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Improved Digital Redesign for Fuzzy Systems: Compensated Bilinear Transform Approach

김도완*, 주영훈**, 박진배*
*Yonsei University **Kunsan National University

Abstract

This paper presents a new intelligent digital redesign (IDR) method via the compensated bilinear transformation to design the digital controller such that the digital fuzzy system is equivalent to the analog fuzzy system in the sense of the state-matching. This paper especially consider a multirate control scheme with a predictive feature, where the digital control input is held constant N times between the sampling points. More precisely, the multirate control scheme is proposed that utilizes a numerical integration scheme to approximately predict the current state from the state measured at the sampling points, the delayed measurements. For this system, the IDR conditions incorporated with stabilizability in the format of the linear matrix inequalities (LMIs) are derived. The superiority of the proposed technique is convincingly visualized through a numerical example.

Key words : Intelligent digital redesign (IDR), fuzzy control, digital control, fuzzy system, linear matrix inequalities (LMIs).

1. 서 론

Intelligent Digital redesign (IDR) has gained tremendously increasing attention as yet another efficient design tool of the digital fuzzy control [1]-[6]. The IDR problem is the problem of designing a digital controller such that the digital fuzzy system is equivalent to the analog fuzzy system in the sense of the state matching.

There have been fruitful researches in the digital control system focusing on IDR method. Historically, Joo et al. first attempted to develop some intelligent digital redesign methodology for complex nonlinear systems [1]. They synergistically merged both the Takagi - Sugeno (T - S) fuzzy-model-based control and the digital redesign technique for a class of nonlinear systems. Chang et al. extended the intelligent digital redesign to uncertain T - S fuzzy systems [2]. These approach [1], [2] to IDR are so called as local approach. The local approach can allows to match the states of the continuous-time and the sampled-data closed-loop fuzzy systems in the analytic way, but it may lead to undesirable and/or inaccurate results. The major reason is that the redesigned digital control gain matrices are obtained by considering only the local state-matching of each sub-closed-loop system [6]. To overcome this weakness, Lee et al. a global state-matching technique

based on the convex optimization method, the linear matrix inequalities (LMIs) method, proposed in [6]. Specifically, their method is to globally match the states of the overall closed-loop T - S fuzzy system with the predesigned analog fuzzy-model-based controller and those with the digitally redesigned fuzzy-model-based controller, and further to examine the stabilizability by the redesigned controller in the sense of Lyapunov. However. the IDR problem becomes the overdamped problem according as transferring the local approach to the global one in IDR problem. It may lead to undesirable and/or inaccurate results.

Motivated by the above observations, we proposea new intelligent digital redesign (IDR) method via the compensated bilinear transformation to design the digital fuzzy controller such that the digital fuzzy control system is equivalent to the analog fuzzy control system in the sense of the state-matching. We especially consider a multirate control scheme with a predictive feature, where the digital control input is held constant N times between the sampling points (see, e.g.,[10-18]). More precisely, we propose the multirate control scheme that utilizes a numerical integration scheme to approximately predict the current state from the state measured at the sampling points, the delayed measurements. For this system, we derive the IDR conditions incorporated with stabilizability in the format of the linear matrix inequalities (LMIs). The superiority of the proposed technique is convincingly visualized through a numerical example.

2. Preliminaries and Problem Statements

In this section, we consider the problem of matching the responses of an existing analog fuzzy control system with those of the digital fuzzy control system for the same initial conditions.

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Consider the system described by the following Takagi-Sugeno fuzzy model [7-9]:

$$\dot{x}(t)=\sum_{i=1}^{r}\theta_i(z(t))(A_ix(t)+B_iu(t)) \tag{1}$$

where $x(t)\in R^n$ and $u_a(t)\in R^m$, r is the number of model rules, $z(t)=[z_1(t)\cdots z_p(t)]^T$ is the premise variable vector that is a function of states, and $\theta_i(z(t))$ is the normalized weight for each rule, that is $\theta_i(z(t))\geq 0$ and $\sum_{i=1}^{r}\theta_i(z(t))=1$.

For the fuzzy system (1), an existing analog fuzzy controller takes the following form:

$$u_a(t)=\sum_{i=1}^{r}\theta_i(z(t))K_ix_a(t) \tag{2}$$

where the subscript 'a'means the analog control. By substituting (2) into (1), we obtain

$$\dot{x}_a(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(t))\theta_j(z(t))(A_i+B_iK_j)x_a(t) \tag{3}$$

It follows from (3) that

$$\begin{aligned}x_a(t)=&\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(t_0))\theta_j(z(t_0))\mathrm{e}^{(A_i+B_iK_j)(t-t_0)}x_a(t_0)\\&+\Theta_1(x_a(\mu),x_a(t_0))\end{aligned} \tag{4}$$

where

$$\begin{aligned}&\Theta_1(x_a(\mu),x_a(t_0))\\&=\int_{t_0}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(\mu))\theta_j(z(\mu))(A_i+B_iK_j)x_a(\mu)d\mu\\&-\int_{t_0}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(t_0))\theta_j(z(t_0))(A_i+B_iK_j)\mathrm{e}^{(A_i+B_iK_j)(t-\mu)}x_a(t_0)d\mu\end{aligned}$$

We consider a multirate digital fuzzy controller where the digital control input $u_d(t)$ is held constant N times between the sampling points. Let T and τ be the sampling time and the control update time, respectively. The relation between T and τ can be defined as $\tau=T/N$. Then, the digital fuzzy controller is implemented by

$$u_d(t)=\sum_{i=1}^{r}\theta_i(z(kT+\kappa\tau))F_ix_d(kT+\kappa\tau) \tag{5}$$

for the time interval $[kT+\kappa\tau,kT+\kappa\tau+\tau)$, $k\times\kappa\in Z_0\times Z_{[0,N-1]}$, where the subscript '$d$'denotes the digital control.

Remark 1. *Within a sampling time T, the single-rate controller is static, while the multirate controller is periodically time-varying, i.e., the control action is updated at a small period τ. Clearly, for the single-rate case, this control update period τ is equal to the sampling period T. Specifically, setting $N=1$ in (5) leads to the following single-rate fuzzy controller:*

$$u_d(t)=\sum_{i=1}^{r}\theta_i(z(kT))F_ix_d(kT) \tag{6}$$

By interfacing an ideal sampler and a zero-order holder between (1) and (5), the closed-loop system is represented as

$$\begin{aligned}\dot{x}_d(t)=&\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(t))\theta_j(z(kT+\kappa\tau))\\&\times(A_ix_d(t)+B_iF_jx_d(kT+\kappa\tau))\end{aligned} \tag{7}$$

for the time interval $[kT+\kappa\tau,kT+\kappa\tau+\tau)$, $k\times\kappa\in Z_0\times Z_{[0,N-1]}$.

It follows from (7) that

$$\begin{aligned}x_d(kT+\kappa\tau+\tau)=&\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau))\\&\times(\Phi_i+\Gamma_iF_j)x_d(kT+\kappa\tau)\\&+\Theta_2(x_d(\mu),x_d(kT+\kappa\tau))\end{aligned} \tag{8}$$

where $\Phi_i=\mathrm{e}^{A_i\tau}$, $\Gamma_i=(\Phi_i-I)A_i^{-1}B_i$, and

$$\begin{aligned}&\Theta_2(x_d(\mu),x_d(kT+\kappa\tau))\\&=\int_{kT+\kappa\tau}^{kT+\kappa\tau+\tau}\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(\mu))\theta_j(z(kT+\kappa\tau))\\&\quad\times(A_ix_d(\mu)+B_iF_jx_d(kT+\kappa\tau))d\mu\\&-\int_{kT+\kappa\tau}^{kT+\kappa\tau+\tau}\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau))\\&\quad\times\left[A_i\mathrm{e}^{A_i(kT+\kappa\tau+\tau-\mu)}+\mathrm{e}^{A_i(kT+\kappa\tau+\tau-\mu)}B_iF_j\right]x_d(kT+\kappa\tau)d\mu\end{aligned}$$

Now letting $t_0=kT+\kappa\tau$ and $t=kT+\kappa\tau+\tau$ in (4), we have

$$\begin{aligned}x_a(kT+\kappa\tau+\tau)=&\sum_{i=1}\sum_{j=1}\theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau))\\&\times\Xi_{ij}x_a(kT+\kappa\tau)\\&+\Theta_1(x_a(\mu),x_a(kT+\kappa\tau))\end{aligned} \tag{9}$$

where $\Xi_{ij}(t-t_0)=\mathrm{e}^{(A_i+B_iK_j)\tau}$.

From (8) and (9), the IDR problem is to find the digital gains F_i under the assumption that $x_c(kT+\kappa\tau)=x_d(kT+\kappa\tau)$ such that

$$\Xi_{ij}=\Phi_i+\Gamma_iF_j \tag{10}$$

and

$$\Theta_1(x_a(\mu),x_a(kT+\kappa\tau))=\Theta_2(x_d(\mu),x_d(kT+\kappa\tau)) \tag{11}$$

are satisfied. However, it may be impossible to solve to (11) because the condition (11) is highly complex nonlinear matrix equality. The following assumption is introduced for ease of control synthesis.

Assumption 1. Assume that $\Theta_1(x_a(\mu), x_a(kT+\kappa\tau))=0$ and $\Theta_2(x_d(\mu), x_d(kT+\kappa\tau))=0$ for sufficiently small sampling period. Then the equations (8) and (9) can be simplified as

$$x_d(kT+\kappa\tau+\tau)=\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau)) \times(\Phi_i+\Gamma_i F_j)x_d(kT+\kappa\tau) \quad (12)$$

and

$$x_a(kT+\kappa\tau+\tau)=\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau)) \times\Xi_{ij}x_a(kT+\kappa\tau) \quad (13)$$

respectively.

Remark 2. *It is found in [1,2,3,4,5] that the discrete-time models of (3) and (7) have been described by (13) and (12), respectively.*

Then, in principle, the digital gain F_i can be determined from (10).

Remark 3. *It is noted that the previous results [1,2,3,4,5] have been interested in the single-rate control problem, which refers only to special case, $N=1$ in the multirate control problem. Meanwhile, in several results in the linear control system, it is shown that the multirate control scheme is more realistic approach, which allows us to consider the intersampling points between sampling points.*

3. New IDR method via the compensated Bilinear Transformation

To relax the ESM condition (10), we first obtain a new discretized version of the analog control system (13) by applying the bilinear transform, and then matches the resulting analog system and the digital system in the discrete-time domain.

Proposition 1. IDR based on the block-pulse method The responses of the digital fuzzy system (8) and the analog one (9) will closely match at $t=kT+\kappa\tau+\tau$ for an arbitrary initial state $x_a(kT+\kappa\tau)=x_d(kT+\kappa\tau)$ if there exist the redesigned digital feedback gains F_i such that

$$\frac{1}{2}(I-\frac{1}{2}K_j\Gamma_i)^{-1}K_j(\Phi_i+I)=F_j, \quad (i,j)\in I_R\times I_R \quad (14)$$

Then, the overall digital control system is redesigned as

$$\dot{x}_d(t)=\sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(t))\theta_j(z(kT)) \times\left[\Phi_i x_d(t)+\Gamma_i\frac{1}{2}(I-\frac{1}{2}K_j\Gamma_i)^{-1}K_j(\Phi_i+I)x_d(kT)\right]. \quad (15)$$

Remark 4. *Note that Proposition 1 is more relaxed condition than (10) in the general case, $m<rn$. Equation (10) consists of $(rn)^2$ scalar equations with rmn unknown elements in $F_i, i\in I_R$, while (14) is composed of r^2mn scalar equations with rmn unknown elements in $F_i, i\in I_R$. However, we still do not obtain a solution to (14) in an analytic way , except in the case of common H, i.e., $\Gamma_1=\Gamma_2=\cdots=\Gamma_r$. Also, the error involved in the bilinear transform causes a performance decline of the state-matching, especially in the slow sampling frequency.*

To avoid the difficulties on the Remark 4, we proposed a new compensated block-pulse function method, and then we reformulate the IDR problem as the minimization problem (MP). The following theorem is the main results of this paper

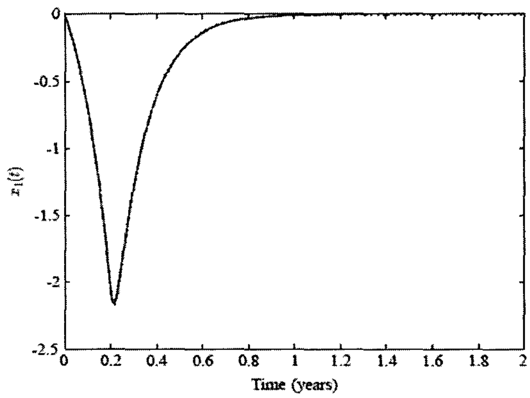

Fig. 1. Comparison of time responses x_1 of the controlled fuzzy system: analog (solid), proposed with N=5 (dashed) , proposed with N=2 (dash-dot), proposed with N=1 (dotted).

Theorem 1. *System (1) is stabilizable by the digital feedback gains F_i and the norm conditions of realizing the conditions (14) of the corresponding closed-loop system is smaller than a given γ^2 if there exist a matrix $Q=Q^T\succ 0$, and matrices $X_{ij}=X_{ij}^T=X_{ji}=X_{ji}^T$, E_{ij}, S_j such that the following two MPs:*

$MP1$: Minimize$_{E_{ij},F_{Ai}}$ γ_1 subject to

$$\begin{bmatrix} -\gamma_1 I & (\bullet)^T \\ \Xi_{ij}-\Phi_i-\Gamma_i 0.5(I-0.5K_j\Gamma_i)^{-1}K_j(\Phi_i+E_{ij}) & -\gamma_1 I \end{bmatrix}\prec 0,$$

$$\begin{bmatrix} -\gamma_1 I & (\bullet)^T \\ \Gamma_i 0.5(1-0.5K_j\Gamma_i)^{-1}K_j(\Phi_i+E_{ij})-\Gamma_i F_{Aj} & -\gamma_1 I \end{bmatrix} \prec 0,$$

$MP\,2$: $Minimize_{Q,S_i,X_{ij}}$ γ_2 *subject to*

$$\begin{bmatrix} -\gamma_2 Q & (\bullet)^T \\ S_i & -\gamma_2 I \end{bmatrix} \prec 0$$

$$\begin{bmatrix} -I & (\bullet)^T \\ x(0) & -Q \end{bmatrix} \prec 0,$$

$$\begin{bmatrix} -Q+X_{ij} & (\bullet)^T \\ \dfrac{\hat{G}_{ij}Q+\Gamma_i S_j+\hat{G}_{ji}Q+\Gamma_j S_i}{2} & -Q \end{bmatrix} \prec 0$$

$$[X_{ij}]_{r\times r} \prec 0, \quad (i,j)\in I_J \times I_R$$

are feasible, where $Q=P^{-1}$, $S_i=F_{Bi}Q$, and $F_i=F_{Ai}+F_{Bi}$.

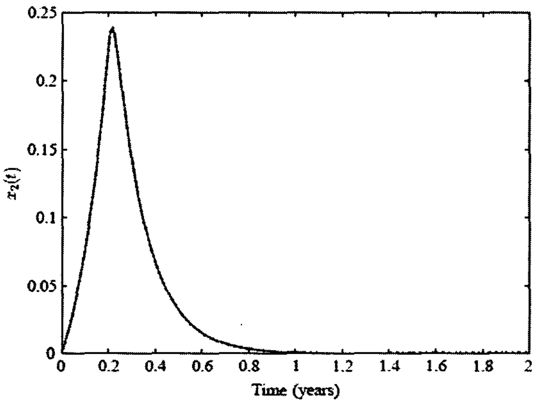

Fig. 2. Comparison of time responses x_2 of the controlled fuzzy system: analog (solid), proposed with N=5 (dashed) , proposed with N=2 (dash-dot), proposed with N=1 (dotted).

4. Computer Simulations

We present in this section a numerical application in order to show the applicability and the effectiveness of our approach.

Consider the following numerical model.

R_1 : IF $x_3(t)$ is about Γ_{11}, THEN $\dot{x}(t) = A_1x(t) + B_1u(t)$

R_2 : IF $x_3(t)$ is about Γ_{21}, THEN $\dot{x}(t) = A_2x(t) + B_2u(t)$

where

$$\left[\begin{array}{c|c} A_1 & B_1 \\ \hline A_2 & B_2 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} -a_1-a_2x_{3min} & 0 & -a_2b_1 & 0 \\ 0 & -a_3+a_4x_{3min} & a_4b_2 & 0 \\ a_5x_{3min} & -a_6x_{3min} & a_5b_1-a_6b_2 & 1 \\ \hline -a_1-a_2x_{3max} & 0 & -a_2b_1 & 0 \\ 0 & -a_3+a_4x_{3max} & a_4b_2 & 0 \\ a_5x_{3max} & -a_6x_{3max} & a_5b_1-a_6b_2 & 1 \end{array}\right],$$

$a_1=0.25$, $a_2=50$, $a_3=0.25$, $a_4=10.0$, $a_5=0.01$, $a_6=0.006$, $b_1=1000$, and $b_2=550$, Here, we can reasonably determine $[x_{3\min}, x_{3\max}]$ as [-0.006,0.006].

We design a stabilizing analog fuzzy controller as follows:

$$K_1^c = \begin{bmatrix} 0.0045 & -0.0004 & -27.5484 \end{bmatrix}$$

$$K_2^c = \begin{bmatrix} 0.0010 & -0.0001 & -20.7420 \end{bmatrix}$$

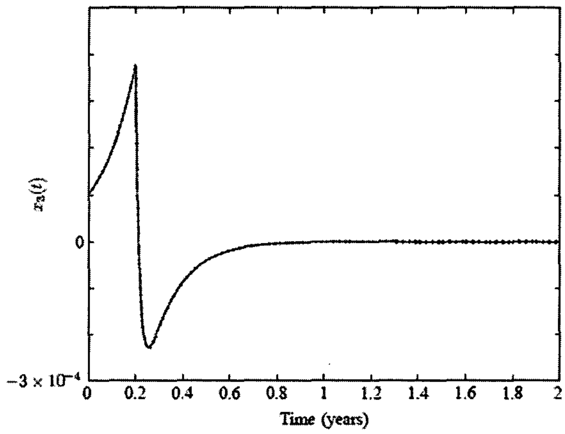

Fig. 3. Comparison of time responses x_3 of the controlled fuzzy system: analog (solid), proposed with N=5 (dashed) , proposed with N=2 (dash-dot), proposed with N=1 (dotted).

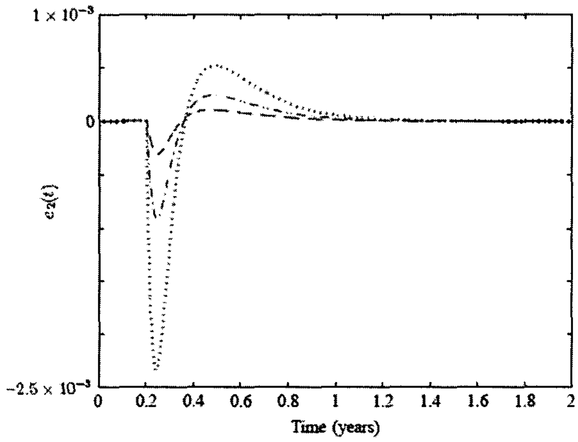

Fig. 4. Comparison of time responses e_2 of the controlled fuzzy system: analog (solid), proposed with N=5 (dashed) , proposed with N=2 (dash-dot), proposed with N=1 (dotted).

We wish to digitally redesign the analog controller for $T=0.01$ as increasing $N=1, N=2$, and $N=5$. From Theorem 1, IDR problem is feasible for three cases:

$$\left[\frac{K_{1d}}{K_{2d}}\right]=\left[\frac{0.0031 \quad -0.0003 \quad -20.2755}{0.0007 \quad -0.0001 \quad -15.8075}\right]$$

for $T=0.01$ and $N=1$.

$$\left[\frac{K_{1d}}{K_{2d}}\right]=\left[\frac{0.0037 \quad -0.0003 \quad -23.3984}{0.0008 \quad -0.0001 \quad -17.9767}\right]$$

for $T=0.01$ and $N=2$.

$$\left[\frac{K_{1d}}{K_{2d}}\right]=\left[\frac{0.0041 \quad -0.0004 \quad -25.7394}{0.0009 \quad -0.0001 \quad -19.5405}\right]$$

for $T=0.01$ and $N=5$.

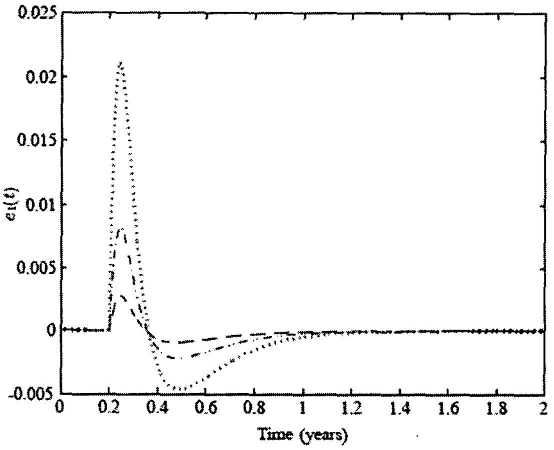

Fig. 5. Comparison of time responses e_1 of the controlled fuzzy system: analog (solid), proposed with N=5 (dashed) , proposed with N=2 (dash-dot), proposed with N=1 (dotted).

Figure 1, 2, and 3. show the state responses of the simulation, and Fig. 4, 5, and 6. show the error responses of the simulation, Control input is activated at t=0.2. Before the control input is activated, the trajectories do not converge to their equilibrium points. After the control input is activated, all trajectories are guided to the equilibrium at the origin. Furthermore, as shown in the figure, the state-matching error effectively decreases as N increases. As one can immediately witnesses, the state trajectory by our approach is almost identical to that of the original analog system.

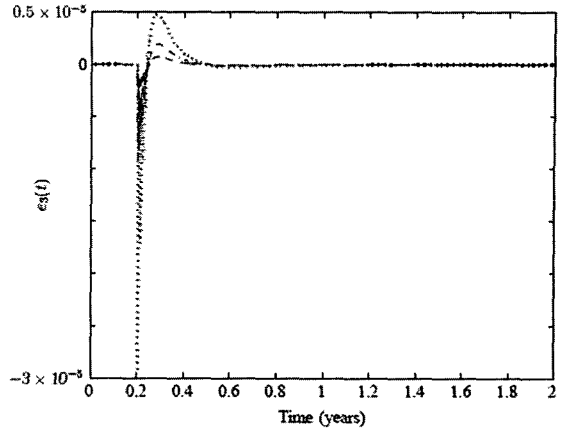

Fig. 6. Comparison of time responses e_3 of the controlled fuzzy system: analog (solid), proposed with N=5 (dashed) , proposed with N=2 (dash-dot), proposed with N=1 (dotted).

5. Conclusions

This paper proposed the multirate control design using the LMI approach for the fuzzy system. Some sufficient conditions were derived for stabilization and state matching of the discretized model by the compensated bilinear transformation. The proposed multirate control scheme can improve the state-matching performance in the long sampling limit.

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저 자 소 개

김도완(Do Wan Kim)

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주영훈(Young Hoon Joo)

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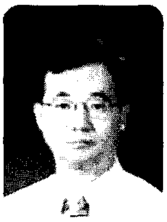

박진배(Jin Bae Park)

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