}_{\nu-1}\mathbf{j}_\nu) \in \Re^{3\times v}$; a matrix having up to ν th columns of $\mathbf{J}_n$

$\boldsymbol{\xi}_\nu = \mathbf{H}_{\nu-1}^{-1}\mathbf{j}_\nu \in \Re^3$;

$\boldsymbol{\zeta}_\nu = \mathbf{J}_{\nu-1}^T \boldsymbol{\xi}_\nu \in \Re^{(v-1)}$;

$\boldsymbol{\phi}_\nu = \mathbf{j}_\nu^T \boldsymbol{\xi}_\nu$;

$\boldsymbol{\chi}_\nu = \boldsymbol{\xi}_\nu \Delta\mathbf{x}$;

$\boldsymbol{\delta}_\nu = \boldsymbol{\zeta}_\nu^T \Delta\psi_{\nu-1}$;

$\boldsymbol{d}_\nu = \mathrm{sgn}(\phi_\nu)\tilde{d}_\nu$; $\tilde{d}_\nu$ is a predetermined scalar value

$$\mathrm{sgn}(\phi_\nu) = \begin{cases} 1, & \phi_\nu \geq 0 \\ -, & \phi_\nu < 0 \end{cases}$$

$\mathbf{D}_\nu = \begin{pmatrix} \mathbf{D}_{\nu-1} & 0 \\ 0 & d_\nu \end{pmatrix} \in \Re^{v\times v}$; ν th principal matrix of $\mathbf{D}_\nu$

$$\mathbf{H}_\nu^{-1} = \mathbf{H}_{\nu-1}^{-1} - \frac{d_\nu}{1 + d_\nu\phi_\nu} \boldsymbol{\xi}_\nu \boldsymbol{\xi}_\nu^T\ ;$$

$$\Delta\tilde{\theta}_\nu = \begin{pmatrix} \Delta\tilde{\boldsymbol{\theta}}_{\nu-1} - \dfrac{d_\nu\chi_\nu}{1 + d_\nu\phi_\nu}\mathbf{D}_{\nu-1}\zeta_\nu \\ \dfrac{d_\nu\chi_\nu}{1 + d_\nu\phi_\nu} \end{pmatrix};$$

$$\Delta\mathbf{r}_\nu = \begin{pmatrix} \Delta\mathbf{r}_{\nu-1} - \dfrac{\Delta\psi_\nu - d_\nu\delta_\nu}{1 + d_\nu\phi_\nu}\mathbf{D}_{\nu-1}\zeta_\nu \\ \dfrac{\Delta\psi_\nu - d_\nu\delta_\nu}{1 + d_\nu\phi_\nu} \end{pmatrix};$$

}

$$\Delta\boldsymbol{\theta}_n = \Delta\tilde{\boldsymbol{\theta}}_n + \Delta\mathbf{r}_n\ ;\ \text{Final result} \qquad (12)$$

For the details of deriving this algorithm, see the reference (Koganezawa, 1997). The excellent properties of this algorithm are as follows.

(1) The fast computation.

(2) It never falls into a computational ill-conditioning by virtue of weights determined by $d_\nu = \mathrm{sgn}(\phi_\nu)\tilde{d}_\nu$.

And the most important reason to adopt this algorithm resides in the third property.

(3) The result of the ν th recursion ($\nu \leq n$), $\Delta\theta_\nu$, gives us an optimal solution when we assume to control the end-point only using up to the th DOF.

The third property implies that we can terminate the calculation after th recursion if the end-point arrives at the target position in allowable error margin. The example below shows one example of the number of recursion during the movement.

3. Kinematical Simulation

3.1 Forward kinematical motion

The motion of the end-point of the proposed HRM is strongly affected by the rotation of the active joints that are proximally located. The following simulation demonstrates this property. It is assumed in the subsequent simulation study that the HRM consists of 9 CTL units, which means the HRM has 10 actuators (one more actuator besides ones for controlling 9 CTL units is needed for controlling the angle α)

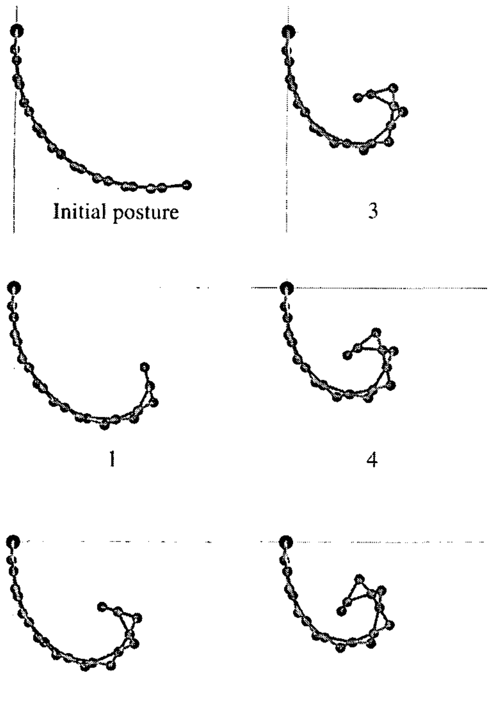


Fig. 3 Motin of the HRM when θ_1 rotates 1.5°

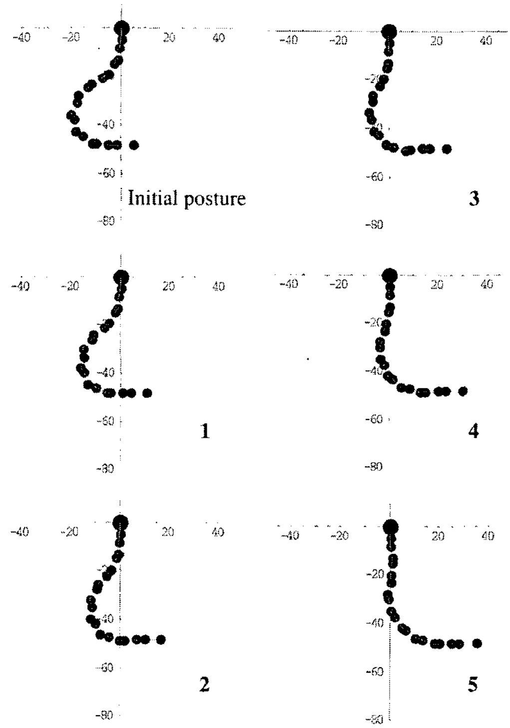


Fig. 4 Motion of HRM when the end-point translates 20 cm along x-axis

Fig. 3 shows the motion of the HRM when θ_1 rotates only 0.3° from figure to figure, totally 1.5°. As shown a winding motion of the distal part gives rise due to a slight rotation of the joint located in the proximal part.

3.2 Kinematical control of the end-point

The recursive algorithm defined by Eq. (12) is applied under the condition $\Delta\psi=\mathbf{0}$. The weight matrix **D** is specified as follows,

$$\mathbf{D}=diag\{\rho^{(n+1)-1}\ \rho^{(n+1)-2}\ \cdots\ \rho\}\ \text{for}\ \rho<1$$

Using the above weight matrix joints locating more proximal part take lighter weight. This means the motion rate of proximal joint is slower than those of the distal joint. Since motion of the end-point is more sensitive to the rotation of more proximal joints as shown in the previous section. ρ is evaluated by the error rate of the end-point's motion as described below. In the subsequent results of simulation it takes $\rho=0.8$.

In Figs. 4, 5, 6, motions of the HRM are shown when the motion of the end-point is specified as translate 20 cm along x-axis (Fig. 4), translate 20 cm along y-axis (Fig. 5) and rotate π rad respectively.

Table 1 specified destination and simulation result of the end-point's motion

		X-coordinate cm	Y-coordinate cm	Direction rad
Fig. 4	destination	35.045	-48.237	6.2805
	result	35.073	−48.069	6.2867
Fig. 5	destination	2.5256	−54.846	4.8801
	result	2.4951	−54.494	4.8855
Fig. 6	destination	0.0650	−25.580	9.4218
	result	0.2625	−26.056	9.3683

Figs. 4, 5 and 6 are the results of 400 step calculations. The destination and the location of the end-point after 400 step calculations are shown in Table 1.

As shown in Table 1 the end-point reaches the destination precisely. Slight destination error can be deleted by summing it to the translation

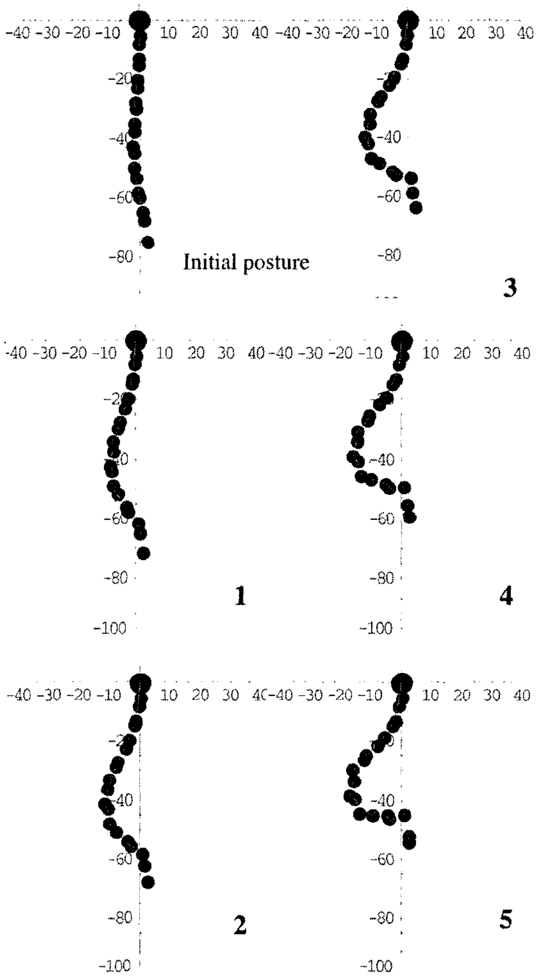


Fig. 5 Motion of HRM when the end-point translates 20 cm along y-axis

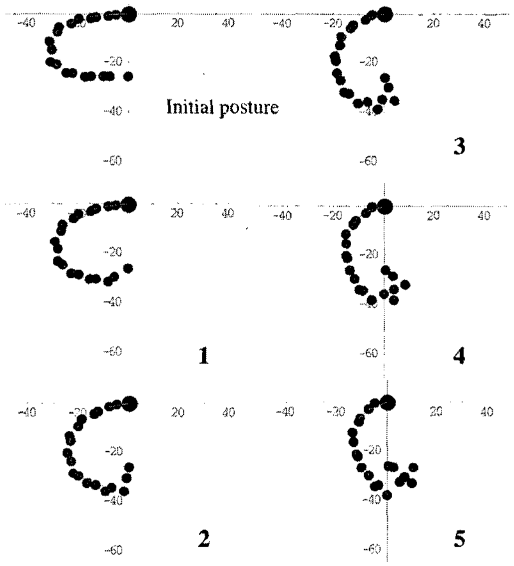


Fig. 6 Motion of HRM when the end-point rotates π[rad]

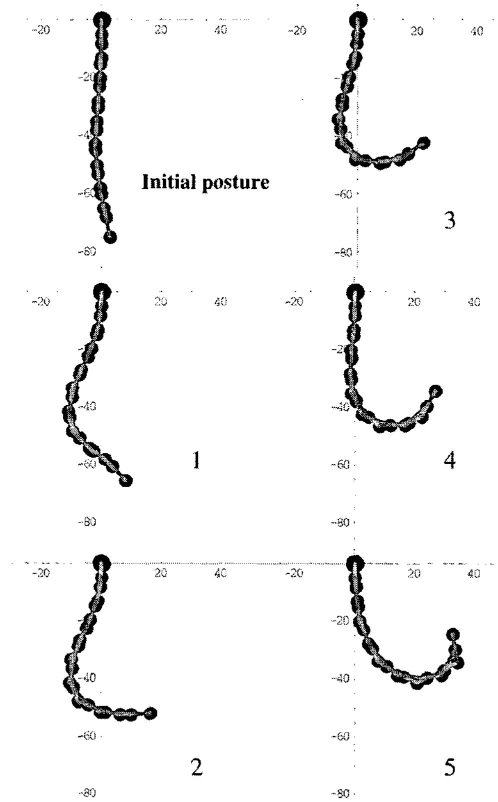


Fig. 7 Motion of HRM when the end-point translates along x- and y- axes and rotates

(or rotation) of the subsequent step.

2.3 Number of recursion of the recursive algorithm

The recursive algorithm presented above has one useful property described the following proposition.

[proposition]

> The result of the ν th recursion $\Delta\boldsymbol{\theta}_\nu = \Delta\boldsymbol{\theta}_\nu + \Delta\mathbf{r}_\nu$ gives us an optimal solution of $\Delta\theta_\nu$ for the end-point reaching the specified destination.

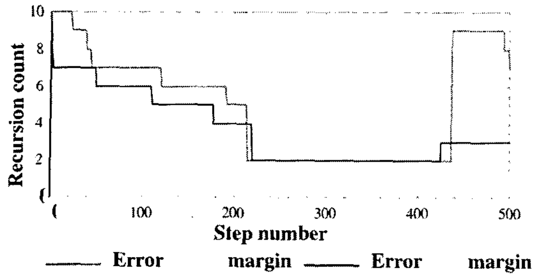


Fig. 8 Recursion count of each step during the motion shown in Fig. 7

This proposition implies that if the end-point reaches the destination in an allowable error margin at the ν th recursion the inverse kinematical calculation can be terminated. It also means that for the practical control of the HRM only ν actuators are suffice to drive.

In the motion illustrated in Fig. 7 the end-point translate and rotate simultaneously (30 cm and 50 cm along x- and y- axes respectively and π rad) in 500 steps. The recursion number at each step is shown in Fig. 8.

Fig. 8 shows that partially the driving of only just two proximal joints are suffice for the end-point to reach the specified destination. This result comes from the intrinsic structure of the proposed HRM; the rotations of proximal joints strongly affect motions of the end-point

4. Dynamic Simulation of Control of the HRM

Since one unit of the CTL is a closed links with one DOF, it is expressed by two angular variables with one holonomic kinematical constraint, which is provided by the slider shown in Fig. 1.

The $2k-1$ th and $2k$ th constraint equation of the k th ($k=1, 2, \cdots, n$) CTL unit are described by Eq. (6).

A method of solving the dynamic equations with multiple DOF with constraints is developed (Mochiyama et al., 1999)

First the Lagrangian with an energy term with respect to the constraint is constructed as,

$$L=K-P-\sum_{k=1}^{2n}(\varphi_k+\nu\dot{\varphi}_k)\lambda_k \tag{13}$$

where, K and P represents the kinematic and the potential energy respectively and λ_k is a Lagrange multiplier. φ_k are constraint equations defined by,

$$\begin{aligned}\varphi &\equiv (\varphi_1,\ \varphi_2,\ \cdots \varphi_{2n-1},\ \varphi_{2n})^T \\ &= (C_{1,x},\ C_{1,y},\ \cdots,\ C_{n,x},\ C_{n,y})^T\end{aligned}$$

ν is a scalar value. The ordinary procedure for deriving the Lagrange's equations gives us,

$$\boldsymbol{M}\ddot{\boldsymbol{q}}+\dot{\boldsymbol{M}}\dot{\boldsymbol{q}}-\boldsymbol{L}_q^T+\nu\boldsymbol{\varphi}_q^T\dot{\boldsymbol{\lambda}}-\boldsymbol{\varphi}_q^T\boldsymbol{\lambda}=\boldsymbol{Q}_{ex} \tag{8}$$

where, $\boldsymbol{q}$ represents a vector of joints variables,

$$\boldsymbol{q}\equiv(\alpha\ \theta_1\ \beta_1\ r_1\ \cdots\ \theta_{n-1}\ \beta_{n-1}\ r_{n-1}\ \theta_n\ \beta_n)^T$$

and $L_q \triangleq \frac{\partial L}{\partial \boldsymbol{q}} \in \Re^{3n}$, $\boldsymbol{\varphi}_q \equiv \frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{q}}$, $\boldsymbol{\lambda}\equiv(\lambda_1\ \cdots\ \lambda_{2n})^T$ is a Lagrange multiplier vector and $\boldsymbol{Q}_{ex}\in\Re^{n+1}$ is an external torque vector. Eq. (8) and time differentiation of constraint equations twice comprises the following matrix differential equation,

$$\begin{pmatrix}\boldsymbol{M} & \nu\boldsymbol{\varphi}_q^T \\ \boldsymbol{\varphi}_q & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\ddot{\boldsymbol{q}} \\ \dot{\boldsymbol{\lambda}}\end{pmatrix}=\begin{pmatrix}\boldsymbol{Q}_{ex}+\boldsymbol{L}_q^T-\dot{\boldsymbol{M}}\dot{\boldsymbol{q}}+\boldsymbol{\varphi}_q^T\boldsymbol{\lambda} \\ -\dot{\boldsymbol{\varphi}}_q^T\dot{\boldsymbol{q}}-2g_1\boldsymbol{\varphi}_q^T\dot{\boldsymbol{q}}-g_2^2\boldsymbol{\varphi}\end{pmatrix} \tag{9}$$

in which terms including scalar parameters and are Baumgarte's stabilizing terms (Baumgarte, 1972).

For the detail of above derivation, see the reference (Koganezawa and Kaneko, 1999). Here, we just state some remarks.

(1) Introducing the Lagrange multipliers derived multipliers, dynamical equations can be systematically derived even for the serial closed links mechanism.

(2) In the conventional procedure Lagrange multipliers are solved algebraically on the differential-algebraic equations. However its instability of sustaining constrains has been reported (Haug, 1989).

The method employed here solves Lagrange multipliers from their differential equations as well as the dynamic equations of the system. This method accompanied by the Baumgarte's stabilizing terms assures a great stabilization of holding constraints.

5. Conclusions

In this paper a new mechanism for hyper redundant manipulator (HRM) is proposed. It has serially connected closed linkage unit (CTL unit), by which it takes peculiar property: the motion of the end-point is strongly affected by the rotations of joints located on the proximal part of the HRM. The forward and the inverse kinematics of the proposed mechanism are derived. Especially a new inverse kinematics algorithm developed by the author is employed. Its recursive form gives us rotation angles of some joints (not full joints), which are sufficient for the end-point to reach the specified location and orientation. This property is desirable for the HRM since it implies that the HRM can be controlled by driving a portion of joints. The simulation study reveals precise motions of the end-point by using the developed Jacobian and also reveals rotations of a few proximal joints is sufficient for the end-point's control. A method of dynamical simulation is described. Proposed HRM has a lot of kinematical constraints due to its closed linkage structure, which comprise many holonomic constraints. The presented method for solving ODE includes a set of multipliers that are obtained as a solution of ODE not of DAE. This procedure brings out a tremendously stable holding of constraints.

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