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INTUITIONISTIC FUZZY REES CONGRUENCES ON A SEMIGROUP

Young Sin Ahn[1], Kul Hur[2] and Jang Hyun Ryou[3]

[1] **Department of Computer Information, Dong Kang College**
[2] **Division of Mathematics and Informational Statistic, Wonkwang University**
[3] ***Bong Young Girls' Middle School***

Abstract

We introduce two concepts of intuitionistic fuzzy Rees congruence on a semigroup and intuitionistic fuzzy Rees congruence semigroup. As an important result, we prove that for a intuitionistic fuzzy Rees congruence semigroup S, the set of all intuitionistic fuzzy ideals of S and the set of all intuitionistic fuzzy congruences on S are lattice isomorphic. Moreover, we show that a homomorphic image of an intuitionistic fuzzy Rees congruence semigroup is an intuitionistic fuzzy Rees congruence semigroup.

Key Words : Intuitionistic fuzzy ideal, intuitionistic fuzzy congruence, intuitionistic fuzzy Rees congruence, intuitionistic fuzzy congruence semigroup.

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0. Introduction

In 1965, Zadeh [28] introduced the concept of fuzzy sets as the generalization of ordinary subsets. After that time, several researchers [22,24-27] have applied the notion of fuzzy sets to congruence theory. In particular, Xie [27] introduced the concept of fuzzy Rees congruences on a semigroup and studied some of its properties.

In 1986, Abanassov[1] introduced the concept of intuitionistic fuzzy sets as the generalization of fuzzy sets. Since then, many researchers [2,4-9,11-17] applied the notion of intuitionistic fuzzy sets to relation, algebra, topology and topological group. In particular, Hur and his colleagues [18-21] investigated intuitionistic fuzzy equivalence relations and various intuitionistic fuzzy congruences.

In this paper, we introduce two concepts of intuitionistic fuzzy Rees congruence on a semigroup and intuitionistic fuzzy Rees congruence semigroup. As an important result, we prove that for a intuitionistic fuzzy Rees congruence semigroup S, the set of all intuitionistic fuzzy ideals of S and the set of all intuitionistic fuzzy congruences on S are lattice isomorphic. Moreover, we show that a homomorphic image of an intuitionistic fuzzy Rees congruence semigroup is an intuitionistic fuzzy Rees congruence semigroup.

1. Preliminaries

In this section, we list some basic concepts one result which are needed in the later sections.

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For sets X, Y and $Z, f = (f_1, f_2) : X \to Y \times Z$ is called a *complex mapping* if $f_1 : X \to Y$ and $f_2 : X \to Z$ are mappings.

Throughout this paper, we will denote the unit interval $[0, 1]$ as I. And for a general background of lattice theory, we refer to [3].

Definition 1.1[1,6]. Let X be a nonempty set. A complex mapping $A = (\mu_A,\ \nu_A) : X \to I \times I$ is called an *intuitionistic fuzzy set* (in short, *IFS*) in X if $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$, where the mapping $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each $x \in X$ to A, respectively. In particular, $0_\sim$ and $1_\sim$ denote the *intuitionistic fuzzy empty set* and the *intuitionistic fuzzy whole set* in X defined by $0_\sim(x) = (0, 1)$ and $1_\sim(x) = (1, 0)$ for each $x \in X$, respectively.

We will denote the set of all IFSs in X as IFS(X).

Definitions 1.2[6]. Let X be a nonempty set and let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be IFSs on X. Then

(1) $A \subset B$ iff $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.
(2) $A = B$ iff $A \subset B$ and $B \subset A$.
(3) $A^c = (\nu_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$.
(6) $[\ ]A = (\mu_A, 1 - \mu_A)$, $< > A = (1 - \nu_A, \nu_A)$.

Definition 1.3[6]. Let $\{A_i\}_{i \in J}$ be an arbitrary family of IFSs in X, where $A_i = (\mu_{A_i}, \nu_{A_i})$ for each $i \in J$. Then

(1) $\bigcap A_i = (\bigwedge \mu_{A_i}, \bigvee \nu_{A_i})$.
(2) $\bigcup A_i = (\bigvee \mu_{A_i}, \bigwedge \nu_{A_i})$.

Definition 1.4[5]. Let X be a set. Then a complex mapping $R = (\mu_R, \ \ \nu_R) : X \times X \to I \times I$ is called an *intuitionistic fuzzy relation* (in short, *IFR*) on X if $\mu_R(x,y) + \nu_R(x,y) \leq 1$ for each $(x,y) \in X \times X$, i.e., $R \in$ IFS $(X \times X)$.

We will denote the set of all IFRs on a set X as IFR(X).

Definition 1.5[8] Let X be a set and let $R, Q \in \text{IFR}(X)$. Then the *composition* of R and Q, $Q \circ R$, is defined as follows : for any $x, y \in X$,

$$\mu_{Q\circ R}(x,y) = \bigvee_{z \in X} [\mu_R(x,z) \wedge \mu_Q(z,y)]$$

and

$$\nu_{Q\circ R}(x,y) = \bigwedge_{z \in X} [\nu_R(x,z) \vee \nu_Q(z,y)].$$

Definition 1.6. An Intutionistic fuzzy Relation R on a set X is called an *intutionsitic fuzzy equivalence relation* (in short, IFER) on X if it satisfies the following conditions :

(i) it is *intutionsitic fuzzy reflexive*, i.e.,$R(x,x) = (1,0)$ for each $x \in X$.

(ii) it is *intutionsitic fuzzy symmetric*, i.e.,$R(x,y) = R(y,x)$ for any $x, y \in X$.

(iii) it is *intutionsitic fuzzy transitive*, i.e., $R \circ R \subset R$.

We will denote the set of all IFERs on X as IFE(X).

Let R be an intuitionistic fuzzy equivalence relation on a set X and let $a \in X$. We define a complex mapping $Ra : X \to I \times I$ as follows : for each $x \in X$

$$Ra(x) = R(a,x).$$

Then clearly $Ra \in \text{IFS}(X)$. The intuitionistic fuzzy set Ra in X is called an *intuitionistic fuzzy equivalence class* of R containing $a \in X$. The set $\{Ra : a \in X\}$ is called the *intuitionistic fuzzy quotient set* of R by X as denoted by X/R.

Result 1.A[19, Theorem 2.15]. Let R be an intuitionistic fuzzy equivalence relation on a set X. Then the followings hold :

(1) $Ra = Rb$ if and only if $R(a,b) = (1,0)$ for any $a, b \in X$.

(2) $R(a,b) = (0,1)$ if and only if $Ra \cap Rb = 0_\sim$ for any $a, b \in X$.

(3) $\bigcup_{a \in X} Ra = 1_\sim$.

(4) There exists the surjection $p : X \to X/R$ defined by $p(x) = Rx$ for each $x \in X$.

Definition 1.7[19]. We define two IFRs on a set X, $\triangle$ and $\bigtriangledown$ as follows, respectively : for any $x, y \in X$,

$$\triangle(x,y) = \begin{cases} (1,0), & \text{if } x = y; \\ (0,1), & \text{if } x \neq y. \end{cases}$$

and

$$\bigtriangledown(x,y) = (1,0).$$

It is clear that $\triangle, \ \bigtriangledown \in \text{IFE}(X)$.

Let S be a semigroup and let A be a nonempty set. Then, A is called an *ideal* of S if AS, $SA \subset A$ (See [10]).

Definition 1.8[11]. Let $A \in \text{IFS}(S)$. Then A is called *an intuitionistic fuzzy ideal* (in short, IFI) of S if for any $x, y \in S$,

$$\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y) \ \ \textit{and} \ \ \nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y).$$

We will denote the set of all IFI$_s$ of S as IFI(S). Then, it is clear that $(\text{IFI}(S), \cap, \cup)$ is a distributive lattice having the greatest element 1_S and the least element $0_\sim$ or 0_S, where $1_S = 1_\sim$ and we use $0_\sim$ if S has no zero element and 0_S if S a zero element 0. In fact, $0_S(x) = (0,1)$ for each $0 \neq x \in S$. It is well-known (Proposition 2.6 in [12]) that if S has a zero element 0, then for each $A \in \text{IFI}(S)$ and each $x \in S$, $\mu_A(x) \leq \mu_A(0)$ and $\nu_A(x) \geq \nu_A(0)$. In this paper, we define $A(0) = (1,0)$ for each $A \in \text{IFI}(S)$.

2. Intuitionistic fuzzy Rees congruences

Definition 2.1[19]. Let X be a set, let $R \in \text{IFR}(X)$ and let $\{R_\alpha\}_{\alpha \in \Gamma}$ be the family of all the IFERs on X containing R. Then $\bigcap_{\alpha \in \Gamma} R_\alpha$ is called the IFER *generated* by R and denoted by R^e.

It is easily seen that R^e is the smallest intuitionistic fuzzy equivalence relation containing R.

Definition 2.2[19]. Let X be a set and let $R \in \text{IFR}(X)$. Then the *intutionsitic fuzzy transitive closure* of R, denoted by R^∞, is defined as follows :

$$R^\infty = \bigcup_{n \in \mathbb{N}} R^n, \quad \text{where} \quad R^n = R \circ R \circ \cdots \circ R(\text{n factors}).$$

Definition 2.3[20]. An IFR R on a groupoid S is said to be:

(1) *intuitionistic fuzzy left compatible* if $\mu_R(x,y) \leq \mu_R(zx, zy)$ and $\nu_R(x,y) \geq \nu_R(zx, zy)$, for any $x, y, z \in S$.

(2) *intuitionistic fuzzy right compatible* if $\mu_R(x,y) \leq \mu_R(xz, yz)$ and $\nu_R(x,y) \geq \nu_R(xz, yz)$, for any $x, y, z \in S$.

(3) *intuitionistic fuzzy compatible* if $\mu_R(x,y) \wedge \mu_R(z,t) \leq \mu_R(xz, yt)$ and $\nu_R(x,y) \vee \nu_R(z,t) \geq \nu_R(xz, yt)$, for any $x, y, z, t \in S$.

Definition 2.4[20]. An IFER R on a groupoid S is called an:

(1) *intuitionistic fuzzy left congruence* (in short, $IFLC$) if it is intuitionistic fuzzy left compatible.

(2) *intuitionistic fuzzy right congruence* (in short, $IFRC$) if it is intuitionistic fuzzy right compatible.

(3) *intuitionistic fuzzy congruence* (in short, IFC) if it is intuitionistic fuzzy compatible.

We will denote the set of all IFCs [resp. IFLCs and IFRCs] on a groupoid S as IFC(S) [resp. IFLC(S) and IFRC(S)]. Then it is clear that $\triangle, \bigtriangledown \in$ IFC(S).

Let R be an intuitionistic fuzzy congruence on a semigroup S and let $a \in S$. The intuitionistic fuzzy set Ra in S is called an *intuitionistic fuzzy congruence class of R containing $a \in S$* and we will denote the set of all intuitionistic fuzzy congruence classes of R as S/R.

Result 2.A[20, Theorem 2.22]. Let R be on intuitionistic fuzzy congruence on a semigroup S. We define the binary operation $*$ on S/R as follows : for any $a, b \in S$,

$$Ra * Rb = Rab.$$

Then $(S/R, *)$ is a semigroup.

For a semigroup S, it is clear that IFC(S) is a partially ordered set by the inclusion relation " $\subset$ ". Moreover, for any $P, Q \in$ IFC(S), $P \cap Q$ is the greatest lower bound of P and Q in (IFC(S),$\subset$) but $P \cup Q \notin$ IFC(S) in general(See Example 2.11 in [19]).

Result 2.B[21, Lemma 2.3]. Let S be a semigroup and let $P, Q \in$ IFC(S). We define $P \vee Q$ as follows: $P \vee Q = \widehat{P \cup Q}$, i.e., $P \vee Q = \bigcup_{n \in \mathbb{N}} (P \cup Q)^n$. Then $P \vee Q \in$ IFC(S).

Result 2.C[21, Proposition 2.5]. Let S be a semigroup. If $P, Q \in$ IFC(S), then $P \vee Q = (P \circ Q)^\infty$.

For a semigroup S, we define two binary operations $\vee$ and $\wedge$ on IFC(S) as follows : for any $P, Q \in$ IFC(S),

$$P \vee Q = \widehat{P \cup Q} \quad \text{and} \quad P \wedge Q = P \cap Q.$$

Result 2.D[21, Theorem 2.6]. Let S be a semigroup. Then (IFC(S),$\wedge, \vee$) is a complete lattice with $\triangle$ and $\bigtriangledown$ as the least and greatest elements of IFC(S).

Let A be an IFI of a semigroup S. Let us define a complex mapping $R_A = (\mu_{R_A}, \nu_{R_A}) : S \times S \longrightarrow I$ as follows: for ant $x, y \in S$,

$$\mu_{R_A}(x,y) = \begin{cases} \mu_A(x) \wedge \mu_A(y), & \text{if } x \neq y; \\ 1, & \text{if } x = y. \end{cases}$$

and

$$\nu_{R_A}(x,y) = \begin{cases} \nu_A(x) \vee \nu_A(y)), & \text{if } x \neq y; \\ 0, & \text{if } x = y. \end{cases}$$

Then clearly $R_A = (\mu_{R_A}, \nu_{R_A})$ is an intuitionistic fuzzy relation on S.

Proposition 2.5. Let A be an IFI of a semigroup S. Then R_A is an IFC on S. In this case, R_A is called the *intuitionistic fuzzy congruence iduced by A on S.*

Proof. By the definition of R_A, it is clear that R_A is intuitionistic fuzzy reflexive and intuitionistic fuzzy symmetric . Let $x, y \in S$. Then

$$\mu_{R_A \circ R_A}(x,y) = \bigvee_{z \in S} [\mu_{R_A}(x,z) \wedge \mu_{R_A}(z,y)]$$

and

$$\nu_{R_A \circ R_A}(x,y) = \bigwedge_{z \in S} [\nu_{R_A}(x,z) \vee \nu_{R_A}(z,y)].$$

Case(i): Suppose $x = y$. Then

$$\begin{aligned} \mu_{R_A \circ R_A}(x,x) &= \bigvee_{z \in S} [\mu_{R_A}(x,z) \wedge \mu_{R_A}(z,x)] \\ &= \bigvee_{z \in S} \mu_{R_A}(x,z) \\ &\text{(Since } R_A \text{ is intuitionistic fuzzysymmetric)} \\ &\geq \mu_{R_A}(x,x) = 1 \end{aligned}$$

and

$$\begin{aligned} \nu_{R_A \circ R_A}(x,x) &= \bigwedge_{z \in S} [\nu_{R_A}(x,z) \vee \nu_{R_A}(z,x)] \\ &= \bigwedge_{z \in S} \nu_{R_A}(x,z) \\ &\leq \nu_{R_A}(x,x) = 0. \end{aligned}$$

Thus $R_A \circ R_A(x,x) = (1,0) = R_A(x,x)$.

Case(ii) : Suppose $x \neq y$. Then

$$\begin{aligned} &\mu_{R_A \circ R_A}(x,y) \\ &= \bigvee_{z \in S - \{x,y\}} [\mu_{R_A}(x,z) \wedge \mu_{R_A}(z,y)] \\ &\quad \vee [\mu_{R_A}(x,x) \wedge \mu_{R_A}(x,y)] \vee [\mu_{R_A}(x,y) \wedge \mu_{R_A}(y,y)] \\ &= \mu_{R_A}(x,y) \vee \bigvee_{z \in S - \{x,y\}} [\mu_A(x) \wedge \mu_A(z) \wedge \mu_A(z) \wedge \mu_A(y)] \\ &\leq \mu_{R_A}(x,y) \vee \bigvee_{z \in S - \{x,y\}} [\mu_A(x) \wedge \mu_A(y)] \\ &= \mu_{R_A}(x,y) \vee \mu_{R_A}(x,y) = \mu_{R_A}(x,y) \end{aligned}$$

and

$$\begin{aligned}
&\nu_{R_A \circ R_A}(x,y)\\
&= \bigwedge_{z\in S-\{x,y\}} [\nu_{R_A}(x,z) \vee \nu_{R_A}(z,y)]\\
&\quad \wedge\ [\nu_{R_A}(x,x) \vee \nu_{R_A}(x,y)] \wedge [\nu_{R_A}(x,y) \vee \nu_{R_A}(y,y)]\\
&= \nu_{R_A}(x,y) \wedge \bigwedge_{z\in S-\{x,y\}} [\nu_{R_A}(x,z) \vee \nu_{R_A}(y,z)]\\
&= \nu_{R_A}(x,y) \wedge \bigwedge_{z\in S-\{x,y\}} [\nu_A(x) \vee \nu_A(z) \vee \nu_A(y) \vee \nu_A(z)]\\
&\geq \nu_{R_A}(x,y) \wedge \bigwedge_{z\in S-\{x,y\}} [\nu_A(x) \vee \nu_A(y)]\\
&= \nu_{R_A}(x,y) \wedge \nu_{R_A}(x,y) = \nu_{R_A}(x,y).
\end{aligned}$$

Thus $\mu_{R_A \circ R_A}(x,y) \leq \mu_{R_A}(x,y)$ and $\nu_{R_A \circ R_A}(x,y) \geq \nu_{R_A}(x,y)$. In either case, $R_A \circ R_A \subset R_A$. So $R_A \in$ IFE(S).

Now let $x, y, t \in S$.

Case (i) : Suppose $tx = ty$. Then

$$\mu_{R_A}(tx,ty) = 1 \geq \mu_{R_A}(x,y)$$

and

$$\nu_{R_A}(tx,ty) = 0 \leq \nu_{R_A}(x,y).$$

Case (ii) : Suppose $tx \neq ty$. Then $x \neq y$. Since $A \in$ IFI(S),

$$\mu_{R_A}(tx,ty) = \mu_A(tx) \wedge \mu_A(ty) \geq \mu_A(x) \wedge \mu_A(y)$$

and

$$\nu_{R_A}(tx,ty) = \nu_A(tx) \vee \nu_A(ty) \leq \nu_A(x) \vee \nu_A(y).$$

So R_A is intuitionistic fuzzy left compatible. In the same way, we can see that R_A is intuitionistic fuzzy right compatible. Hence $R_A \in$ IFI(S). This completes the proof. ■

Definition 2.6. Let S be a semigroup and let $0_\sim \neq A \in$ IFI(S). Then R_A is called an *intuitionistic fuzzy Rees congruence* (in short, IFRC) on S.

Let A be an IFI of a semigroup S and let

$$\overline{supp}A = \{x \in S : A(x) = (1,0)\}.$$

Then it is clear that $\overline{supp}A$ is an ideal of S.

Theorem 2.7. Let A be an IFI of a semigroup S. Let $\mathcal{A}$ be the set of all ideal of S containing $\overline{supp}A$ and let $\mathcal{B}$ be the set of all ideals of the quotient semigroup $(S/R_A, *)$. We define the mapping $f : \mathcal{A} \longrightarrow \mathcal{B}$ as follows : for each $J \in \mathcal{A}$,

$$f(J) = JR_A,$$

where $JR_A = \{bR_A : b \in J\}$. Then f is an inclusion - preserving bijection.

Proof. Let $J \in \mathcal{A}$. Let $K \in S/R_A$ and let $H \in JR_A$. Then there exist $a \in S$ and $b \in J$ such that $K = aR_A$ and $H = bR_A$. Thus $K * H = aR_A * bR_A = abR_A$ and $H * K = bR_A * aR_A = baR_A$. Since J is an ideal of S, $ab \in J$ and $ba \in J$. So $K * H \in JR_A$ and $H * K \in JR_A$. Hence $JR_A \in \mathcal{B}$.

Suppose $J_1 \neq J_2$ for any $J_1, J_2 \in \mathcal{A}$. Then there exists an $a \in S$ such that $a \in J_1 \setminus J_2$ or $a \in J_2 \setminus J_1$.

Case (i) : Suppose $a \in J_1 \setminus J_2$. Assume that $f(J_1) = f(J_2)$, i.e, $J_1R_A = J_2R_A$. Then there exists a $b \in J_2$ such that $aR_A = bR_A$. Thus, by Result 1.A, $R_A(a,b) = (1,0)$. Since $a \notin J_2$, $a \neq b$. Then $\mu_{aR_A}(b) = \mu_{R_A}(a,b) = \mu_A(a) \wedge \mu_A(b) = 1$ and $\nu_{aR_A}(b) = \nu_{R_A}(a,b) = \nu_A(a) \vee \nu_A(b) = 0$. Thus $\mu_A(a) = \mu_A(b) = 1$ and $\nu_A(a) = \nu_A(b) = 0$, i.e, $A(a) = A(b) = (1,0)$. So $a \in \overline{supp}A \subset J_2$ and thus $a \in J_2$. This contradicts the fact that $a \notin J_2$. Hence $f(J_1) \neq f(J_2)$.

Case (ii) : Suppose $a \in J_2 \setminus J_1$. By the similar arguments of Case (i), we also have $f(J_1) \neq f(J_2)$. Therefore f is injective.

Now let $X \in \mathcal{B}$. Then there exists a $K \subset S$ such that $X = KR_A$. Let $K_1 = \{x \in S : xR_A \in KR_A\}$ and let $z \in SK_1$. Then there exists $y \in S$ and $x \in K_1$ such that $z = yx$. Since $x \in K_1, xR_A \in KR_A$. Since KR_A is an ideal of $S \backslash R_A$, $zR_A = yxR_A = yR_A * xR_A \in KR_A$. Thus $z \in K_1$. So $SK_1 \subset K_1$. By the similar arguments, we have $K_1S \subset K_1$. Hence K_1 is an ideal of S.

Let $a \in \overline{supp}A$ and let $x \in K_1$.

Case(i) : Suppose $a = ax$. Since K_1 is an ideal of $S, a \in K_1$.

Case(ii) : Suppose $a \neq ax$. Let $z \in S$.

(1) If $z \neq a$ and $z \neq ax$, then

$$\begin{aligned}
\mu_{aR_A}(z) &= \mu_{R_A}(a,z) = \mu_A(a) \wedge \mu_A(z)\\
&= \mu_A(ax) \wedge \mu_A(z) \qquad \text{(Since } A(a) = (1,0))\\
&= \mu_{(ax)R_A}(z)
\end{aligned}$$

and

$$\begin{aligned}
\nu_{aR_A}(z) &= \nu_{R_A}(a,z) = \nu_A(a) \vee \nu_A(z)\\
&= \nu_A(ax) \vee \nu_A(z) = \nu_{(ax)R_A}(z).
\end{aligned}$$

(2) If $z = a$, then

$$\mu_{aR_A}(z) = \mu_{R_A}(a,z) = 1 = \mu_A(ax) \wedge \mu_A(z) = \mu_{(ax)R_A}(z)$$

and

$$\nu_{aR_A}(z) = \nu_{R_A}(a,z) = 0 = \nu_A(ax) \vee \nu_A(z) = \nu_{(ax)R_A}(z).$$

(3) If $z = ax$, then , by the similar arguments of (2), we have

$$\mu_{aR_A}(z) = \mu_{(ax)R_A}(z) \text{ and } \nu_{aR_A}(z) = \nu_{(ax)R_A}(z).$$

In all, $aR_A = (ax)R_A \subset KR_A$. By the definition of $K_1, a \in K_1$. Thus $K_1 \in \mathcal{A}$. It is clear that $K_1R_A = KR_A = X$. So f is surjective.

We can easily check that f is an inclusion preserving. This completes the proof. ■

Proposition 2.8. Let S be a semigroup with 0. We define the mapping g : IFI(S) $\rightarrow$ IFC(S) by $g(A) = R_A$

for each $A \in \text{IFI}(S)$. Then g is an order-preserving injection.

Proof. Suppose $A \neq B$ for any $A, B \in \text{IFI}(S)$. Then there exists an $x \in S$ such that $A(x) \neq B(x)$. Clearly $x \neq 0$ $A(0) = B(0) = (1,0)$. Thus

$$\mu_{R_A}(x,0) = \mu_A(x) \wedge \mu_A(0) = \mu_A(x),$$
$$\nu_{R_A}(x,0) = \nu_A(x) \vee \nu_A(0) = \nu_A(x)$$

and

$$\mu_{R_B}(x,0) = \mu_B(x) \wedge \mu_B(0) = \mu_B(x),$$
$$\nu_{R_B}(x,0) = \nu_B(x) \vee \nu_B(0) = \nu_B(x).$$

So $R_A \neq R_B$ and thus g is injective. It is easily seen that g is an order-preserving. This completes the proof.■

3. Intuitionistic fuzzy Rees congruence semigroups

Definition 3.1. A semigroup S is called an *intuitionistic fuzzy Rees congruence semigroup* (in short, *IFRC-semigroup*) if every IFC on S is an IFRC.

Proposition 3.2. Let S be an IFRC-semigroup. Then
(1) S has a zero element 0.
(2) If R is an IFC on S, then $R_A = R$, where $A(x) = R(x,0)$ for each $x \in S$.

Proof. (1) Clearly, $\triangle_S \in \text{IFC}(S)$. Since S is an IFRC-semigroup, $\triangle_S$ is an IFRC on S. Then there exists an $0_\sim \neq A \in \text{IFI}(S)$ such that $\triangle_S = R_A$. Since $A \neq 0_\sim$, there exists an $x \in S$ such that $\mu_A(x) > 0$ and $\nu_A(x) < 1$. Let $y \in S$ such that $y \neq x$. Then

$$\mu_{\triangle_S}(y,x) = \mu_{R_A}(y,x) = \mu_A(y) \wedge \mu_A(x) = 0$$

and

$$\nu_{\triangle_S}(y,x) = \nu_{R_A}(y,x) = \nu_A(y) \vee \nu_A(x) = 1.$$

Since $\mu_A(x) > 0$ and $\nu_A(x) < 1$, $\mu_A(y) = 0$ and $\nu_A(y) = 1$. Thus $A(y) = (0,1)$ for each $y \in S$ with $y \neq x$. Since A is an IFI of S, $\mu_A(zx) \geq \mu_A(x), \nu_A(zx) \leq \nu_A(x)$ and $\mu_A(xz) \geq \mu_A(x), \nu_A(xz) \leq \nu_A(x)$ for each $z \in S$. Thus $zx = xz = x$. Hence x is a zero element of S.

(2) Suppose R be an IFC on S. Since S is an IFRC-semigroup, there exists an $0_\sim \neq A \in \text{IFI}(S)$ such that $R = R_A$. By (1), S has a zero element, say 0. We define a complex mapping $B : S \to I \times I$ by $B(x) = R(x,0)$ for each $x \in S$. Then clearly $B \in \text{IFS}(S)$. Let $x, y \in S$. Then

$$\mu_B(yx) = \mu_R(yx,0) \geq \mu_R(x,0) = \mu_B(x),$$
$$\nu_B(yx) = \nu_R(yx,0) \leq \nu_R(x,0) = \nu_B(x),$$
$$\mu_B(yx) = \mu_R(yx,0) \geq \mu_R(x,0) = \mu_B(y),$$
$$\nu_B(yx) = \nu_R(yx,0) \leq \nu_R(x,0) = \nu_B(y)$$

and

$$B(0) = R(0,0) = (1,0).$$

So $B \in \text{IFI}(S)$. Now let $y \in S$ with $y \neq x$. Then

$$\mu_B(y) = \mu_R(y,0) = \mu_{R_A}(y,0) = \mu_A(y) \wedge \mu_A(0) = \mu_A(y)$$

and

$$\nu_B(y) = \nu_R(y,0) = \nu_{R_A}(y,0) = \nu_A(y) \vee \nu_A(0) = \nu_A(y).$$

Hence $B = A$. This completes the proof.■

Theorem 3.3. Let S be an IFRC-semigroup. Then $\text{IFI}(S)$ and $\text{IFC}(S)$ are isomorphic.

Proof. By Proposition 3.2(1), S has a zero element 0. Then, by Proposition 2.8, that exists an order-preserving injection $g : \text{IFI}(S) \to \text{IFC}(S)$ defined by $g(A) = R_A$ for each $A \in \text{IFI}(S)$. Moreover, by Proposition 3.2(2), g is surjective. Thus g is an order-preserving bijection.

Let $A, B \in \text{IFI}(S)$ and let $x, y \in S$ with $x \neq y$. Then

$$\begin{aligned}
\mu_{g(A\cap B)}(x,y) &= \mu_{R_{A\cap B}}(x,y) = \mu_{A\cap B}(x) \wedge \mu_{A\cap B}(y) \\
&= [\mu_A(x) \wedge \mu_B(x)] \wedge [\mu_A(y) \wedge \mu_B(y)] \\
&= [\mu_A(x) \wedge \mu_A(x)] \wedge [\mu_B(y) \wedge \mu_B(y)] \\
&= \mu_{R_A}(x,y) \wedge \mu_{R_A}(x,y) = \mu_{R_A \cap R_B}(x,y)
\end{aligned}$$

and

$$\begin{aligned}
\nu_{g(A\cap B)}(x,y) &= \nu_{R_{A\cap B}}(x,y) = \nu_{A\cap B}(x) \vee \nu_{A\cap B}(y) \\
&= [\nu_A(x) \vee \nu_B(x)] \vee [\nu_A(y) \vee \nu_B(y)] \\
&= [\nu_A(x) \vee \nu_A(x)] \vee [\nu_B(y) \vee \nu_B(y)] \\
&= \nu_{R_A}(x,y) \vee \nu_{R_A}(x,y) = \nu_{R_A \cap R_B}(x,y).
\end{aligned}$$

Moreover, $\mu_{g(A\cap B)}(x,x) = \mu_{R_{A\cap B}}(x,x) = 1 = \mu_{R_A \cap R_B}(x,x)$ and $\nu_{g(A\cap B)}(x,x) = \nu_{R_{A\cap B}}(x,x) = 0 = \nu_{R_A \cap R_B}(x,x)$. So $g(A \cap B) = g(A) \cap g(B)$.

Clearly, $A \subset A \vee B$ and $B \subset A \vee B$. Since g is an order-preserving, $g(A) \subset g(A \vee B)$ and $g(B) \subset g(A \vee B)$, i.e., $R_A \subset R_{A\vee B}$ and $R_B \subset R_{A\vee B}$. So $R_A \vee R_B \subset R_{A\vee B}$. Let $x, y \in S$ with $x \neq y$. Then

$$\begin{aligned}
\mu_{R_{A\vee B}}(x,y) &= \mu_{A\vee B}(x) \wedge \mu_{A\vee B}(y) \\
&= [\mu_A(x) \vee \mu_B(x)] \wedge [\mu_A(y) \vee \mu_B(y)] \\
&= [\mu_A(x) \wedge \mu_A(y)] \vee [\mu_A(x) \wedge \mu_B(y)] \\
&\quad \vee [\mu_A(y) \wedge \mu_B(x)] \vee [\mu_B(x) \wedge \mu_B(y)]
\end{aligned}$$

and

$$\begin{aligned}
\nu_{R_{A\vee B}}(x,y) &= \nu_{A\vee B}(x) \vee \nu_{A\vee B}(y) \\
&= [\nu_A(x) \wedge \nu_B(x)] \vee [\nu_A(y) \wedge \nu_B(y)] \\
&= [\nu_A(x) \vee \nu_A(y)] \wedge [\nu_A(x) \vee \nu_B(y)] \\
&\quad \wedge [\nu_A(y) \vee \nu_B(x)] \wedge [\nu_B(x) \vee \nu_B(y)].
\end{aligned}$$

On the other hand,

$$\begin{aligned}
&\mu_A(x) \wedge \mu_A(y) \\
&= \mu_{R_A}(x,y) \leq \mu_{R_A \circ R_B}(x,y) \\
&\leq \mu_{(R_A \circ R_B)^\infty}(x,y) = \mu_{R_A \vee R_B}(x,y) \quad \text{(By Result 2.C)} \quad (1)
\end{aligned}$$

and

$$\begin{aligned} \nu_A(x)\vee\nu_A(y) &= \nu_{R_A}(x,y)\geq\nu_{R_A\circ R_B}(x,y)\\ &\geq \nu_{(R_A\circ R_B)^\infty}(x,y)\\ &= \nu_{R_A\vee R_B}(x,y). \qquad (1)' \end{aligned}$$

Also,

$$\begin{aligned} \mu_B(x)\wedge\mu_B(y) &= \mu_{R_B}(x,y)\leq\mu_{R_A\circ R_B}(x,y)\\ &\leq \mu_{(R_A\circ R_B)^\infty}(x,y)\\ &= \mu_{R_A\vee R_B}(x,y)\\ &\qquad \text{(By Result 2.C)} \qquad (2) \end{aligned}$$

and

$$\begin{aligned} \nu_B(x)\vee\nu_B(y) &= \nu_{R_B}(x,y)\geq\nu_{R_A\circ R_B}(x,y)\\ &\geq \nu_{(R_A\circ R_B)^\infty}(x,y)\\ &= \nu_{R_A\vee R_B}(x,y). \qquad (2)' \end{aligned}$$

On the other hand,

$$\mu_A(x)\wedge\mu_B(y)\leq\mu_A(xy)\wedge\mu_B(xy)\wedge\mu_A(x)\wedge\mu_B(y) \quad (3)$$

and

$$\nu_A(x)\vee\nu_B(y)\geq\nu_A(xy)\vee\nu_B(xy)\vee\nu_A(x)\vee\nu_B(y). \quad (3)'$$

Also,

$$\mu_B(x)\wedge\mu_A(y)\leq\mu_A(xy)\wedge\mu_B(xy)\wedge\mu_B(x)\wedge\mu_A(y) \quad (4)$$

and

$$\nu_B(x)\vee\nu_A(y)\geq\nu_A(xy)\vee\nu_B(xy)\vee\nu_B(x)\vee\nu_A(y). \quad (4)'$$

In (3) and (3)′,

Case (i) : Suppose $xy=x$. Then

$$\begin{aligned} \mu_A(x)\wedge\mu_B(y) &\leq \mu_A(x)\wedge\mu_B(x)\wedge\mu_B(y)\\ &\leq \mu_B(x)\wedge\mu_B(y)\\ &\leq \mu_{R_A\vee R_B}(x,y) \qquad \text{(By (2))} \end{aligned}$$

and

$$\begin{aligned} \nu_A(x)\vee\nu_B(y) &\geq \nu_A(x)\vee\nu_B(x)\vee\nu_B(y)\\ &\geq \nu_B(x)\vee\nu_B(y)\\ &\geq \nu_{R_A\vee R_B}(x,y). \qquad \text{(By (2)}'\text{)} \end{aligned}$$

Case (ii) : Suppose $xy=y$. Then

$$\begin{aligned} \mu_A(x)\wedge\mu_B(y) &\leq \mu_A(y)\wedge\mu_B\wedge\mu_B(x)\\ &\leq \mu_A(x)\wedge\mu_A(y)\\ &\leq \mu_{R_A\vee R_B}(x,y) \qquad \text{(By (1))} \end{aligned}$$

and

$$\begin{aligned} \nu_A(x)\vee\nu_B(y) &\geq \nu_A(y)\vee\nu_B\vee\nu_B(x)\\ &\geq \nu_A(x)\vee\nu_A(y)\\ &\geq \nu_{R_A\vee R_B}(x,y). \qquad \text{(By (1)}'\text{)} \end{aligned}$$

Case (iii) : Suppose $xy\neq x$ and $xy\neq y$. Then

$$\begin{aligned} \mu_A(x)\wedge\mu_B(y) &\leq \mu_{R_A}(x,xy)\wedge\mu_{R_B}(xy,y)\\ &\leq \mu_{R_A\circ R_B}(x,y)\leq\mu_{(R_A\circ R_B)^\infty}(x,y)\\ &= \mu_{R_A\vee R_B}(x,y) \end{aligned}$$

and

$$\begin{aligned} \nu_A(x)\vee\nu_B(y) &\geq \nu_{R_A}(x,xy)\vee\nu_{R_B}(xy,y)\\ &\geq \nu_{R_A\circ R_B}(x,y)\\ &\geq \nu_{(R_A\circ R_B)^\infty}(x,y)=\nu_{R_A\vee R_B}(x,y). \end{aligned}$$

By the similar arguments, from (4) and (4)′, we obtain $\mu_A(x)\wedge\mu_B(y)\leq\mu_{R_A\vee R_B}(x,y)$ *and* $\nu_A(x)\vee\nu_B(y)\geq\nu_{R_A\vee R_B}(x,y)$.

In all, $\mu_{R_{A\vee B}}(x,y)\leq\mu_{R_A\vee R_B}(x,y)$ and $\nu_{R_{A\vee B}}(x,y)\geq\nu_{R_A\vee R_B}(x,y)$. So, by (*) and (**), $R_{A\vee B}\subset R_A\vee R_B$. Hence $R_{A\vee B}=R_A\vee R_B$, i.e., $g(A\vee B)=g(A)\vee g(B)$. Therefore g is lattice-order preserving, i.e., g is a lattice isomorphism. This completes the proof.■

Since IFI(S) is a distributive lattice, by Theorem 3.3, we have the following result.

Corollary 3.4. Let S be an IFRC-semigroup. Then IFC(S) is a distributive lattice.

Definition 3.5[6]. Let X and Y be nonempty sets and let $f:X\to Y$ be a mapping. Let $A=(\mu_A,\nu_A)$ be an IFS in X and $B=(\mu_B,\nu_B)$ be an IFS in Y. Then

(1) the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by:

$$f^{-1}(B)=(f^{-1}(\mu_B),f^{-1}(\nu_B)),$$

where $f^{-1}(\mu_B)=\mu_B\circ f$.

(2) the *image* of A under f, denoted by $f(A)$, is the IFS in Y defined by:

$$f(A)=(f(\mu_A),f(\nu_A)),$$

where for each $y\in Y$

$$\mu_{f(A)}(y)=f(\mu_A)(y)=\begin{cases}\bigvee_{x\in f^{-1}(y)}\mu_A(x), & \text{if } f^{-1}(y)\neq\emptyset\\ 0, & \text{if } f^{-1}(y)=\emptyset\end{cases}$$

and

$$\nu_{f(A)}(y)=f(\nu_A)(y)=\begin{cases}\bigwedge_{x\in f^{-1}(y)}\nu_A(x) & \text{if } f^{-1}(y)\neq\emptyset;\\ 1, & \text{if } f^{-1}(y)=\emptyset.\end{cases}$$

Definition 3.6[11]. Let A be an IFS in a set X. Then A is said to *have the sup property* if for each subset T of X, there exists a $t_0\in T$ such that $\mu_A(t_0)=\bigvee_{t\in T}\mu_A(t)$ and $\nu_A(t_0)=\bigwedge_{t\in T}\nu_A(t)$.

Result 3.A[11, Proposition 4.4]. Let $f:G\to G'$ be a groupoid homomorphism and let $A\in$ IFS(G) have sup property. If $A\in$ IFI(G), then $f(A)\in$ IFI(G').

By using the process of the proof of Proposition 2.19 in [17], we can easily show that the following result holds without the condition having the sup property.

Lemma 3.7. Let $f : S \to S'$ be a semigroup homomorphism and let $A \in \text{IFS}(S)$. If $A \in \text{IFI}(S)$, then $f(A) \in \text{IFI}(S')$.

Proposition 3.8. The homomorphic image of an IFRC-semigroup is an IFRC-semigroup.

Proof. Let $f : S \to T$ be a semigroup epimorphism and let S be an IFRC-semigroup. Let $H \in IFC(T)$. Define a complex mapping $R = (\mu_R, \nu_R) : S \times S \to I \times I$ by $R(x, y) = H(f(x), f(y))$ for any $x, y \in S$. Then clearly $R \in \text{IFR}(S)$. Since $H \in \text{IFR}(T)$, $\mu_R(x,y) + \nu_R(x,y) = \mu_H(f(x), f(y)) + \nu_H(f(x), f(y)) \leq 1$. Thus $R \in \text{IFR}(S)$. Moreover, R is intuitionistic fuzzy reflexive and intuitionistic fuzzy symmetric from the definition of R. Let $x, y \in S$. Then

$$\begin{aligned} \mu_{R\circ R}(x,y) &= \bigvee_{z\in S} [\mu_R(x,z) \wedge \mu_R(x,y)] \\ &= \bigvee_{z\in S} [\mu_H(f(x), f(z)) \wedge \mu_H(f(z), f(y))] \\ &\leq \bigvee_{z\in S} [\mu_H(f(x), z) \wedge \mu_H(z, f(y))] \\ &= \mu_{H\circ H}(f(x), f(y)) \leq \mu_H(f(x), f(y)) \\ &= \mu_R(x,y) \end{aligned}$$

and

$$\begin{aligned} \nu_{R\circ R}(x,y) &= \bigwedge_{z\in S} [\nu_R(x,z) \vee \nu_R(x,y)] \\ &= \bigwedge_{z\in S} [\nu_H(f(x), f(z)) \vee \nu_H(f(z), f(y))] \\ &\geq \bigwedge_{z\in S} [\nu_H(f(x), z) \vee \nu_H(z, f(y))] \\ &= \nu_{H\circ H}(f(x), f(y)) \geq \nu_H(f(x), f(y)) \\ &= \nu_R(x,y). \end{aligned}$$

Thus R is intuitionistic fuzzy transitive. So $R \in \text{IFE}(S)$.
Let $x, y, a, b \in S$. Then

$$\begin{aligned} \mu_R(xa, yb) &= \mu_H(f(xa), f(yb)) \\ &= \mu_H(f(x)f(a), f(y)f(b)) \\ &\geq \mu_H(f(x), f(y)) \wedge \mu_H(f(a), f(b)) \\ &= \mu_R(x,y) \wedge \mu_R(a,b) \end{aligned}$$

and

$$\begin{aligned} \nu_R(xa, yb) &= \nu_H(f(xa), f(yb)) \\ &= \nu_H(f(x)f(a), f(y)f(b)) \\ &\leq \nu_H(f(x), f(y)) \vee \nu_H(f(a), f(b)) \\ &= \nu_R(x,y) \vee \nu_R(a,b). \end{aligned}$$

Thus R is intuitionistic fuzzy compatible. So $R \in \text{IFC}(S)$. Since S is an IFRC-semigroup, there exists an $0_\sim \neq A \in \text{IFI}(S)$ such that $R = R_A$. By Lemma 3.7, $f(A) \in \text{IFI}(T)$.

We will show that $H = H_{f(A)}$. Let $x, y \in T$. Then
Case (i) : Suppose $x = y$. Then, clearly $H_{f(A)}(x,y) = (1, 0) = H(x,y)$. Case (ii) : Suppose $x \neq y$. Since f is surjective, there exist $a, b \in S$ such that $x = f(a)$ and $y = f(b)$. Thus

$$\begin{aligned} \mu_H(x,y) &= \mu_H(f(a), f(b)) = \mu_R(a,b) = \mu_{R_A}(a,b) \\ &= \mu_A(a) \wedge \mu_A(b) \\ &\leq (\bigvee_{z\in f^{-1}(x)} \mu_A(z)) \wedge (\bigvee_{z\in f^{-1}(y)} \mu_A(z)) \\ &= f(\mu_A)(x) \wedge f(\mu_A(y) = \mu_{f(A)}(x) \wedge \mu_{f(A)}(y) \\ &= \mu_{H_{f(A)}}(x,y) \end{aligned}$$

and

$$\begin{aligned} \nu_H(x,y) &= \nu_H(f(a), f(b)) = \nu_R(a,b) = \nu_{R_A}(a,b) \\ &= \nu_A(a) \vee \nu_A(b) \\ &\geq (\bigvee_{z\in f^{-1}(x)} \nu_A(z)) \vee (\bigvee_{z\in f^{-1}(y)} \nu_A(z)) \\ &= f(\nu_A)(x) \vee f(\nu_A(y) = \nu_{f(A)}(x) \vee \nu_{f(A)}(y) \\ &= \nu_{H_{f(A)}}(x,y). \end{aligned}$$

Thus $H \subset H_{f(A)}$. On the other hand,

$$\begin{aligned} \mu_{H_{f(A)}}(x,y) &= \mu_{f(A)}(x) \wedge \mu_{f(A)}(y) \\ &= f(\mu_A)(x) \wedge f(\mu_A)(y) \\ &= (\bigvee_{z\in f^{-1}(x)} \mu_A(z)) \wedge (\bigvee_{w\in f^{-1}(y)} \mu_A(w)) \\ &= \bigvee_{z\in f^{-1}(x), w\in f^{-1}(y)} [\mu_A(z) \wedge \mu_A(w)] \\ &= \bigvee_{z\in f^{-1}(x), w\in f^{-1}(y)} \mu_{R_A}(z,w) \\ &= \bigvee_{z\in f^{-1}(x), w\in f^{-1}(y)} \mu_R(z,w) \\ &= \bigvee_{z\in f^{-1}(x), w\in f^{-1}(y)} \mu_H(f(z), f(w)) \\ &\leq \mu_H(x,y) \end{aligned}$$

and

$$\begin{aligned} \nu_{H_{f(A)}}(x,y) &= \nu_{f(A)}(x) \vee \nu_{f(A)}(y) \\ &= f(\nu_A)(x) \vee f(\nu_A)(y) \\ &= (\bigwedge_{z\in f^{-1}(x)} \nu_A(z)) \vee (\bigwedge_{w\in f^{-1}(y)} \nu_A(w)) \\ &= \bigwedge_{z\in f^{-1}(x), w\in f^{-1}(y)} [\nu_A(z) \vee \nu_A(w)] \\ &= \bigwedge_{z\in f^{-1}(x), w\in f^{-1}(y)} \nu_{R_A}(z,w) \\ &= \bigwedge_{z\in f^{-1}(x), w\in f^{-1}(y)} \nu_R(z,w) \\ &= \bigwedge_{z\in f^{-1}(x), w\in f^{-1}(y)} \nu_H(f(z), f(w)) \\ &\geq \nu_H(x,y). \end{aligned}$$

Thus $H_{f(A)} \subset H$. Hence $H = H_{f(A)}$. This completes the proof.■

Definition 3.9. A semigroup S is said to be *intuitionistic fuzzy congruences free* if S has no intuitionistic fuzzy congruences other then ∇_S and $\triangle_S$.

Definition 3.10. A A semigroup S is said to be *intuitionistic fuzzy 0-simple* if $S^2 \neq \{0\}$, and 0_S and 1_S are the only intuitionistic fuzzy ideals.

Theorem 3.11. Let S be an IFRC-semigroup and $S^2 \neq \{0\}$. Then S is intuitionistic fuzzy congruences free if and only if S is intuitionistic fuzzy 0-simple.

Proof. $(\Rightarrow)$: Suppose S is intuitionistic fuzzy congruences free. Let $A(\neq 0_\sim)$ be any IFI of S. Then $R_A \in$ IFC(S). Thus, by Definition 3.9, $R_A = \nabla_S$ or $R_A = \triangle_S$.

Case(i) : Suppose $R_A = \nabla_S$. Let $0 \neq x \in S$. Then

$$\mu_{R_A}(0,x) = \mu_{\nabla_S}(0,x) = 1 = \mu_A(0) \wedge \mu_A(x) = \mu_A(x)$$

and

$$\nu_{R_A}(0,x) = \nu_{\nabla_S}(0,x) = 0 = \nu_A(0) \vee \nu_A(x) = \nu_A(x).$$

So, $A = 1_S$.

Case(ii) : Suppose $R_A = \triangle_S$. Let $0 \neq x \in S$. Then

$$\mu_{R_A}(0,x) = \mu_A(x) = \mu_{\triangle_S}(0,x) = 0$$

and

$$\nu_{R_A}(0,x) = \nu_A(x) = \nu_{\triangle_S}(0,x) = 1.$$

So, $A = 0_S$. Hence, in all, S is intuitionistic fuzzy 0-simple.

$(\Leftarrow)$: Suppose S is intuitionistic fuzzy 0-simple and let $R \in$ IFC(S). Then, by Theorem 3.3, there exists an $0_\sim \neq A \in$ IFI(S) such that $R = R_A$. Since S is intuitionistic fuzzy 0-simple, either $A = 0_S$ or $A = 1_S$.

Case(i) : Suppose $A = 1_S$. Let $x \neq y \in S$. Then

$$\mu_R(x,y) = \mu_{R_A}(x,y) = \mu_A(x) \wedge \mu_A(y) = 1$$

and

$$\nu_R(x,y) = \nu_{R_A}(x,y) = \nu_A(x) \vee \nu_A(y) = 0.$$

So, $R = \nabla_S$.

Case(ii) : Suppose $A = 0_S$. By a routine verification, we have $R = \triangle_S$. Hence, in all, S is intuitionistic fuzzy congruences free. This completes the proof.■

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저 자 소 개

Kul Hur
제 13권 5호(2003년 10월호) 참조

Jang Hyun Ryou
제 13권 5호(2003년 10월호) 참조

Young Sin Ahn
graduated from Chosun University in 1983, and received Ph. D. at Wonkwang University in 1997. Since 1988, she has been at Dongkang College, Her research interests focus on fuzzy topology, algebraic structure and mathematical education.
Dept. of Computer Information Science, Dongkang College, Kwangju, Korea 500-714
e-mail : youngsin@dongkang.ac.kr