Accuracy Analysis of Optimal Trajectory Planning Methods Based on Function Approximation for a Four-DOF Biped Walking Model

Chunye Peng
Department of Mechanical Engineering and Science, Tokyo Institute of Technology, 2-12-1, Ookayama, Meguro-ku, Tokyo 152-8552, Japan
Kyosuke ONO*
Department of Mechanical Engineering and Science, Tokyo Institute of Technology, 2-12-1, Ookayama, Meguro-ku, Tokyo 152-8552, Japan

Based on an introduced optimal trajectory planning method, this paper mainly deals with the accuracy analysis during the function approximation process of the optimal trajectory planning method. The basis functions are composed of Hermit polynomials and Fourier series to improve the approximation accuracy. Since the approximation accuracy is affected by the given orders of each basis function, the accuracy of the optimal solution is examined by changing the combinations of the orders of Hermit polynomials and Fourier series as the approximation basis functions. As a result, it is found that the proper approximation basis functions are the 5^{th} order Hermit polynomials and the 7^{th}-10^{th} order of Fourier series.

Key Words : Optimal Trajectory Planning, Biped Walking, Accuracy Analysis, Function Approximation, Forward Dynamic Simulation

1. Introduction

Recently, a growing community of researchers is working towards a better understanding of human walking locomotion, and many biped walking humanoid robots have been developed. Among them, some representative ones are P2, P3 and ASIMO constructed by HONDA (Hirose et al., 2001), WABIN built by Waseda University (Yamaguchi et al.,1999), H6 and H7 by Tokyo University (Nishiwaki et al., 2000), HRP-1 and HRP-2 by National Institute of Advanced Industrial Science and Technology (Kaneko et al., 2002), JOHNNIE by Technical University of Munich (Gienger et al., 2001), and SDR-3X by SONY (Kuroki et al., 2001). However, comparing with human walking, most of these robots have a problem in common : they consume great amount of energy because of lacking energy-efficient walking gaits.

Besides robots, energy-efficient walking gaits are also meaningful for developing prosthetics and artificial walking. It has been measured that lower limb amputee consumes about two times of metabolic energy larger than that of non-amputee (Jessica and James, 1993). Clearly, those with a lower limb amputation suffer a severe handicap. Therefore, powered prosthetics combined with energy-efficient walking gaits should be developed to reduce the metabolic cost of locomotion and thus help the amputees walk better. For the case of artificial walking, without energy-efficient walking gaits, the metabolic energy needed by the paraplegic to ambulate with a synthetic gait is still too high to make it a practical alternative to

* Corresponding Author,
E-mail : ono@mech.titech.ac.jp
TEL : +81-3-5734-2171; **FAX** : +81-3-5734-2171
Department of Mechanical Engineering and Science, Tokyo Institute of Technology, 2-12-1, Ookayama, Meguro-ku, Tokyo 152-8552, Japan. (Manuscript **Received** November 29, 2004; **Revised** December 15, 2004)

the wheelchair now.

Experimental studies of human locomotion also support the hypothesis that the choice of human gait is influenced by energy consideration. In our previous paper (Peng and Ono, 2003), we calculated the energy-optimal walking gaits for a 4-DOF planar biped model with flexed knee at foot exchange by using a proposed trajectory planning method based on function approximation method. The selected basis functions of the trajectory planning method are composed of Hermit polynomials and Fourier series to improve the approximation accuracy. We calculated the optimal walking gaits for the biped model under both full-actuated and under-actuated conditions, and the obtained energy-optimal walking gaits seem natural and closely like that of human walking. Also, the validity of the gait generating method had been confirmed by forward dynamics simulation.

During the analyzing process, we found that the approximation accuracy is definitely affected by the given orders of selected basis functions-Hermit polynomials and Fourier series. Therefore, it is interesting to examine the effects of combinations of different orders of Hermit polynomials and Fourier series as approximation basis function on approximation accuracy.

In this paper, by combining different orders of Hermit polynomials and Fourier series as the approximation basis function in the proposed trajectory planning method, we calculated the energy-efficient walking gaits for the biped model. Then, these generated walking gaits are verified by forward dynamics simulation and some of them are judged false. After then, the approximation accuracy of these results are evaluated by the error of the forward dynamic motion from the gait solution and the smoothness of the joint torque. Subsequently, by summing up the above analysis, the best approximation basis function is concluded.

2. Biped Walking Model and Its Dynamic Equations

In this study, we modeled the planar biped walking mechanism as a 4-DOF links system as shown in Figure 1. Here we disregarded the upper body because it has little effect on walking gaits (Ono and Liu, 2002). The two legs are assumed to be directly connected to each other through an actuator at the hip joint. Also, we assumed that the knee and ankle joints have actuators. The ankle of the stance leg is modeled as a rotating joint fixed to the ground, while the small feet of both legs are neglected.

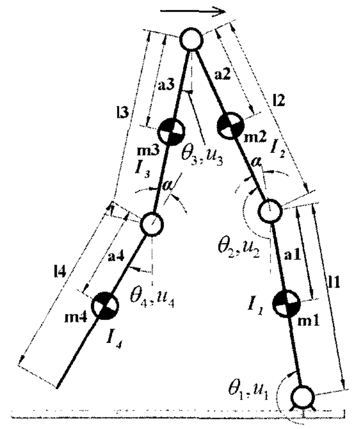

Fig. 1 Planar 4-DOF biped model

Biped walking is a periodic phenomenon. A complete step cycle in human walking can be divided into two phases: single-support phase and double-support phase. During the single-support phase, one foot swings from the rear to the front while the other foot keeps stationary on the ground. During the double-support phase, both feet keep contact with the ground. However, if we neglect the foot length, we have to assume the period of the double-support phase to be zero. Figure 2 shows the analytical model of one step walking locomotion. In this study the single-support phase (posture 1- posture 2) is treated in only one time period, and the time interval of this phase is denoted by T. The double-support phase is assumed to be instantaneous: the foot exchange takes place instantly (posture 2- posture 3) once the swing leg touches the ground. Thus T is the whole step period. From posture 3, the next swing phase begins. This cyclic pattern of walking movements is repeated over and over, step after step, with a reasonable assumption that successive

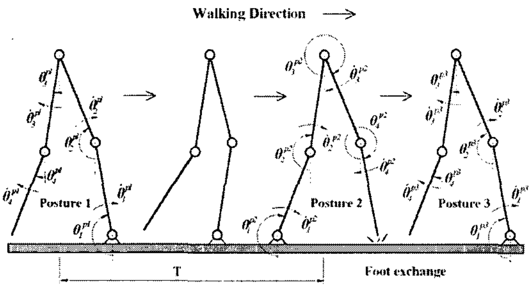

Fig. 2 Postures of biped model during one step walking

steps are all the same.

The equation of motion of the biped model can be written as follows:

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{\theta}}\}+[\boldsymbol{C}]\{\dot{\boldsymbol{\theta}}^2\}+[\boldsymbol{K}]\{\boldsymbol{\theta}\}=\{\boldsymbol{u}\} \quad (1)$$

where, $[\boldsymbol{M}]$, $[\boldsymbol{C}]$, and $\{\boldsymbol{K}\}$ are calculated by the parameter values of the biped model and the angular positions of the links, and $\{\boldsymbol{u}\}$ is the input torque vector.

We assumed that at the instant of foot exchange the collision between the swing leg and the ground is perfectly inelastic without any slippage. By using the impulse-momentum equations for translation and rotation, the relationship of the link angular speeds between right before and right after the collision can be obtained as follows:

$$[\boldsymbol{H}]\{\boldsymbol{\theta}^{p2}\}=[\boldsymbol{Z}]\{\dot{\boldsymbol{\theta}}^{p3}\} \quad (2)$$

where, $[\boldsymbol{H}]$, and $[\boldsymbol{Z}]$ are calculated by the parameter values of the biped model and the angular positions of the links.

From posture 3, the next swing phase begins. With the assumption that the successive step is the same as the current step, the motion state variables of posture 3 must be the same as those of Posture 1. This can be expressed as:

$$\begin{aligned}\boldsymbol{\theta}^{p3}&=\boldsymbol{\theta}^{p1}\\ \dot{\boldsymbol{\theta}}^{p3}&=\dot{\boldsymbol{\theta}}^{p1}\end{aligned} \quad (3)$$

By plugging Eq. (3) back into Eq. (2), the angular velocities of posture 2 can be expressed by those of posture 1 as follows:

$$\{\dot{\boldsymbol{\theta}}\}^{p2}=[\boldsymbol{H}]^{-1}[\boldsymbol{Z}]\{\dot{\boldsymbol{\theta}}\}^{p1} \quad (4)$$

In addition, from Fig. 2 and Eq.(3), the relationship of angular positions between posture 2 and posture 1 can be calculated as follows:

$$\boldsymbol{\theta}^{p2}=\boldsymbol{\theta}^{p1}+\pi \quad (5)$$

Equations (4) and (5) are the cyclic constraint conditions of walking motion and can be rewritten in the general form:

$$c_b(\boldsymbol{\theta}^{p1}, \dot{\boldsymbol{\theta}}^{p1}, \boldsymbol{\theta}^{p2}, \dot{\boldsymbol{\theta}}^{p2})=0 \quad (6)$$

3. Optimal Trajectory Planning Method

In this study, the single-support phase of walking locomotion is solved by optimal trajectory planning method. In order to obtain a natural walking locomotion with the lowest possible input torque, the performance index J here is defined as the sum of the integration of the square input torque at all joints during one step period:

$$J=\sum_{i=1}^{4}\int_0^T u_i^2(t)\,dt \quad (7)$$

Rearranging Eq. (1), u_i can be expressed as a function of the trajectory variables:

$$u_i=\Gamma_i(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}}) \qquad (i=1\sim4) \quad (8)$$

By using the function approximation method, the joint angle trajectory can be approximated by a linear combination of the basis function as follows:

$$\boldsymbol{\theta}(\boldsymbol{p}, t)=\boldsymbol{h}^T(t)\boldsymbol{p} \quad (9)$$

where $\boldsymbol{h}(t)$ is the basis function vector, and $\boldsymbol{p}$ is the coefficient vector to be optimized.

In this study, to improve the approximation accuracy, Hermite polynomials and Fourier series are grouped together as the basis functions (Peng and Ono, 2003). Hermit polynomials that can easily specify the initial and terminal conditions of the trajectory are used to represent the non-cyclic components and act as â coarse approximation. Fourier series can efficiently approximate the periodical components and act as precise approximation. Thus, the approximation accuracy is improved. Then the trajectory of the j^{th} joint is of the following form:

$$\theta_j(\mathrm{p},\, t) = \sum_{i_h=1}^{nh} h_{hi_h}(t)\, p_{i_h j} + \sum_{i_f=1}^{nf} h_{fi_f}(t)\, p_{i_f j}$$
$$= (h_{h1}(t),\, h_{h2}(t),\, \cdots) \begin{pmatrix} \theta_j(0) \\ \dot{\theta}_j(0) \\ \cdots \end{pmatrix} \tag{10}$$
$$+ \sum_{i_{fc}=0}^{nf} \cos(i_{fc}\omega t)\, c_{i_{fc}j} + \sum_{i_{fs}=1}^{nf} \sin(i_{fs}\omega t)\, s_i$$

where, nh is the orders of Hermit polynomials and nf is the orders of Fourier series.

Substituting Eqs. (8) and (9) back into Eq. (7), we will get a performance index J related to $\boldsymbol{p}$:

$$J = J(p) \tag{11}$$

If the biped model walks with unactuated joints and joint j is a passive one, the following dynamic constraint should be added to the constraint conditions: in the basis function space, the projection of the joint torque $u_j(\boldsymbol{p},\, t)$ onto the basis axes should be zero (Ono and Liu, 2002):

$$c_d(p) = \int_0^T u_j(\boldsymbol{p},\, t)\, \boldsymbol{h}(t)\, dt = \boldsymbol{0} \tag{12}$$

The most generic optimal walking gait will be obtained by solving the coefficient vector $\boldsymbol{p}$ by using the minimizing condition of J in Eq. (11) under the constraint Eqs. (6) and (12). However, it was found difficult to solve this problem within a reasonable computing time. By examining the classification of the locomotion variables into the unknown variables and given variables in optimization calculation, we found that the computing time can be remarkably reduced if the boundary posture 1 and 3 are given. In addition, if the other variables are determined from the J minimum condition, the sufficient foot clearance (the maximum elevation of the swing foot above the ground) of the swing leg can not be assured. Instead, we found that the foot clearance can be directly tuned by changing the beginning boundary velocities of the swing leg, $\dot{\theta}_3^{p1}$ and $\dot{\theta}_4^{p1}$. Therefore, in this study, the values of the beginning boundary displacements $\boldsymbol{\theta}^{p1}$ are given. Then the end boundary displacements $\boldsymbol{\theta}^{p2}$ are also determined from Eq. (6). The velocities of the swing leg $\dot{\theta}_3^{p1}$ and $\dot{\theta}_4^{p1}$ are properly adjusted outside of the optimization routine so as to give desired foot clearance of the swing leg.

Because the values of $\boldsymbol{\theta}^{p1}$, $\boldsymbol{\theta}^{p2}$ and $\dot{\theta}_3^{p1}$, $\dot{\theta}_4^{p1}$ are all given before optimization, they should be taken out from the optimizing parameter set and the boundary constraints. Thus the remained optimizing parameter vector and boundary constraints are of following forms:

$$\boldsymbol{q} = \{\dot{\theta}_1^{p1},\, \dot{\theta}_2^{p1},\, \dot{\boldsymbol{\theta}}^{p2T},\, \ddot{\boldsymbol{\theta}}^{p1T},\, \ddot{\boldsymbol{\theta}}^{p2T},\, \boldsymbol{\theta}^{(3)p1T},\, \boldsymbol{\theta}^{(3)p2T},\, \boldsymbol{c}_{i_{fc}j}^T,\, \boldsymbol{s}_{i_{fs}j}^T\}^T \tag{13}$$

Dynamics constraint Eq. (12) and boundary condition Eq. (6) are put together to the following equation:

$$\boldsymbol{c}(\boldsymbol{q}) = \begin{pmatrix} \boldsymbol{c}_d(\boldsymbol{q}) \\ \boldsymbol{c}_b(\boldsymbol{q}) \end{pmatrix} = \boldsymbol{0} \tag{14}$$

Thus this trajectory planning problem is transformed from the calculus of variation subject to the differential equation constraints into a parameter optimization problem for the basis function coefficients, that is, to find proper $\boldsymbol{q}$ value in Eq. (13) which minimize the objective function J in Eq. (11) in a system described by Eq. (1), and subject to the constraint conditions of Eq. (14).

Since Eq. (14) is nonlinear with respect to $\boldsymbol{q}$, it is solved by Newton-Raphson iteration method in the following form:

$$\frac{\partial \boldsymbol{c}(\boldsymbol{q}_k)}{\partial \boldsymbol{q}}(\boldsymbol{q}_{k+1} - \boldsymbol{q}_k) = -\boldsymbol{c}(\boldsymbol{q}) \tag{15}$$

In Eq. (15), the number of optimizing variables (which is 102) is larger than that of constraint equations (four for full-actuated condition or 31 for under-actuated condition). Therefore, a solution that satisfies the constraints has redundant freedom for optimization. This optimization problem is solved by Singular Value Decomposition (SVD) method and line search method as introduced in our previous paper (Peng and Ono, 2003).

4. Results and Discussions

4.1 Full-actuated case

Using the trajectory planning method stated above, we calculated the optimal trajectories for the biped model that has the same limb parameters as those of human beings as shown in Table 1.

Table 1 Link parameter values

Parameters	1st link	2nd link	3rd link	4th link
Length l_i [m]	0.45	0.45	0.45	0.45
Mass m_i [kg]	4.0	8.0	8.0	4.0
Center of mass a_i [m]	0.2	0.15	0.15	0.2
Moment of inertia I_i [kgm^2]	0.067	0.135	0.135	0.067

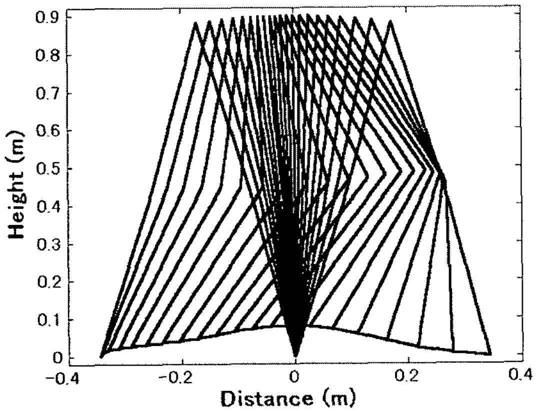

Fig. 3 Trajectory planned walking gait (Full-actuated, Hermit=5, Fourier=10)

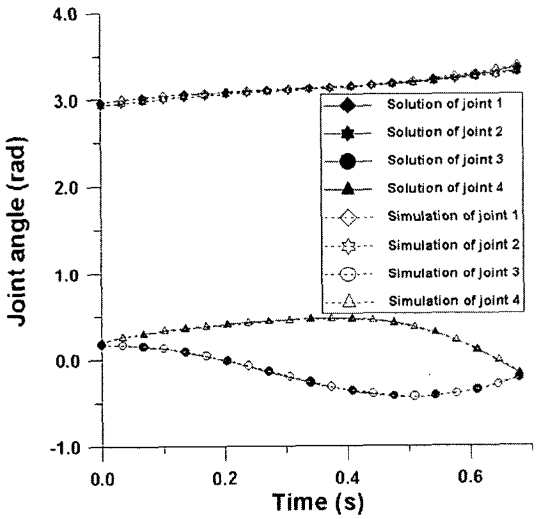

Fig. 4 Joint angle trajectories (Full-actuated, Optimized solution and Forward simulation, Hermit=5, Fourier=10)

First, we performed the trajectory optimization for the biped model when all joints are actuated

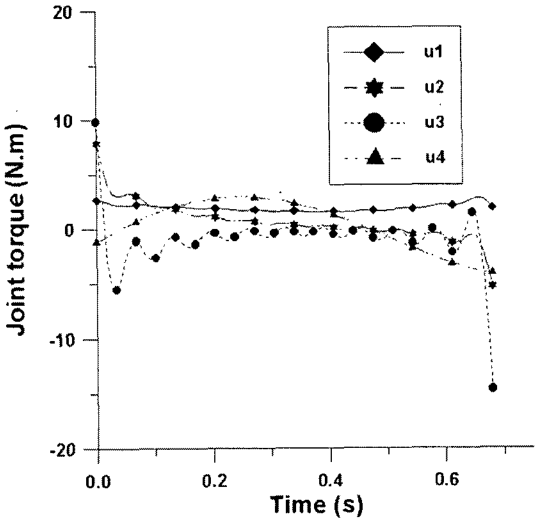

Fig. 5 Calculated joint torque (Full-actuated, Hermit=5, Fourier=10)

and used the 5th order Hermit polynomials and 10th order Fourier series as the basis functions. Figure 3 shows the stick figure of the trajectory-planned walking gait. The optimized trajectories of the joint angles are shown in Fig. 4 by black symbols. And Fig. 5 shows the calculated joint torque.

Adopting the calculated joint torque shown in Fig. 5 as the feed-forward input torque, a forward dynamic simulation has also been done to check the validity of this trajectory planning method. The simulated trajectories of the joint angles are shown in Fig. 4 by white symbols. It is clear that they are almost the same as those trajectory-planned ones (black ones).

Next, to examine the effect of the chosen order of each basis function on approximation accuracy, we first fixed the order of Fourier series to 10, and calculated the optimal trajectories for the biped model by changing the order of Hermit polynomials from 3 to 11. The terminal joint angle errors (at the end of one period) between the optimized trajectories and the forward simulated ones are shown in Fig. 6.

From Fig. 6, we can see that excepts the order of Hermit polynomials less than 5, which means lacking of enough approximation accuracy, we can get accurate solutions for all the other cases.

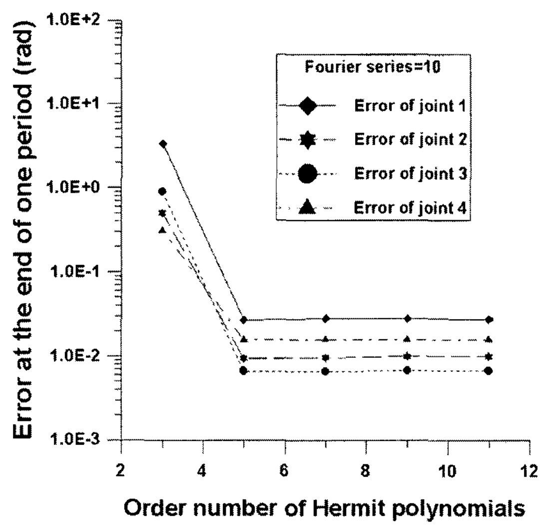

Fig. 6 Terminal simulation errors (Full-actuated, Fourier=10)

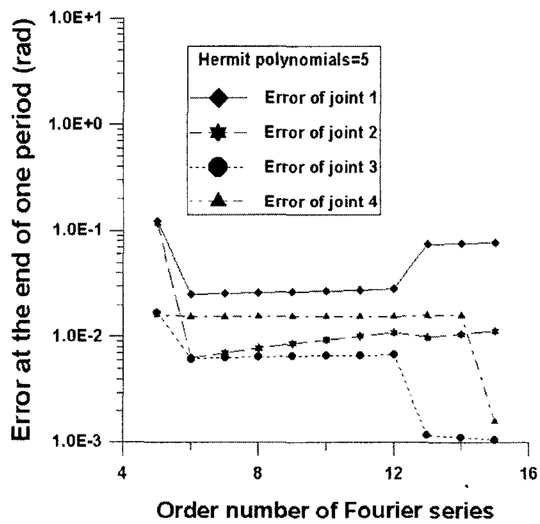

Fig. 7 Terminal simulation errors (Full-actuated, Hermit=5)

Similarly, by fixing the order of Hermit polynomials to 5, we calculated the optimal solutions by changing the order of Fourier series from 5 to 15. The terminal error of each case is shown in Fig. 7.

As seen in Fig. 7, when lacking of enough approximation accuracy (Fourier<5), the terminal errors are quite large as can be expected. But, when the order of Fourier series being larger than 12, the terminal error of the first link becomes large again. The reason of this phenomenon will be explained later.

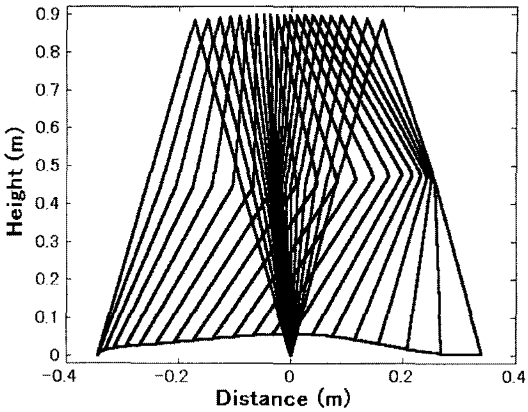

Fig. 8 Trajectory planned walking gait (ul=0, Hermit=5, Fourier=10)

4.2 Under-actuated case (ul=0)

Because it seems that the stance ankle of human being is not easy to produce the input torque, particularly at the beginning of the stance phase, we also calculated the optimal trajectories for the biped model walking with a passive stance ankle ($u_1=0$). In the calculation, we also used the 5th order Hermit polynomials and the 10th order Fourier series as the basis functions. Figure 8 shows the stick figure of the trajectory-planned walking gait. And the optimized trajectories of the joint angles are shown in Fig. 9 by black symbols. Fig. 10 shows the optimized joint torque solutions. Note that u g$_1$ is almost zero in the entire range of time.

Similar to the full-actuated case, a forward dynamic simulation has also been done, and the simulated trajectories of the joint angles are shown in Fig. 9 by white symbols. In the forward dynamic simulation, $u_1=0$ is used for forward dynamic calculation instead of the solution u_1 shown in Fig. 10. As we can see, they are also the same as the trajectory planned ones (black ones).

Next, as the same as the full-actuated case, we first fixed the order of Fourier series to 10, and calculated the optimal trajectories for the biped model by changing the order of Hermit polynomials from 3 to 11. The terminal joint angle errors between the optimized trajectories and the

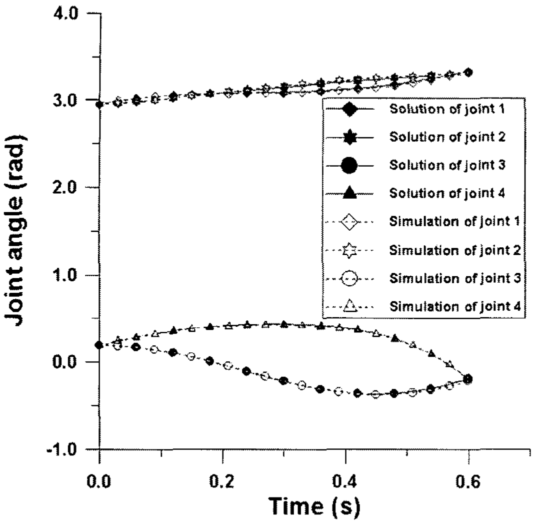

Fig. 9 Joint angle trajectories (u1=0, Optimized solution and Forward simulation, Hermit=5, Fourier=10)

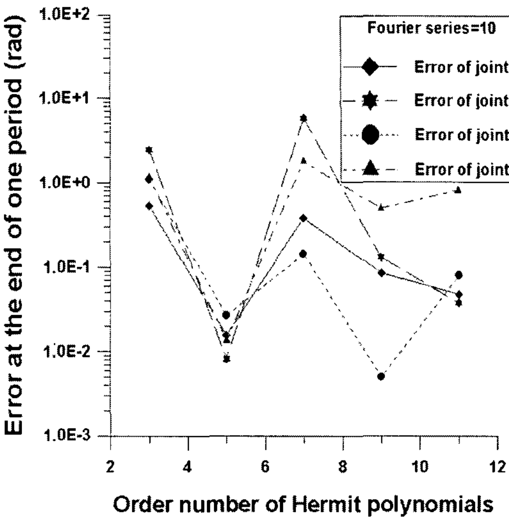

Fig. 11 Terminal simulation errors (u1=0, Fourier=10)

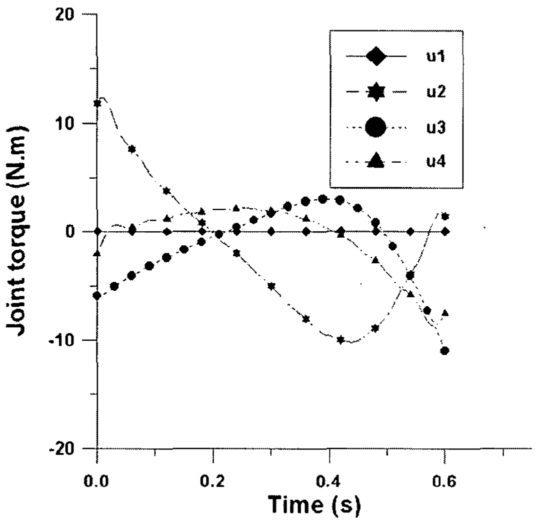

Fig. 10 Calculated joint torque (u1=0, Hermit=5, Fourier=10)

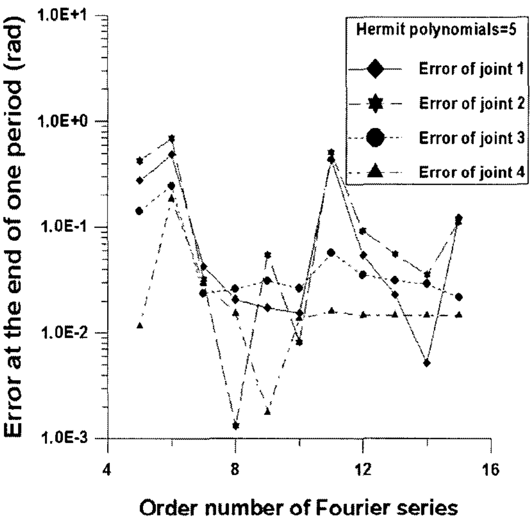

Fig. 12 Terminal simulation errors (u1=0, Hermit=5)

forward-simulated ones are shown in Fig. 11. Different from Fig. 6, we can see in Fig.11 that excepts the order of Hermit polynomials equals to 5, we can not get accurate solutions for the other cases.

Similarly, by fixing the order of Hermit polynomials to 5, we calculated the optimal trajectories by changing the order of Fourier series from 5 to 15. And the terminal error of each case is shown in Fig. 12. From Fig. 12, we found that when lacking of enough approximation accuracy (Fourier<7), the terminal errors are quite large. Also, when the order of Fourier series is larger than 10, the terminal errors become large again.

4.3 Discussion

From Figs. 6, 7, 11 and 12, we found that there exists a common phenomenon : To get good approximation accuracy, the orders of the two basis functions must be set within a proper regionneither too small nor too large. This phenomenon conflicts with the usual thought that the higher the order of the approximation function is set, the higher the approximation accuracy could be got.

To find the reason of the above phenomenon, we analyzed the Singular Value Decomposition (SVD) method used in solving Eq. (15). SVD method is a very powerful set of techniques for dealing with sets of equations or matrices that are either singular or else numerically very close to singular. It is based on the following theorem of linear algebra : Any $M \times N$ matrix A can be written as the product of an $M \times N$ column-orthogonal matrix U, an $N \times N$ diagonal matrix W with positive or zero elements (the singular values), and the transpose of an $N \times N$ orthogonal matrix V. The various shapes of these matrices will be made clearly by the following form :

$$(\boldsymbol{A}) = (\boldsymbol{U}) \cdot \begin{pmatrix} w_1 & & & \\ & w_2 & & \\ & & \cdots & \\ & & & w_N \end{pmatrix} \cdot (V^T) \qquad (16)$$

When using SVD method to solve the simultaneous equations like $\boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{b}$, a particular solution-set can be simply calculated by :

$$\boldsymbol{x} = \boldsymbol{V} \cdot [diag(1/w_j)] \cdot (\boldsymbol{U}^T \cdot \boldsymbol{b}) \qquad (17)$$

note that if $w_j = 0$, $1/w_j$ is replaced by zero before calculation.

But SVD method can not be applied blindly. For our cases, in Eq. (15), $\frac{\partial \boldsymbol{c}(\boldsymbol{q}_k)}{\partial \boldsymbol{q}}$ corresponds to A, $(\boldsymbol{q}_{k+1} - \boldsymbol{q}_k)$ corresponds to $\boldsymbol{x}$, and $-\boldsymbol{c}(\boldsymbol{q}_k)$ corresponds to $\boldsymbol{b}$. When the order of the basis function is set larger than required, the number of optimizing variables and the dimension of matrix $\boldsymbol{A}$ increase accordingly. This unavoidably brings in some very small but nonzero singular values (w_j's) to matrix $\boldsymbol{W}$. That is, the matrix $\boldsymbol{A}$ is ill-conditioned. In this case, because of the round-off errors during inverse calculation, we may get a very large residual $|\boldsymbol{A} \cdot \boldsymbol{x} - b|$

A method for solving this problem is that : the solution vector $\boldsymbol{x}$ obtained by zeroing the small

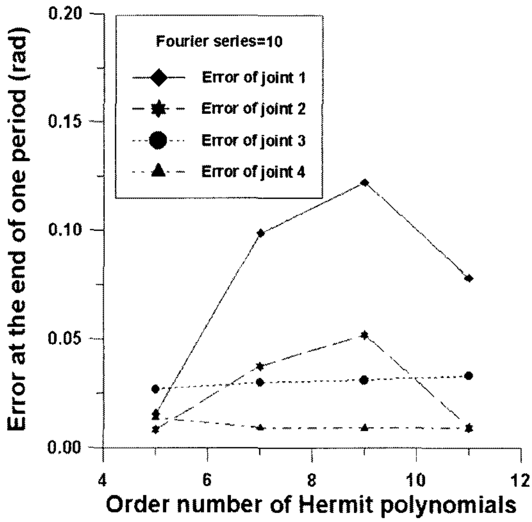

Fig. 13 Terminal simulation errors (ul=0, Fourier=10, Zeroing solution)

w_j's will be better (in the sense of the residual $|\boldsymbol{A} \cdot \boldsymbol{x} - \boldsymbol{b}|$ being smaller) than the SVD solution where the small w_j's are left nonzero (William et al., 1992).

By using the above "zeroing" method, we re-calculated the optimal trajectories for the cases of the orders of Hermit polynomials larger than 5 in Fig. 11. The new-calculated terminal joint angle errors of these cases together with the errors of the case of Hermit polynomials=5 (the same as those in Fig. 11 because there has no small w_j's needed for "zeroing") are shown in Fig. 13. Note that ordinate is illustrated in linear scale that different from the case in Fig. 11.

Comparing Fig. 13 with Fig. 11, we found that the approximation accuracy is improved to a certain extent. However, it is still not good enough because of the irremovable residual $|\boldsymbol{A} \cdot \boldsymbol{x} - \boldsymbol{b}|$. The reason is that the "zeroing" method is actually another kind of round-off operation, it brought some small errors into the residual $|\boldsymbol{A} \cdot \boldsymbol{x} - \boldsymbol{b}|$ when avoiding the larger ones.

The above results indicate that, the 5^{th} order of Hermit polynomials combined with the $7^{th} \sim 10^{th}$ order of Fourier series as approximation basis functions has an enough approximation accuracy for this trajectory planning problem. Higher orders of approximation basis functions (Hermit>

5 or Fourier > 10) are not efficient to approximate the constraint conditions better.

5. Conclusion

Summing up the above analysis, we can make a conclusion that the approximation accuracy of this optimal trajectory planning method is definitely affected by the given orders of each basis function (Hermit polynomials and Fourier series). For full-actuated case, the combinations of over the 5^{th} order of Hermit polynomials and the $5^{th}\sim 12^{th}$ order of Fourier series has good approximation accuracy. As to the under-actuated ($u_1=0$) case, the combinations of exactly 5^{th} order of Hermit polynomials and $7^{th}\sim 10^{th}$ order of Fourier series has good approximation accuracy.

For both the above two cases, lower orders of basis functions lead to larger approximation error because of lacking enough approximation accuracy. On the other hand, higher orders of basis functions may also result in larger approximation error due to the introduced small but nonzero singular values (w_j's). Although it can be mended to a certain extent by using the "zeroing" method, the approximation accuracy is still not improved.

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