A Time Integration Method for Analysis of Dynamic Systems Using Domain Decomposition Technique

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This paper presents a precise and stable time integration method for dynamic analysis of vibration or multibody systems. A total system is divided into several subsystems and their responses are calculated separately, while the coupling effect is treated equivalently as constant force during time steps. By using iterative procedure to improve equivalent coupling forces, a precise and stable solution is obtained. Some examples such as a seismic response and multibody analyses were carried out to demonstrate its usefulness.

Key Words : Multibody Dynamics, Time History Analysis, Substructure Method, Domain Decomposition Mothod

1. Introduction

The domain decomposition method (DDM) is useful for solving a very large equation. Usually the term DDM is used as a technique of solving a linear equation of a total system which is divided into many subdomains, and calculations are carried out separately in each domain accompanying the interface equation connecting subdomains (Yagawa and Sioya, 1998 ; Akiba et al., 2003 ; Smith et al., 1996).

However, in this paper DDM is used as the same meaning of partitioned or substructure method for dynamic analyses of a system having not so large degree of freedom and consisting of a few substructures. There are some merits to apply DDM to dynamic systems. One is that a suitable expression of equation or solver can be chosen in complicated systems consisting of different field substructures such as structure, control or hydraulic systems. In addition check or recognition is easy because the response behavior can be observed in coupled and decoupled conditions. Of course a large system is solved effectively.

So, the target of this study is to develop an effective dynamic analysis method, and at the first step ,we consider a case that subsystems are only coupled by springs, and they are solved by the stable Newmark method.

2. DDM with Constant Coupling Force (CCF) During a Time Step

At first a DDM with an approximation of CCF (Imanishi, 1992) is introduced in which coupling spring forces are treated as constant. For simplicity we consider a dynamic system consisting of two substructures shown in Fig. 1 and explain the CCF method.

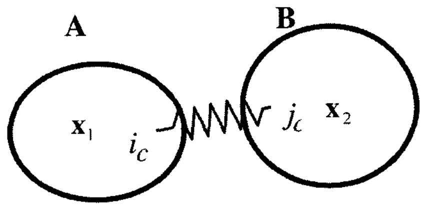

Fig. 1 A system coupled by springs

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2.1 Theory of the CCF method

Equation of motion of a linear system is expressed by

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{f} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are mass, damping, stiffness matrices, and $\mathbf{x}$, $\mathbf{f}$ are displacement and force vectors respectively. In the two subdomain system, Eq. (1) can be represented by

$$\begin{bmatrix}\mathbf{M}_{11} & 0\\ 0 & \mathbf{M}_{22}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{x}}_1\\ \ddot{\mathbf{x}}_2\end{Bmatrix}+\begin{bmatrix}\mathbf{C}_{11} & 0\\ 0 & \mathbf{C}_{22}\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{x}}_1\\ \dot{\mathbf{x}}_2\end{Bmatrix}+\begin{bmatrix}\mathbf{K}_{11} & \mathbf{K}_{12}\\ \mathbf{K}_{21} & \mathbf{K}_{22}\end{bmatrix}\begin{Bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{Bmatrix}=\begin{Bmatrix}\mathbf{f}_1\\ \mathbf{f}_2\end{Bmatrix} \tag{2}$$

Eq. (2) is rewritten as

$$\begin{aligned}\mathbf{M}_{11}\ddot{\mathbf{x}}_1+\mathbf{C}_{11}\dot{\mathbf{x}}_1+\mathbf{K}_{11}\mathbf{x}_1&=\mathbf{f}_1-\mathbf{K}_{12}\tilde{\mathbf{x}}_2\\ \mathbf{M}_{22}\ddot{\mathbf{x}}_2+\mathbf{C}_{22}\dot{\mathbf{x}}_2+\mathbf{K}_{22}\mathbf{x}_2&=\mathbf{f}_2-\mathbf{K}_{21}\tilde{\mathbf{x}}_1\end{aligned} \tag{3}$$

Here coupling terms $\mathbf{K}_{12}\mathbf{x}_2$ and $\mathbf{K}_{21}\mathbf{x}_1$ are approximated by $\mathbf{K}_{12}\tilde{\mathbf{x}}_2$ and $\mathbf{K}_{21}\tilde{\mathbf{x}}_1$, where $\tilde{\mathbf{x}}_1$ and $\tilde{\mathbf{x}}_2$ are equivalent displacements treated as constant during a time step width. They may be decided as

$$\tilde{\mathbf{x}}_1=\mathbf{x}_{1n},\ \tilde{\mathbf{x}}_2=\mathbf{x}_{2n} \tag{4a}$$

or

$$\tilde{\mathbf{x}}_1=\mathbf{x}_{1n}+0.5h\dot{\mathbf{x}}_{1n},\ \tilde{\mathbf{x}}_2=\mathbf{x}_{2n}+0.5h\dot{\mathbf{x}}_{2n} \tag{4b}$$

Where subscript 1 and 2 mean subdomain, and n means time step. In Eq. (4a) $\tilde{\mathbf{x}}$ is approximated by the displacement at the beginning of time step width h, while $\tilde{\mathbf{x}}$ in Eq. (4b) is a mid point value predicted by using displacement $\mathbf{x}_n$ and velocity $\dot{\mathbf{x}}_n$.

2.2 Time integration by the Newmark method

In order to solve Eq. (1) or Eq. (3) the Newmark-β method with $\beta=1/4$ is used, which is unconditionally stable and has no numerical damping. The algorithm is

$$\begin{aligned}\ddot{\mathbf{x}}_{n+1}&=\ddot{\mathbf{x}}_n+\Delta\mathbf{a}\\ \dot{\mathbf{x}}_{n+1}&=\dot{\mathbf{x}}_n+h\ddot{\mathbf{x}}_n+0.5h\Delta\mathbf{a}\\ \mathbf{x}_{n+1}&=\mathbf{x}_n+h\dot{\mathbf{x}}_n+0.5h^2\ddot{\mathbf{x}}_n+\beta h^2\Delta\mathbf{a}\end{aligned} \tag{5}$$

Where $\Delta\mathbf{a}$ is acceleration change during h. Substituting Eq. (5) into Eq. (1) at time t_{n+1} yields

$$\begin{aligned}&[\mathbf{M}+0.5h\mathbf{C}+\beta h^2\mathbf{K}]\Delta\mathbf{a}\\ &=\mathbf{f}_{n+1}-\{\mathbf{M}\ddot{\mathbf{x}}_n+\mathbf{C}(\dot{\mathbf{x}}_n+h\ddot{\mathbf{x}}_n)+\mathbf{K}(\mathbf{x}_n+h\dot{\mathbf{x}}_n+0.5h^2\ddot{\mathbf{x}}_n)\}\end{aligned} \tag{6}$$

Once $\Delta\mathbf{a}$ is obtained by solving Eq. (6), $\ddot{\mathbf{x}}_{n+1}$, $\dot{\mathbf{x}}_{n+1}$, $\mathbf{x}_{n+1}$ are calculated in Eq. (5).

In the case of DDM, following equations are derived and used in each domain.

$$\begin{aligned}&[\mathbf{M}_{11}+0.5h\mathbf{C}_{11}+\beta h^2\mathbf{K}_{11}]\Delta\mathbf{a}_1=\mathbf{f}_{1,n+1}-\mathbf{K}_{12}\tilde{\mathbf{x}}_2\\ &-\{\mathbf{M}_{11}\ddot{\mathbf{x}}_{1n}+\mathbf{C}_{11}(\dot{\mathbf{x}}_{1n}+h\ddot{\mathbf{x}}_{1n})+\mathbf{K}_{11}(\mathbf{x}_{1n}+h\dot{\mathbf{x}}_{1n}+0.5h^2\ddot{\mathbf{x}}_{1n})\}\end{aligned} \tag{7a}$$

$$\begin{aligned}&[\mathbf{M}_{22}+0.5h\mathbf{C}_{22}+\beta h^2\mathbf{K}_{22}]\Delta\mathbf{a}_2=\mathbf{f}_{2,n+1}-\mathbf{K}_{21}\tilde{\mathbf{x}}_1\\ &-\{\mathbf{M}_{22}\ddot{\mathbf{x}}_{2n}+\mathbf{C}_{22}(\dot{\mathbf{x}}_{2n}+h\ddot{\mathbf{x}}_{2n})+\mathbf{K}_{22}(\mathbf{x}_{2n}+h\dot{\mathbf{x}}_{2n}+0.5h^2\ddot{\mathbf{x}}_{2n})\}\end{aligned} \tag{7b}$$

After solving $\Delta\mathbf{a}_i$ by Eq. (7), Eq. (5) is used to obtain $\ddot{\mathbf{x}}_{i,n+1}$, $\dot{\mathbf{x}}_{i,n+1}$, $\mathbf{x}_{i,n+1}$ for i=1, 2.

2.3 Calculation examples

2.3.1 Model

The validity of this method is examined thorough some calculations for a two degree of freedom vibration system shown in Fig. 2. The model constants, natural frequencies, and conditions used in the analysis are

mass : $m_1=m_2=1$ kg,
damping : $c_1=c_2=0$ Ns/m
stiffness : $k_1=k_2=100$ N/m,
$k_{12}=100$ or $k_{12}=1000$ N/m

Natural angular frequency

when $k_{12}=100$: $\omega_1=10$, $\omega_2=\sqrt{300}$ rad/s
when $k_{12}=1000$: $\omega_1=10$, $\omega_2=\sqrt{2100}$ rad/s
initial condition : $x_1=1$, $x_2=-1$, $\dot{x}_1=\dot{x}_2=0$
time step width : h=0.01s
integration method : Newmark ($\beta=1/4$)

Free vibration responses from initial displacement are calculated by the CCF method. The

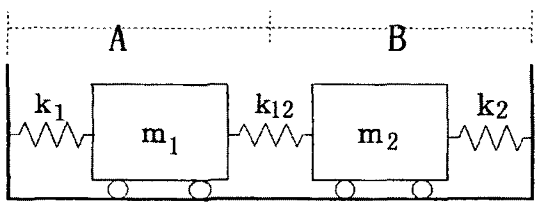

Fig. 2 Two mass system connected by spring

initial condition is set so that only 2nd mode response occurs, for it is unstable mode. Its exact solution is given by

$$x_1=\cos\omega_2 t,\ x_2=-\cos\omega_2 t$$

2.3.2 Calculation results and discussion

Calculation results are shown in Figs. 3, 4, 5. Fig. 3 shows the response waves when $k_{12}=100$, $\tilde{x}=x_n$. Undesirable growth in amplitude appeared due to the approximation of CCF. In Fig. 4 equivalent displacements are replaced to $\tilde{x}=x_n+0.5h\dot{x}_n$, which improved stability and accuracy. In Fig. 5 harder spring with $k_{12}=1000$ is used in stead of $k_{12}=100$. Remarkable instability appeared even though $\tilde{x}=x_n+0.5h\dot{x}_n$ is used.

The CCF method is reported by Fujikawa et al. (1985). According to their results and the above examples, characteristics of the CCF method is summarized as

(1) The CCF method is available when the coupling stiffness is not strong.

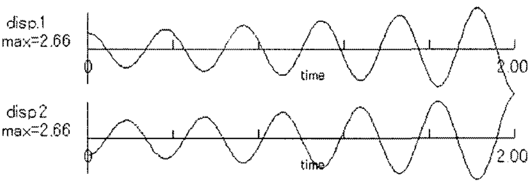

Fig. 3 Calculated response ($k_{12}=100$, $\tilde{x}=x_n$)

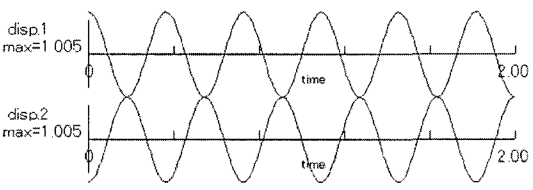

Fig. 4 Calculated response ($k_{12}=100$, $\tilde{x}=x_n+0.5h\dot{x}_n$)

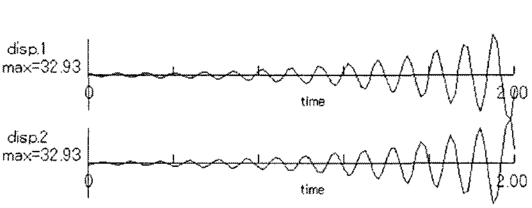

Fig. 5 Calculated response ($k_{12}=1000$, $\tilde{x}=x_n+0.5h\dot{x}_n$)

(2) Stability and accuracy may be spoiled by the approximation of CCF treatment.

(3) The use of equivalent displacement $\tilde{x}=x_n+0.5h\dot{x}_n$ improves stability and accuracy compared with $\tilde{x}=x_n$.

(4) Stability becomes worse as coupling stiffness is lager or time step width becomes larger.

3. DDM with Iterations

As shown in the previous section, the CCF method is available when subsystems are coupled by soft springs, however numerical instability appears and response grows up, when springs are strong. In order to improve them an iterative procedure is introduced in the CCF method. The algorithm is explained based on Eqs. (8a) (8b), which are almost same as Eq. (7), but iterative scripts are added

$$\begin{aligned}&[\mathbf{M}_{11}+0.5h\mathbf{C}_{11}+\beta h^2\mathbf{K}_{11}]\Delta\mathbf{a}_1^{(k+1)}=\mathbf{f}_{1,n+1}-\mathbf{K}_{12}\tilde{\mathbf{x}}_2^{(k)}\\&-\{\mathbf{M}_{11}\ddot{\mathbf{x}}_{1n}+\mathbf{C}_{11}(\dot{\mathbf{x}}_{1n}+h\ddot{\mathbf{x}}_{1n})+\mathbf{K}_{11}(\mathbf{x}_{1n}+h\dot{\mathbf{x}}_{1n}+0.5h^2\ddot{\mathbf{x}}_{1n})\}\end{aligned}\quad(8a)$$

$$\begin{aligned}&[\mathbf{M}_{22}+0.5h\mathbf{C}_{22}+\beta h^2\mathbf{K}_{22}]\Delta\mathbf{a}_2^{(k+1)}=\mathbf{f}_{2,n+1}-\mathbf{K}_{21}\tilde{\mathbf{x}}_1^{(k)}\\&-\{\mathbf{M}_{22}\ddot{\mathbf{x}}_{2n}+\mathbf{C}_{22}(\dot{\mathbf{x}}_{2n}+h\ddot{\mathbf{x}}_{2n})+\mathbf{K}_{22}(\mathbf{x}_{2n}+h\dot{\mathbf{x}}_{2n}+0.5h^2\ddot{\mathbf{x}}_{2n})\}\end{aligned}\quad(8b)$$

At first (k=0), approximate $\Delta\mathbf{a}^{(k+1)}$ is calculated using $\tilde{\mathbf{x}}^{(k)}=\tilde{\mathbf{x}}$ in Eq. (4a) or (4b), and the displacement $\mathbf{x}^{(k+1)}$ is calculated. This is the same process of the CCF method. Next $\tilde{\mathbf{x}}^{(k)}$ is renewed by using $\mathbf{x}^{(k+1}$, and carried out the calculation again to obtain the new $\Delta\mathbf{a}_1^{(k+1)}$ and $\mathbf{x}^{(k+1)}$. This procedure is continued until converge.

Two kinds of method are considered. One is that both $\tilde{\mathbf{x}}_2^{(k)}$ and $\tilde{\mathbf{x}}_1^{(k)}$ are renewed after calculations are finished in all subdomains, which is called 'BlkJacobi' here. Another is that $\tilde{\mathbf{x}}_2^{(k)}$ or $\tilde{\mathbf{x}}_1^{(k)}$ is replaced immediately if it is obtained in the analysis of another subsystem. This method is called 'BlkSeidel' here, as mentioned at next section.

Fig. 6 shows the results by these iterative methods. The same result are obtained in both BlkJacobi and BlkSeidel, though the number of iterations until convergence is different. Calculation condition is the same as Fig. 5. It is seen that very good result is obtained.

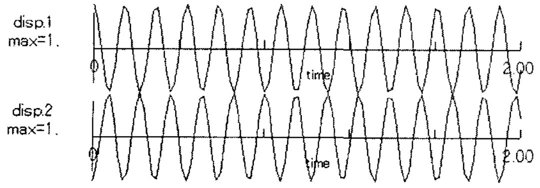

Fig. 6 Calculated response ($k_{12}=1000$, $\tilde{x}=x_n+0.5h\dot{x}_n$)

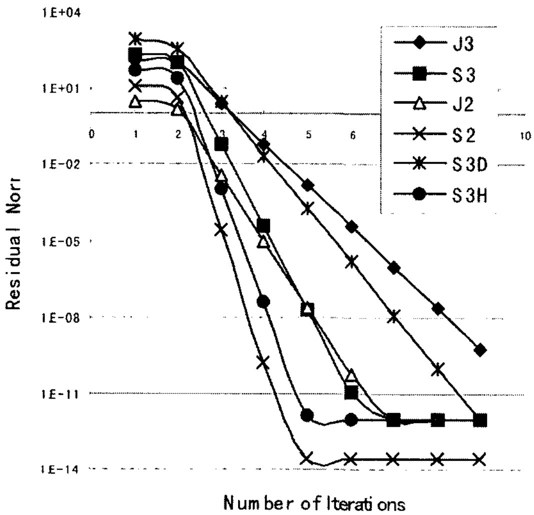

Fig. 7 Convergence behavior

Convergence behavior is investigated at the various analysis condition that $k_{12}=100$, 1000 N/m in stiffness, BlkJacobi, BlkSeidel in method, $h=0.005$, 0.01, 0.02s in step width. Residual error against iteration is plotted in Fig. 7. Meaning of symbols in the figure is listed in Table 1. As for the initial displacement choosing way of Eq. (4a) or Eq. (4b) gives little difference in numbers of iteration.

Table 1 Analysis conditions and spectrum radii

Symbol	Method	k_{12}, N/m	h, s	S.radius
J2	BlkJacobi	100	0.01	2.50×10^{-3}
S2	BlkSeidel	100	0.01	0.006×10^{-3}
J3	BlkJacobi	1000	0.01	24.9×10^{-3}
S3	BlkSeidel	1000	0.01	0.622×10^{-3}
S3H	BlkSeidel	1000	0.005	0.039×10^{-3}
S3D	BlkSeidel	1000	0.02	9.80×10^{-3}

Concerning the convergence, it is found from Fig. 7 that

(1) Blkseidel is better than BlkJacobi.
(2) It is faster as the stiffness is weaker.
(3) It is faster as the time step width is smaller.

4. Discussion on Convergence

4.1 Jacobi and Seidel methods

As the above iterative method is related to the Jacobi or Gauss-Seidel (Hereinafter denoted as Seidel) methods for solving linear equations, we review their algorithms and characteristics. A linear equation

$$\mathbf{Ax}=\mathbf{b} \tag{9}$$

can be solved by the Jacobi or Seidel method iteratively.

In the Jacobi method, $\mathbf{A}$ is separated as $\mathbf{A}=\mathbf{D}+\mathbf{E}$, and Eq. (9) is rewritten by

$$\mathbf{Dx}^{(k+1)}=-\mathbf{Ex}^{(k)}+\mathbf{b} \tag{10}$$

where $\mathbf{D}=\mathrm{diag}(\mathbf{A})$, $\mathbf{E}=\mathbf{A}-\mathbf{D}$, and superscript (k) denotes iteration number. It is cleared that this iteration procedure is stable and converge if $\mathbf{A}$ is diagonally dominant (Mori, 1984).

In the Seidel method $\mathbf{A}$ is divided into $\mathbf{L}+\mathbf{D}+\mathbf{U}$, and Eq. (9) is expressed by

$$\mathbf{Dx}^{(k+1)}=-\mathbf{Lx}^{(k+1)}-\mathbf{Ux}^{(k)}+\mathbf{b} \tag{11}$$

where $\mathbf{L}$ is lower and $\mathbf{U}$ is upper triangular matrices. When i-th row of $\mathbf{x}^{(k+1)}$ is calculated, components of $\mathbf{x}^{(k+1)}$ used in the calculation of $\mathbf{Lx}^{(k+1)}$ are already obtained. So Eq. (11) is explicitly solved. This Seidel method is proved to be stable if $\mathbf{A}$ is positive and symmetrical (Mori, 1984).

In the case that $\mathbf{A}$ is 2×2 matrix, they are written as

$$\mathbf{D}=\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \quad \mathbf{E}=\begin{bmatrix} 0 & a_{12} \\ a_{21} & 0 \end{bmatrix}$$
$$\mathbf{L}=\begin{bmatrix} 0 & 0 \\ a_{21} & 0 \end{bmatrix} \quad \mathbf{U}=\begin{bmatrix} 0 & a_{12} \\ 0 & 0 \end{bmatrix} \tag{12}$$

4.2 Discussion on convergence and stability

The similar algorithm as Eqs. (10), (11) can be used when $a_{11}, \cdots, a_{22}$ become matrices and $x_1, \cdots, f_2$ become vectors, namely

$$\mathbf{D}=\begin{bmatrix}\mathbf{A}_{11} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}_{22}\end{bmatrix}\ \mathbf{E}=\begin{bmatrix}\mathbf{0} & \mathbf{A}_{12}\\ \mathbf{A}_{21} & \mathbf{0}\end{bmatrix}$$
$$\mathbf{L}=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{A}_{21} & \mathbf{0}\end{bmatrix}\ \mathbf{U}=\begin{bmatrix}\mathbf{0} & \mathbf{A}_{12}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} \quad (12')$$

In the case of dynamic analysis by the Newmark or Newton methods these matrices are

$$\mathbf{A}_{11}=\mathbf{M}_{11}+0.5h\mathbf{C}_{11}+\beta h^2\mathbf{K}_{11}\ \mathbf{A}_{12}=\beta h^2\mathbf{K}_{12}$$
$$\mathbf{A}_{21}=\beta h^2\mathbf{K}_{21}\ \mathbf{K}_{22}=\mathbf{M}_{22}+0.5h\mathbf{C}_{22}+\beta h^2\mathbf{K}_{22} \quad (13)$$

The matrix $\mathbf{D}$ is a block diagonal, So they may be called blocked Jacobi or blocked Seidel methods respectively. (we denote them 'BlkJacobi' and 'BlkSeisel' here). Their stability are better than Jacobi or Seidel method, and they are evaluated by the spectrum radius of iterative matrix $\mathbf{H}$ defined as $\mathbf{H}=\mathbf{D}^{-1}\mathbf{E}$ or $\mathbf{H}=(\mathbf{L}+\mathbf{D})^{-1}\mathbf{U}$. When its value is smaller than unit iterations are stable and $\mathbf{x}^{(k)}$ converge to the exact solution. (Spectrum radius is the absolute maximum of eigenvalues of matrix $\mathbf{H}$, and it can be obtained by eigenvalue analysis.)

It is found by Eq.(13) that the matrix $\mathbf{A}$ in dynamic analysis is advantageous to convergence than in static problems, because masses makes it diagonally dominant.

Spectrum radii used in the calculations of Fig. 7 are shown in the last column of Table 1.

5. Seismic Response Analysis

5.1 MCK matrices of equation of motion

In this section the present method is applied to a vibration analysis of a beam structure. The equation of motion can be derived by the finite element method, using scalar mass, damping, spring, and beam elements. $\mathbf{M}$ and $\mathbf{K}$ in Eq. (1) are built up by assembling element matrices :

$$\mathbf{M}=\sum \mathbf{N}_e\mathbf{L}_e{}^T\mathbf{M}_e\mathbf{L}_e \quad \mathrm{K}=\sum \mathbf{N}_e\mathbf{L}_e{}^T\mathbf{K}_e\mathbf{L}_e \quad (14)$$

where $\mathbf{M}_e$, $\mathbf{K}_e$ are mass and stiffness matrices in each element, $\mathbf{L}_e$ is a coordinate transform matrix and $\mathbf{N}_e$ is a $[0-1]$ matrix that maps the element components to the total system.

In two dimensional problems the element stiffness matrix $\mathbf{K}_e$ of a beam is given by

$$\mathrm{K}_e=\begin{bmatrix} 12EI/l^3 & 0 & 6EI/l^2 & -12EI/l^3 & 0 & 6EI/l^2\\ 0 & AE/l & 0 & 0 & -AE/l & 0\\ 6EI/l^2 & 0 & 4EI/l & 6EI/l^2 & 0 & 2EI/l\\ -12EI/l^3 & 0 & 0 & 12E/l^3 & 0 & -6EI/l^2\\ 0 & -AE/l & 0 & 0 & AE/l & 0\\ 6EI/l^2 & 0 & 2EI/l & -6EI/l^2 & 0 & 4EI/l \end{bmatrix} \quad (15)$$

where l is length, E is Young's modulus, I is sectional moment of inertia, and A is sectional area.

The matrix of a scalar spring with stiffness k_{ij} connecting two freedoms i and j is

$$\mathbf{K}_e=\begin{bmatrix} k_{ij} & -k_{ij}\\ -k_{ij} & k_{ij}\end{bmatrix} \quad (16)$$

The matrix of scalar mass m_i or spring k_i at one freedom i is

$$\mathbf{K}_e=k_i\ \mathbf{M}_e=m_i \quad (17)$$

5.2 Calculation of a seismic response

An earthquake response analysis is often carried out for a seismic design of structures. Here we apply the present method to a seismic response analysis. Fig. 8 shows a model which consists of two towers and a piping. Towers are modeled by beam elements (node 1-4, and node 5-7), and piping is modeled as connecting spring (node 3-7), Mass values are prepared by calculation of weight. Rayleigh damping $\mathbf{C}=\alpha\mathbf{K}$ is used, where α is set to 0.00417 so that damping ratio of 1st mode equals 0.03. Its vibration characteristics is shown in Table 2.

Responses to El Centro NS earthquake were calculated in the condition that the maximum acceleration is 340 Gal (3.4 m/s^2), and time step is 0.02s. The results by the presented method is shown in Fig.9. Waves in the figure indicate the relative displacement between point 3 and 7, the

Table 2 Vibration characteristics

Mode number	Natural frequency, Hz	Damping ratio	Natural frequency without coupling, Hz
1st	2.29	0.030	2.01
2nd	4.39	0.057	2.35
3rd	11.84	0.155	11.58

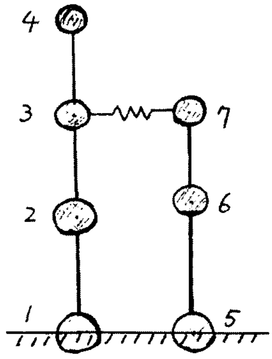

Fig. 8 Two tower model

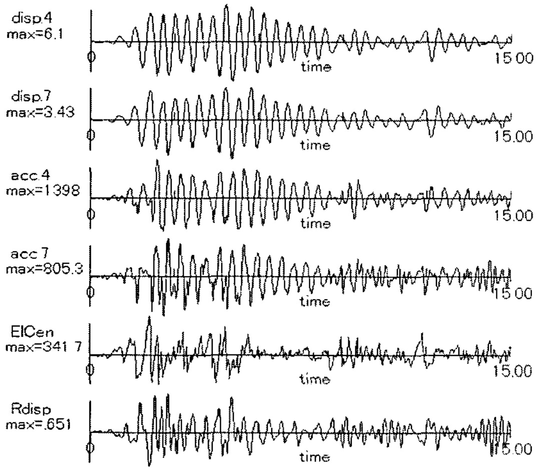

Fig. 9 Seismic response

input acceleration, accelerations at points 7, 4 and displacements at point 7, 4 from bottom to upward, whose maximum values represented by cm or cm/s^2 units are shown in the figure.

For a comparison, the usual analysis treated as total system was done. Its results were perfectly coincident with the Fig. 9, which demonstrates that the present method is reasonable and applicable.

6. Multibody Analysis

Here, in this section the present method is applied to simple multibody systems including a nonlinear large link motion.

6.1 Nonlinear systems

Equation of motion with nonlinear spring forces is represented by

$$\mathbf{g} \equiv \mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f}_{\mathrm{int}} - \mathbf{f} = 0 \tag{18}$$

where $\mathbf{f}_{\mathrm{int}}$ is an element force. Assume the variables $\ddot{\mathbf{x}}_n$, $\dot{\mathbf{x}}_n$, $\mathbf{x}_n$ at time t_n are already known, then those at next step are calculate as follows. Let $\mathbf{a}^{(k)}$, $\mathbf{v}^{(k)}$, $\mathbf{d}^{(k)}$ be the approximate values of $\ddot{\mathbf{x}}_{n+1}$, $\dot{\mathbf{x}}_{n+1}$, $\mathbf{x}_{n+1}$. The error $\mathbf{g}$ at t_{n+1} is

$$\mathbf{g}^{(k)} = \mathbf{M}\mathbf{a}^{(k)} + \mathbf{C}\mathbf{v}^{(k)} + \mathbf{K}\mathbf{d}^{(k)} + \mathbf{f}_{\mathrm{int}}(\mathbf{d}^{(k)}) - \mathbf{f}_{n+1} \tag{19}$$

In the Newmark algorithm $\mathbf{a}^{(k)}$, $\mathbf{v}^{(k)}$, $\mathbf{d}^{(k)}$ are expressed by

$$\begin{aligned} \mathbf{a}^{(k)} &= \ddot{\mathbf{x}}_n + \Delta\mathbf{a}^{(k)} \\ \mathbf{v}^{(k)} &= \dot{\mathbf{x}}_n + h\ddot{\mathbf{x}}_n + 0.5h\Delta\mathbf{a}^{(k)} \\ \mathbf{d}^{(k)} &= \mathbf{x}_n + h\dot{\mathbf{x}}_n + 0.5h^2\ddot{\mathbf{x}}_n + \beta h^2\Delta\mathbf{a}^{(k)} \end{aligned} \tag{20}$$

So, $\mathbf{g}^{(k)}$ in Eq. (19) is regarged as a function of $\Delta\mathbf{a}$. Applying the Newton method to Eq. (19), modification of $\Delta\mathbf{a}$ is given by

$$\Delta\mathbf{a}^{(k+1)} = \Delta\mathbf{a}^{(k)} - [\mathbf{J}^{(k)}]^{-1}\mathbf{g}^{(k)}$$

where

$$\begin{aligned} \mathrm{J}^{(k)} &= \frac{\partial \mathrm{g}}{\partial \Delta \mathrm{a}} = \mathrm{M}\frac{\partial \mathrm{a}}{\partial \Delta \mathrm{a}} + \mathrm{C}\frac{\partial \mathrm{v}}{\partial \Delta \mathrm{a}} + \mathrm{K}\frac{\partial \mathrm{d}}{\partial \Delta \mathrm{a}} + \frac{\partial \mathrm{f}_{\mathrm{int}}}{\partial \mathrm{d}}\frac{\partial \mathrm{d}}{\partial \Delta \mathrm{a}} \\ &= \mathrm{M} + 0.5h\mathrm{C} + \beta h^2(\mathrm{K} + \mathrm{K}_N) \end{aligned} \tag{21}$$

So, the procedure to obtain $\ddot{\mathbf{x}}_{n+1}$, $\dot{\mathbf{x}}_{n+1}$, $\mathbf{x}_{n+1}$ is shown as

(1) At the beginning of $k=0$, set $\Delta\mathbf{a}^{(k)}=0$

(2) Calculate $\mathbf{a}^{(k)}$, $\mathbf{v}^{(k)}$, $\mathbf{d}^{(k)}$ by Eq. (20)

(3) Calculate $\mathbf{g}^{(k)}$ using $\mathbf{a}^{(k)}$, $\mathbf{v}^{(k)}$, $\mathbf{d}^{(k)}$ by Eq. (19)

(4) Calculate matrix
$\mathbf{J}^{(k)} = \mathbf{M} + 0.5h\mathbf{C} + \beta h^2(\mathbf{K} + \mathbf{K}_N)$

(5) Solve
$[\mathbf{M} + 0.5h\mathbf{C} + \beta h^2(\mathbf{K} + \mathbf{K}_N)]\delta\mathbf{a} = -\mathbf{g}^{(k)}$

(6) Modify $\Delta\mathbf{a}^{(k+1)} = \mathbf{a}^{(k)} + \delta\mathbf{a}$

(7) go to (2) until convergence

(8) if converged, replace
$\mathbf{a}^{(k+1)}\mathbf{v}^{(k+1)}\mathbf{d}^{(k+1)}$ to $\ddot{\mathbf{x}}_{n+1}$, $\dot{\mathbf{x}}_{n+1}$, $\mathbf{x}_{n+1}$.

6.2 Element force and matrix of a truss

In order to analyze a motion of a system including truss element, the $\mathbf{f}_{\mathrm{int}}$ and stiffness $\mathbf{K}_N = (\partial\mathbf{f}_{\mathrm{int}}/\partial\mathbf{d})$ of truss in Eqs. (19) and (21) should be calculated. For plane motion with two degree of freedom per node they are obtained as follows.

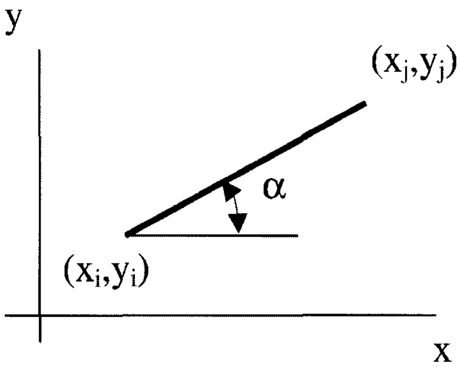

Fig. 10 Truss element

When coordinates of the truss are given, strain energy **U** is expressed by

$$U=(1/2)k(L-L_0)^2 \tag{22}$$

where $\mathbf{k}=AE/L$, $L=\sqrt{(x_j-x_i)^2+(y_j-y_i)^2}$

Element forces are obtained by differentiating **U** with coordinate, for example

$$f_{1x}=\frac{\partial U}{\partial x_1}=-k\Delta L\cos\alpha$$

where $\Delta L=L-L_0$, L_0 is free length, α is truss angle shown in Fig.10. Components of stiffness matrix are obtained by differentiating force with coordinate, for example

$$K_{11}=\frac{\partial f_{1x}}{\partial x_1}=-k\left(\cos\alpha\frac{\partial\Delta L}{\partial x_1}+\Delta L\frac{\partial\cos\alpha}{\partial x_1}\right)$$
$$=k\cos^2\alpha$$

Consequently $\mathbf{f}_e$ and $\mathbf{K}_e$ are given by

$$\mathbf{f}_e=\begin{Bmatrix}-Tc\\-Ts\\0\\Tc\\Ts\\0\end{Bmatrix}\quad \mathbf{K}_e=k\begin{bmatrix}cc & cs & 0 & -cc & -cs & 0\\ sc & ss & 0 & -sc & -ss & 0\\ 0 & 0 & 1 & 0 & 0 & 1\\ -cc & -cs & 0 & cc & cs & 0\\ -sc & -ss & 0 & sc & ss & 0\\ 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix} \tag{1}$$

where $T=k\Delta L$, $c=\cos\alpha$, $s=\sin\alpha$.

6.3 Motion analysis

6.3.1 Analysis of a double pendulum by usual method

A double pendulum is analyzed by the usual method (not DDM) to check the nonlinear analysis with truss elements. The model is shown in Fig. 11. At the initial condition two pendulums are

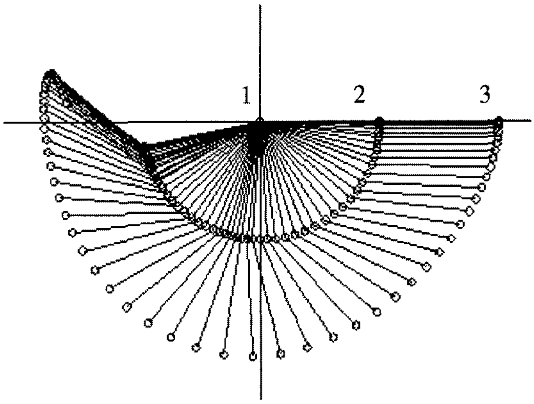

Fig. 11 Double pendulum

located horizontally without velocity. The nodal point 1 is supported with stiff springs. A free fall motion by gravity is calculated. The calculated locus of motion is plotted in Fig. 11. Reasonable results are obtained. Used data are

Length of truss : 0.5 m each
Sectional area : 0.002 m^2
Young's modulus : 207 GPa
Mass : 9.8 kg
Damping : C=0
Time step width : h=0.02 s

6.3.2 Analysis of a pendulum supported by a flexible beam structure by the present method

As a problem to apply DDM, a pendulum supported by a flexible structure is taken. The model is shown in Fig. 12 or 13. A pendulum (node 4-5) and structure (node 1-2-3) are modeled by a truss and beams. Both are connected by springs in x and y directions between node 3 and 4 (they are at same position). The total system is divided into the beam structure (subsystem 1) and the pendulum (subsystem 1), and analyzed by the BlkSeidel method. Used data are

Length of truss : 1.0 m
Length of beams : 2.0, 1.2 m
Sectional area : 0.0006 m^2
Young's modulus : 207 MPa
Coupling stiffness : 106 N/m
Mass : 0.98 kg

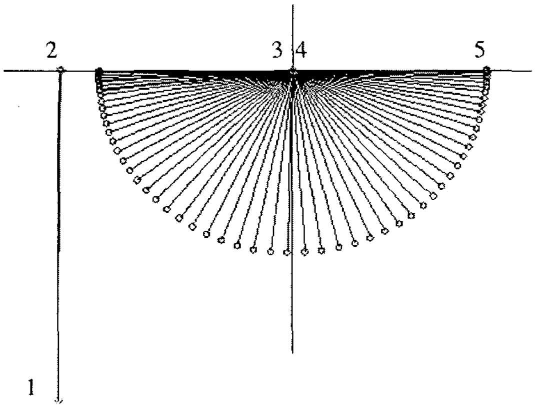

Fig. 12 A pendulum supported by a structure (Free Fall Response)

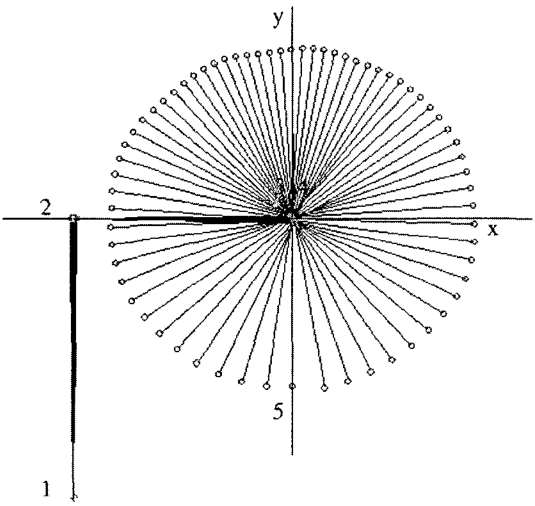

Fig. 13 A pendulum suported by a structure (Whirl Motion Under Initial Velocity Given)

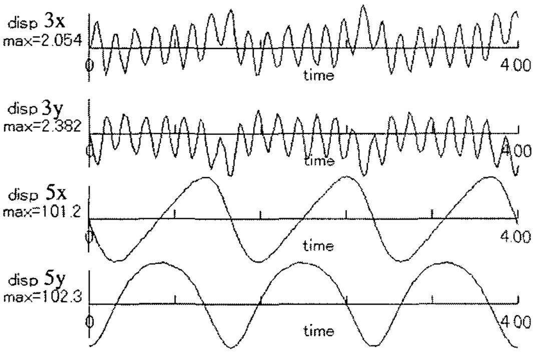

Fig. 14 Response waves

Damping : C=0 Ns/m,
Time step width : h=0.02s

(1) Swing motion

Fig. 12 show a calculation result. Here the truss is located horizontally without velocity at the initial condition. A free fall motion due to gravity is analyzed. Small deflection of the beam is observed.

7. Conclusions

A domain decomposition method for dynamic analysis is proposed. It is an extension of the CCF method by introducing iteration process. Stability and convergence are discussed based on the Jacobi or Seidel methods. It is found that

(1) The BlkSeidel method has better convergence behavior than BlkJacobi.

(2) Convergence of BlkSeidel is assured for positive and symmetric matrices.

(3) Convergence is better as coupling stiffness is weaker, and time step is smaller.

(4) Convergence is much better in dynamic system than in static system because masses improve the matrix characteristics of diagonally dominant.

(5) Choosing way of the initial displacement is not so important.

Some calculations were carried out, and it is demonstrate that this present method is reasonable and useful.

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