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Designing a Supply Chain Coordinating Returns Policies for a Risk Sensitive Manufacturer

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ABSTRACT

In this article we consider a supply chain consisting of a risk-sensitive manufacturer and a risk-neutral retailer. The manufacturer maximizes her individual expected profit by designing a supply chain coordinating returns contract (SCRC) that consists of (i) a channel coordinating returns policy that maximizes the supply chain joint expected profit, and (ii) a profit sharing arrangement that gives the retailer an expected profit only slightly higher than that in the no returns case so that it is just enough to induce the retailer to accept the SCRC. Thus, the manufacturer captures as high a percentage as possible of the jointly maximum supply chain profit. However, this contract can sometimes lead to the manufacturer's resulting realized profit being lower than that in the no returns case when demand is lower than expected. In this context, even though profit is sufficiently attractive on average, will the risk-sensitive manufacturer ever consider applying a SCRC? Our research raises this question and focuses on designing a SCRC that can significantly increase the probability of the manufacturer's resulting realized profit being at least higher than that in the no returns case.

Keywords: Supply Chain Coordination, Newsboy Inventory Control Model, Returns Policy, Risk-Sensitive Agent.

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1. INTRODUCTION

Consider a supply chain that consists of one manufacturer and one retailer operating to meet random demand for a "short life cycle goods" (e.g., personal computers, consumer electronics, fashion items). The retailer initially places a one-time order to the manufacturer. The manufacturer, upon receiving the order, produces and delivers it to the retailer prior to the selling season. The retailer then sells the product to consumers for an exogenously determined price. After the selling season, consumer demand is disclosed, and it is obvious that the retailer, whose products are subject to uncertain demand, frequently faces the risk of overstocking. This then leads to the retailer's aversion to overstock scenarios, depressing the quantity that the manufacturer can sell. Knowing this, the manufacturer designs a schedule of returns to reduce the retailer's risk so as to encourage a larger order quantity. Our study is conducted under this basic scenario, which is more or less similar to that in most previous research.

The pioneering work of Pasternack [9] showed that a manufacturer's returns policy can coordinate a supply chain so as to generate the greatest supply chain joint expected profit. The author revealed that when a supply chain is coordinated by a returns policy, the manufacturer's (retailer's) expected profit function is an increasing (decreasing) function of the returns rebate. Thus, the manufacturer could push the returns rebate as high as possible so as to increase her individual profit share at the expense of the retailer's profit share (see also Lariviere [4]). This insightful result motivates us to consider a supply contract in which a manufacturer could maximize her individual expected profit by designing a supply chain coordinating returns contract (SCRC) that consists of

(i) *a coordinating returns policy that maximizes the supply chain joint expected profit*
(ii) *a profit sharing arrangement that gives the retailer (via a manipulation of the returns contract terms) an expected profit only slightly higher than that in the no returns case so that it is just enough to induce the retailer to accept the SCRC*

In this way, the manufacturer captures as high a percentage as possible of the jointly maximized supply chain profit, and earns an expected profit higher than that earned from any other contracts that are acceptable to the retailer (retailer's expected profit is at least as great as in the no returns case). A numerical study provided in section 2 shows that, when a return is not permitted, the independent manufacturer will most likely set the wholesale price $C = \$75$, since this price range maximizes the manufacturer's individual expected profit (see Table 3). Here, if the manufacturer applies the SCRC she can increase the individual expected profit from \$625 to \$937 (a 49.9% increase).

The Pasterneck's model has since been expanded in many directions. These include extending the model to consider price-sensitive demands (Emmons and Gilbert [1], Marvel and Peck [8], Kandel [2], and Lee [7]) and comparing returns policy with other coordinating mechanisms (markdown allowance in Tsay [12] and two-parts tariff price only contract in Lariviere [4]), among many others. However, nearly all of these works, including the work of Pasternack [9], are based on the assumption that the manufacturer and the retailer are both risk-neutral; hence, the agent's attitude toward risk is not explicitly considered. As noted by Tsay [13], "Any logic that fails to differentiate between certain and uncertain payoffs is fundamentally at odds with the notion of sensitivity to risk, and therefore may offer spurious recommendation." Some of the more recent works have acknowledged these problems and tried to encompass agents' individual agendas and attitudes toward risk into modeling efforts. For example, Lau and Lau [6] and Tsay [13] assumed a risk-sensitive manufacturer and retailer, and modeled an agent's risk-sensitive behavior by a mean standard deviation utility function in which attitude toward risk is explicitly formulated as a risk-aversion parameter. Webster and Weng [14], on the other hand, assumed a risk-neutral retailer and a risk-sensitive manufacturer who preferred to offer a returns policy if every resulting realized profit is no less than that in the no returns case. Let I denote an actualized leftovers quantity, and let K denote a *Maximum Allowable Returns Quantity* (MARQ) designed by a manufacturer to limit her responsibility for the leftovers ($\textit{returns quantity} = \min[I,K]$). Let Q denotes the retailer's order quantity, and let Q_{NO} and Q_{RE}, respectively, denote order quantities without and with a returns option. Denote event $\{Rf\}$:= *manufacturer obtains a profit at least as high as in the no returns case*. Mathematically, their statement can be represented by the following inequality, which specifies that the manufacturer's resulting returns cost must be no more than the additional profit generated by the returns policy:

$$(\textit{unit returns value}) \times \min(K,I) \le (\textit{manufacturer's unit sales profit}) \times (Q_{RE} - Q_{NO})\,. \tag{1}$$

Webster and Weng [14] referred to the situation in which probability of event $\{Rf\}$ occurring = 100% as a "*risk-free*" state. They designed a *quantity returns* *policy* in which an additive returns schedule $K = Q - q$ with $q \in [0, Q]$ is assumed to achieve the risk-free state. They showed that when the system is not coordinated, there always exists a "*risk-free*" quantity returns policy so that the probability of event $\{Rf\} = 100\%$. However, there may be no such risk-free policy that also achieves supply chain coordination. Let $\bar{K} := \max K$ satisfying $(returns\ value) \times K \leq (manufacturer's\ profit) \times (Q_{RE} - Q_{NO})$. Now, given that the system is coordinated, it may result in a retailer's expected profit being lower than that in the no returns situation even for a most beneficial returns contract in which MARQ $= \bar{K}$. Hence, no risk-neutral retailer will accept the returns contract. As a result, the manufacturer may need to grant a more lucrative returns policy with a higher $\tilde{K} > \bar{K}$. In this situation, the condition in (1) can only be met if a realized demand is higher than a specific demand level $\bar{y}$ where upon substitution $I(\bar{y}) = Q - \bar{y} = \bar{K} < \tilde{K}$; thus, the probability of event $\{Rf\}$ occurring $\leq 100\%$.

Now, let us consider again the SCRC. But this time let us assume that the manufacturer is a risk-sensitive agent, and consider the risk involved with the SCRC. In our particular numerical example ($C = \$75$), the requirements of supply chain coordination lead to the probability of event $\{Rf\}$ occurring $= \bar{F}(\bar{y})$ to be barely more than 62%. In other words, in almost 40% of the transactions the manufacturer will be made worse off by employing the SCRC. In this context, even though realization of profit is sufficiently attractive on average (an increase of 49.9%), will a risk-sensitive manufacturer ever consider applying the SCRC when she knows her chance of being worse off is so high? This suspicion led us to study the possibility of designing a SCRC that aims to achieve the following two goals simultaneously

i. Significantly improve the manufacturer's expected profit (via coordination and giving the retailer an expected profit only slightly higher than that in the no returns case), and

ii. Maintaining the probability of event $\{Rf\}$ occurring to be relatively high.

In this article, we extend the work of Webster and Weng [14], and develop a supply chain coordinating returns contract that aims to achieve these two goals simultaneously. We will refer to the contract as a *Multilevel Returns Policy* (MLR) contract. The main difference between *multilevel returns policy* and *quantity returns policy* in Webster and Weng [14] is that MLR is a supply chain coordinating contract that aims to significantly improve the manufacturer's expected profit.

The results of a numerical experiment summarized in Table 3 highlight performances of MLR. For example, at C=$80, the supply chain joint profit is $1,250. Here, by giving the retailer an expected profit slightly higher than that in the no returns case, the manufacturer earns an expected profit as high as $1,050, which is 75% higher than the $600 earned from a no returns policy. We also see that the probability of event $\{Rf\}$ occurring is 96.6%, which is only about 3.4% lower than being risk- free.

2. MODELS AND ANALYSIS

This article focuses on providing a solution to a risk-sensitive manufacturer who seeks to *significantly increase her expected profit with a small decrease in the risk-free probability.* Khouja [3] reported that two approaches were commonly used to extend the classical newsboy model to encompass the agent's attitude toward risk: one way is to use effective criterion, risk tolerance and utility functions as in Lau and Lau [6] and Tsay [13]; the other is to maximize the probability of achieving a target profit level, as in Sankarasubramanian and Kumarraswamy [11] and Lau [5]. Our model follows the second approach. Let Π be the manufacturer's realized profit obtained from applying the system coordinating returns policy, and let R be the profit under a no returns policy. Our purpose is to design a SCRC that aims to maximize *Prob* $\{\Pi \geq R \mid$ retailer's expected profit is at least as great as in the no returns case$\}$.

The problem will be modeled as a Newsboy model (see, for example, Porteus [10] for a review of the Newsboy model). As such, our parameters are the Newsvendor parameters augmented with price and returns credit per unit of product transferred between two agents:

Q = retailer's order quantity or the manufacturer's production quantity

$f(y), F(y)$ = probability density and cumulative function for the random demand Y

I = $\max[Q-y,0]$ actualized leftovers at the end of the selling period

r = returns credit paid by the manufacturer

P = retail price per unit collected from the market by the retailer.

C = unit wholesale price paid by the retailer to the manufacturer

M = unit manufacturing cost incurred by the manufacturer

Let us first consider the full returns policy (hereafter FR policy will refer to full returns policy) addressed in Paterneck [9]. The retailer's expected profit in FR is $\Pi_R(Q) = (P-C)Q - (P-r)\int_0^Q F(y)dy$, and the optimal order quantity that maximizes the objective function is given by $Q_{RE} = F^{-1}((P-C)/(P-r))$. The supply chain can be coordinated by the returns value:

$$r^* = (C-M)/F(Q_J)\text{, where } Q_{RE}(r^*) = Q_J = F^{-1}(1-M/P)\,. \qquad (2)$$

Substituting the coordinating returns value r^* into $Q_{RE}(r^*)$ leads to a supply chain joint optimizing order quantity Q_J. Now, substituting the coordinating returns value in (2) into (1), and the fact that $\min(K,I) = I$ (granting full returns) leads to the simultaneously risk-free and coordinating condition $(Q_J - y) \le F(Q_J)$ $(Q_J - Q_{NO})$, where $Q_{NO} = F^{-1}(1-C/P)$. Hereafter, let $\Phi := Q_J\bar{F}(Q_J) + F(Q_J)Q_{NO}$ This implies that with system coordination, event $\{Rf\}$ occurs only if actualized demands $y \ge \Phi$ (with probability $\bar{F}\{\Phi\}$) since $(Q_J - \Phi) = F(Q_J)(Q_J - Q_{NO})$ upon substitution.

Let us now direct our attention to the quantity returns policy (hereafter QR policy refers to quantity returns policy) proposed in Webster and Weng (2000). Let q be a minimum requirement for demands designed by a manufacturer so that the manufacturer will grant full returns if the actualized demands y > q, and grant MARQ= $Q-q$ if $y \le q$. As such, the returns schedule in QR policy is:

$$returns\ quantity = \begin{cases} I & if\ \ y > q \\ Q\text{-}q & if\ \ y \le q \end{cases}. \qquad (3)$$

With the manufacturer's returns schedule provided in (3), the retailer's expected profit is given by $\Pi_R(Q \mid q) = (P-C)Q - P\int_0^Q F(y)dy + r\int_q^Q F(y)dy$. The optimal order quantity maximizing the objective function satisfies the following expression:

$$\begin{cases} if \quad Q_{NO} \ge q & Q^* = Q_{RE} \\ if \quad Q_{NO} < q & \begin{bmatrix} Q^* = Q_{NO} & if \ \ \Pi_R(Q_{RE} \mid q) < \Pi_R(Q_{NO}) \\ Q^* = Q_{RE} & if \ \ \Pi_R(Q_{RE} \mid q) > \Pi_R(Q_{NO}) \end{bmatrix} \end{cases}. \qquad (4)$$

Here, as in FR, $Q_{NO} = F^{-1}(1 - C/P)$ and $Q_{RE}(r) = F^{-1}((P-C)/(P-r))$, respectively, denote the retailer's order quantities when returns are prohibited and permitted. Webster and Weng [14] suggested that the manufacturer can set $q = Q_{NO}$ in (4) so as to induce the retailer to order $Q^* = Q_{RE}$, and accepts the manufacturer's quantity returns policy. Here, if the manufacturer designs a $r \in (0, C-M]$, then upon substitution into (1) it reveals the policy to also be risk-free (probability of event $\{Rf\}$ occurring = 100%).

Now let us consider the problem of maximizing the probability of event $\{Rf\}$ given a coordinating returns policy. The system joint objective function in QR is $\Pi = (P-M)Q - P\int_0^Q F(y)dy$. The optimal order quantity optimizing the joint objective function is given by $Q_J = F^{-1}(1 - M/P)$; thus, the returns value $r^* = (C-M)/F(Q_J)$ in (2), which is higher than $C - M$, coordinates the supply chain, i.e., $Q_{RE}(r^*) = Q_J = F^{-1}(1 - M/P)$. Substituting the coordinating returns value in (2) into (1) leads to $(Q_J - \max[q, y]) \le F(Q_J)(Q_J - Q_{NO})$; hence, setting $q = Q_{NO}$ as in Webster and Weng [14] will not guarantee a risk-free result. Lemma 1 summarizes requirements for being risk-free.

Lemma 1.

QR Policy*: Substituting the coordinating returns value* $r^* = (C-M)/F(Q_J)$ *into (1) leads to* $(Q_J - \max[q, y]) \le F(Q_J)(Q_J - Q_{NO})$; *thus, the system needs to satisfy the following two conditions in order to be (i) risk-free, and (ii) accepted by the retailer (retailer's expected profit to be no less than in the no returns case):*

(i) $(Q_J - q) \le F(Q_J)(Q_J - Q_{NO}) \Rightarrow q \ge \Phi$, *and*

(ii)

$$\Pi_R(Q_J \mid q) \ge \Pi_R(Q_{NO}) \Rightarrow \left(\frac{P-r}{r}\right)\left\{F(Q_J)(Q_J - Q_{NO}) - \int_{Q_{NO}}^{Q_J} F(y)dy\right\} \ge \int_{Q_{NO}}^{q} F(y)dy.$$

FR Policy*: Substituting the coordinating returns value* $r^* = (C-M)/F(Q_J)$ *into (1) leads to* $(Q_J - y) \le F(Q_J)(Q_J - Q_{NO})$ *; thus, the demand needs to satisfy* $y \ge \Phi$ *(with probability* $\bar{F}\{\Phi\}$*) in order to be risk-free.* □

Lemma 1 shows that violating condition (ii) will result in the retailer rejecting the returns policy; thus, if (i) and (ii) cannot be simultaneously satisfied, the manufacturer will have no choice but to reduce q to $q < \Phi$ (violates (i)). However, under this returns schedule, the event $\{Rf\}$ occurs only if $y \geq \Phi$ (with probability $\bar{F}\{\Phi\}$). Let us furnish a numerical example to demonstrate this situation. Consider the example given in Webster and Weng [14]. The base parameters will take the following values – $P = \$100$ and $M = \$50$ – and the retail is uniformly distributed with a range of $[0,100]$. Without coordination, the independent manufacturer will most likely set the wholesale price $C = \$75$, since this price range maximizes the manufacturer's individual expected profit (see Table 3). When $C = \$75$ the decision variables will take the following values: Q_{NO} =25, Q_J =50, and coordinating returns value r^* = 50. Now, in order to be risk-free, q needs to be greater than or equal to $\Phi = 37.5$. However, even for a minimum level of $q = 37.5$ the retailer's expected profit is still less than that in the no returns case, i.e., $\Pi_R(Q_J \mid q = 37.5) = \$273 < \Pi_R(Q_{NO}) = \$313$; therefore, the retailer will not accept the returns policy. Two options are available here to the manufacturer. First, she can give up coordination of the supply chain and concentrate otherwise on inducing the retailer to accept the returns policy while making sure that she is risk-free. We will refer to this policy as a *Risk Free QR* (RQR) policy. This can be accomplished, as in Webster and Weng [14], by reducing the returns value from r^* =50 (coordinating returns value) to $r \in (0, C-M]$ and equating $q = Q_{NO}$. Table 1 summarizes expected profits in RQR policy under various wholesale prices. $\bar{r}$ in Table 1 is the returns value $r \in (0, C-M]$ that maximizes the manufacturer's expected profit. For example, when $C = \$75$, $r = \$25$ maximizes the manufacturer's expected profit. Second, the manufacturer can design a coordinating returns value r^* = 50 and equate $q = 35.2 < \Phi = 37.5$ so that $\Pi_R(Q_J \mid q = 35.2, r^* = 50) = \Pi_R(Q_{NO}) = \$313$ (the supply chain is coordinated and the retailer's profit is identical to the no returns case). We will refer to this policy as a *Coordinating QR policy* (CQR). We see that the manufacturer's expected profit in CQR (\$937) is significantly higher than that in RQR (\$772). However, in CQR the event $\{Rf\}$ occurs only if demand is at least larger than $\Phi = 37.5$; thus, the probability of event $\{Rf\}$ occurring $= \bar{F}(\Phi) = 62.5\%$. Table 2 provides more examples as a function of wholesale price $C = \$60, \$70, \cdots, \$90$. For example, at $C = 60$, the likelihood of the manufacturer being worse off from CQR is almost as high as 45%; thus, the manufacturer might hesitate to propose a system coordinating CQR policy.

Table 1. Risk Free QR (RQR)

Wholesale Price C	60	70	75	80	90
Returns Values $\bar{r}$ Maximizes Manufacturer's Expected Profit	10	20	25	30	40
Expected Profits (equating $r = \bar{r}$)					
Supply Chain Joint Profit	1,233	1,171	1,111	1,020	694
Retailer	808	472	339	225	63
Manufacturer	425	699	772	795	631
% Increase Ratio*	6.25	16.5	23.5	32.5	57.75

* $100 \times (RQR - No\ Returns)/No\ Returns$

Table 2. Coordinating and Risk-free QR (CQR)

Wholesale Price C	60	70	75	80	90
Expected Profits					
q=Φ (to be risk-free)					
Retailer	797	430	273	133	0
Manufacturer	453	820	977	1117	1250
q<Φ (retailer's expected profit = no returns case)					
Retailer	800	450	313	200	50
Manufacturer	450	800	937	1,050	1,200
Probability of $\{Rf\} = \bar{F}\{\Phi\}$	55%	60%	62.5%	65%	70%

3. THE MULTI-LEVEL MARQ RETURNS POLICY (MLR)

In this section, we propose a SCRC that aims to achieve the following two goals as listed in Section 1. *(i) Significantly improve the manufacturer's expected profit, and (ii) maintain the probability of event $\{Rf\}$ occurring to be relatively high.* Let $\Pi = (C - M)Q_J - r^*(Q_J - \max[q, y])$ be the manufacturer's resulting realized profit when the supply chain is coordinated by a system coordinating returns value r^* given in (2), and let $R = (C - M)Q_{NO}$ be the certain profit attainable by the manufacturer when returns are not permitted. Mathematically, the problem involves finding coordinating returns policies that also maximize $\Pr ob\{\Pi \geq R \mid \Pi_R(Q_J) \geq \Pi_R(Q_{NO})\}$.

We will now propose a returns policy that has multiple levels of MARQs. The returns schedule for a multi-level returns (MLR) policy can be described as follows:

MLR Policy

Let $q_n = \Phi$ $(\Phi := Q_J\bar{F}(Q_J) + F(Q_J)Q_{NO}$*), and* $\{q_j\}_{j=1}^{n-1}$ *satisfy* $0 \le q_1 \le \cdots \le q_n$.

(i) If the retailer orders a quantity $Q \ge q_n$*, then the manufacturer will grant the following MLR policy.*

$$\text{returns quantity} = \begin{cases} Q - q_1 & \text{if } y < q_1 \\ \cdots\cdots & \\ Q - q_n & \text{if } q_{n-1} \le y < q_n \\ I & \text{if } y \ge q_n \end{cases}, \text{ where } 0 \le q_1 \le \cdots \le q_n \text{ and } q_n \le Q. \tag{5}$$

On the other hand,

(ii) If the retailer's order quantity $Q < q_n$*, then the manufacturer will not permit any returns (no returns policy).*

(iii) The retailer's expected profits are thus,

$$\begin{cases} MLR: \Pi_R\left(Q \middle| \{q_j\}_{j=1}^{n}\right) = (P-C)Q - (P-r)\int_0^Q F(y)\,dy + r\left\{\sum_{J=1}^{n-1} F(q_J)(q_{J+1} - q_J) - \int_0^{q_n} F(y)\,dy\right\} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } Q \ge q_n \\ \text{No Returns}: \Pi_R(Q) = (P-C)Q - P\int_0^Q F(y)\,dy \qquad \text{if } Q < q_n \end{cases}$$

Let $Q_{RE}(r)$ *and* Q_{NO} *respectively denote optimal order quantities that maximize* $\Pi_R(Q \mid \{q_j\}_{j=1}^{n})$ *and* $\Pi_R(Q)$*. For an arbitrary* $\delta > 0$*,* $\{q_j\}_{j=1}^{n-1}$ *is chosen to satisfy*

$$\Pi_R\left(Q_{RE}(r) \mid q_n = \Phi, \{q_j\}_{j=1}^{n-1}\right) = \Pi_R(Q_{NO}) + \delta > \Pi_R(Q_{NO}).$$

Proposition 1 states the property of the optimal MLR policy.

Proposition 1. *MLR policy*

(i.1) Both objective functions MLR: $\Pi_R\left(Q \mid \{q_j\}_{j=1}^{n}\right)$ *and No Returns:* $\Pi_R(Q)$ *are concave in* Q*. The order quantity maximizing (a)* $\Pi_R\left(Q \mid \{q_j\}_{j=1}^{n}\right)$ *satisfies* $Q_{RE}(r)$ $= F^{-1}((P-C)/(P-r))$*, and maximizing (b)* $\Pi_R(Q)$ *satisfies* $Q_{NO} = F^{-1}((P-C)/P))$*. Thus, for the retailer,* $Q_{RE}(r)$ *is optimal if* $\Pi_R(Q_{RE}(r) \mid \{q_j\}_{j=1}^{n}(r)) \ge \Pi_R(Q_{NO})$ *and* Q_{NO} *is optimal if* $\Pi_R(Q_{RE}(r) \mid \{q_j\}_{j=1}^{n}(r)) \le \Pi_R(Q_{NO})$*. However, since* $\{q_j\}_{j=1}^{n-1}$ *is chosen to satisfy* $\Pi_R(Q_{RE}(r) \mid \{q_j\}_{j=1}^{n}) = \Pi_R(Q_{NO}) + \delta > \Pi_R(Q_{NO})$*, the risk-neutral retailer will prefer ordering* $Q_{RE}(r)$*.*

(i.2) In MLR, the supply chain can be coordinated by the returns value $r^* =$ $(C-M)/F(Q_J)$*, which upon substitution leads to* $Q_{RE}(r^*) = Q_J \ge \Phi = Q_J\bar{F}(Q_J) +$

$F(Q_J)Q_{NO}$*; thus, it satisfies the requirement of MLR* $Q \geq q_n = \Phi$. *Upon substituting* r^* *and* $\{q_j\}_{j=1}^{n}$

$$\Pi_R\left(Q_{RE}(r^*) = Q_J \mid q_n = \Phi, \{q_j\}_{j=1}^{n-1}\right) = \Pi_R(Q_{NO}) + \delta \Rightarrow$$

$$\left(\frac{P-r^*}{r^*}\right)\left\{F(Q_J)(Q_J - Q_{NO}) - \int_{Q_{NO}}^{Q_J} F(y)\,dy\right\} + \sum_{j=1}^{n-1} F(q_j)(q_{j+1} - q_j) = \int_{Q_{NO}}^{q_n} F(y)\,dy + \delta.$$

(ii) For any MLR policy with n-1 levels of MARQ, there always exists a MLR policy with the same $\{q_j\}_{j=1}^{n-1}$ *and one more* $q > q_{n-1}$ $(q < q_{n-1})$ *in which a manufacturer earns a no lower (no higher) expected profit; hence, for any QR policy with an arbitrary* $\bar{q}$, *there always exists a two- level MLR policy with a* $q = \bar{q}$ *and the other* $q > \bar{q}$ $(q < \bar{q})$ *in which a manufacturer earns a no lower (no higher) expected profit.*
(iii) The probability of event $\{Rf\} = \bar{F}(q_{n-1}) \geq \bar{F}(q_n) = \bar{F}\{\Phi\}$ *in MLR.*
Proof of Proposition 1.ii The difference in the manufacturer's expected profits between n levels and n-1 levels' of MARQ with the same $\{q_j\}_{j=1}^{n-1}$ and one more $q_n > q_{n-1}$ can be shown to be equal to $D = (\Pi_M^n - \Pi_M^{n-1}) = r\left\{\int_{q_{n-1}}^{q_n} F(y)dy - F(q_{n-1})\right.$ $\left.(q_n - q_{n-1})\right\}$. Let ε denote $q_n - q_{n-1}$; then the difference $D(\varepsilon) = -rF(q_{n-1})\varepsilon +$ $r\int_{q_{n-1}}^{q_{n-1}+\varepsilon} F(y)dy$ can be expressed as a function of ε. Here, taking the first derivative reveals $\partial D / \partial \varepsilon = -rF(q_{n-1}) + rF(q_{n-1} + \varepsilon) \geq 0$, and setting ε =0 leads to $D(\varepsilon = 0) = 0$; therefore, we see that $D(\varepsilon) \geq 0 \quad \forall \varepsilon$. This result tells us that the manufacturer's expected profit is increased by one more $q_n > q_{n-1}$. The rest of the cases can be demonstrated similarly.
Proof of Proposition 1.iii The maximum returns quantity for the actualized demand $y \geq q_{n-1}$ (with probability $\bar{F}(q_{n-1})$) is less than or equal to $Q_J - q_n$, which satisfies the risk-free condition upon substituting $q_n = \Phi$. Therefore, any actualized demands $y \geq q_{n-1}$ will lead to the risk-free situation with probability of $y \geq q_{n-1} = \bar{F}(q_{n-1})$. □

The benefit of MLR policy to a manufacturer is clearly seen in Proposition 1. We have shown that a coordinating and risk-free QR (CQR) schedule with a $q \geq \Phi$ cannot guarantee constant acceptance by a retailer. Assuming first that the retailer accepts the coordinating and risk-free QR schedule with a $q \geq \Phi$, that is, $\Pi_R(Q_J \mid q \geq \Phi) = \Pi_R(Q_{NO})$, equating $q_1 \ldots q_{n-1}$=0, $q_n = q$ will make the manufacturer's and the retailer's expected profits in the MLR policy exactly equal to those in the CQR policy. Now assume that the retailer refuses to accept the coordinating and risk-free CQR policy even with the smallest possible $q = \Phi$ (satisfying Lemma 1(i)) because of the resulting expected profit being lower than that in the no returns case. Then, the manufacturer has no other choice but to reduce q to a $q < \Phi$ so as to increase the retailer's expected profit. This will lead to the probability of event $\{Rf\}$ occurring $= \bar{F}\{\Phi\}$. However, in MLR, by setting $q_n = \Phi$ and designing more $\{q_j\}_{j=1}^{n-1}$ in demands $y \in [0, q_n]$, the manufacturer can increase the retailer's expected profit to match that in the no returns case. In doing so, the manufacturer increases the probability of event $\{Rf\}$ occurring to $\bar{F}(q_{n-1} < \Phi) > \bar{F}(q_n = \Phi)$ higher than that in the CQR policy.

We would like to mention that although we have not explicitly considered the possibility of an agent committing irrational or deceptive conduct, nevertheless, it is possible that the arrangement in MLR might induce a retailer to commit intentional sabotage on selling efforts. Note that a retailer can return $Q - q_j$ when demands $y \in (q_{j-1}, q_j]$, but is only allowed to return $Q - q_{j+1}$ when $y \in (q_j, q_{j+1}]$. Therefore, she could gain an unfair advantage by intentionally reducing her selling efforts on those occasions in which demands happens to be slightly more than a threshold point q_j so as to gain a higher returns rebate. Perhaps only an accurate demand estimation and a monitoring system that can closely monitor the retailer's selling operation could reduce the manufacturer's concern.

Let us now consider the upper bound of the probability of event $\{Rf\}$ occurring (or lower bound of q_{n-1}). Assume the manufacturer designs a MLR policy in which infinite levels of MARQs are in $[0, q_1]$, and no MARQs are designed for demands $y \geq q_2$; thus, full returns are granted for $y \geq q_2$ and $y \leq q_1$.

(i) If the retailer's order quantity $Q \geq q_2 = \Phi$, then the manufacturer will grant the following MLR returns policy.

$$\textit{returns quantity} = \begin{cases} I & \textit{if } y \leq q_1 \\ Q - q_2 & \textit{if } q_1 < y < q_2 \\ I & \textit{if } y \geq q_2 \end{cases}, \textit{ where } 0 \leq q_1 \leq q_2 \textit{ and } q_2 \leq Q. \quad (5.1)$$

(ii) If the retailer's order quantity $Q < q_2 = \Phi$, *then the manufacturer will not permit any returns.*

We label this as an Upper Bound MLR (UMLR) returns policy. Given that the coordinating returns policy is employed by the supply chain, the probability $\bar{F}(q_1)$ obtained from UMLR is the maximum attainable probability of $\{Rf\}$ occurring if q_1 are chosen to satisfy $\Pi_R(Q_J \mid q_2 = \Phi, q_1) = \Pi_R(Q_{NO})$. Thus, the probability of event $\{Rf\}$ occurring in any n-level MLR policy $= \bar{F}(q_{n-1}) \in [\bar{F}(\Phi), \bar{F}(q_1)]$. Let us consider again the example given in the previous section. We have computed the probability of event $\{Rf\}$ occurring for UMLR in Table 3. For example, Q_{NO} =20, Q_J =50, and $\Phi = 35$ when C =\$80. In UMLR, the manufacturer designs a returns policy with (q_1, q_2) given as follows:

$$\text{return quantity} = \begin{cases} I & \text{if } \; y \le q_1 = 3.4 \\ Q - 35 & \text{if } \; 3.4 < y < q_2 = 35 \\ I & \text{if } \; y \ge 35 \end{cases}.$$

Table 3 reveals that the probability of event $\{Rf\}$ occurring in UMLR is 96.6%, which is 32% more than that in the Coordinating QR (CQR) policy. Figure 2 reveals that UMLR provides a higher probability of event $\{Rf\}$ occurring in all five cases compared to CQR; however, it performs particularly well at the lower wholesale price range where CQR performs poorly. Table 3 tells us that when no returns are permitted, the manufacturer will design the wholesale price $C = \$75$. We see that, by using the UMLR policy, the manufacturer's profit increases from \$625 to \$937 with the probability of event $\{Rf\}$ occurring equaling 97.8%. Here, in order to induce the retailer to accept an MLR contract, for example, the manufacturer could increase q_1 from 2.2 ($\bar{F}(q_1) = 97.8\%$) to 7 ($\bar{F}(q_1) = 93\%$, see δ in Table 3) so as to increase the retailer's expected profit from \$313 (no returns) to \$393. In this way, the retailer earns an additional expected profit $\delta = \$80$ (the last line in Table 3). While this will be accompanied by a reduction in the probability of event $\{Rf\}$ occurring of 4.8%, it is still significantly higher than the CQR policy in which the probability of event $\{Rf\}$ occurring equals 62.5%. Figures 1 and 2 show that UMLR significantly increases the manufacturer's expected profit with very small decreases in the risk-free probability.

Table 3. Multilevel Returns Policy (MLR)

Wholesale Price C	60	70	75*	80	90
Supply Chain Joint Profit					
Coordinating Returns Policy	1,250	1,250	1,250	1,250	1,250
No Returns Policy	1,200	1,050	938	800	450
Expected Profits					
No Returns Policy					
Retailer	800	450	313	200	50
Manufacturer	400	600	625*	600	400
UMLR					
Probability of $\{Rf\} = \bar{F}\{q_1\}$	99.7%	98. 8%	97.8%	96.6%	92.4%
Retailer	800	450	313	200	50
Manufacturer	450	800	937	1,050	1,200
% Increase Ratio**	12.5	33.3	49.9	75	200
Probability of $\{Rf\} = \bar{F}\{q_1\}$	98%	95%	93%	92%	90%
Retailer	816	505	393	282	90
Manufacturer	434	745	857	968	1160
*** δ	16	55	80	82	40

* The wholesale price that maximizes the manufacturer's expected profit under a no returns policy

** $100 \times (UMLR - NO\ returns)/NO\ returns$

*** δ is an arbitrarily chosen addition rebate that can be used to induce the retailer

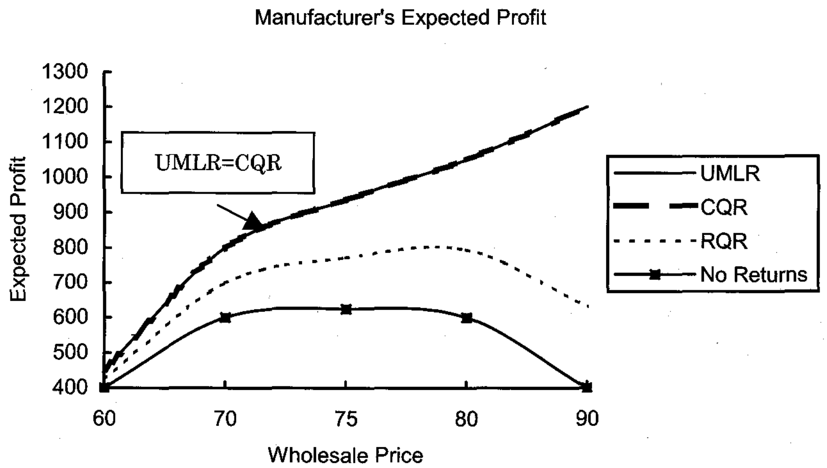

Figure 1. The manufacturer's Expected Profits

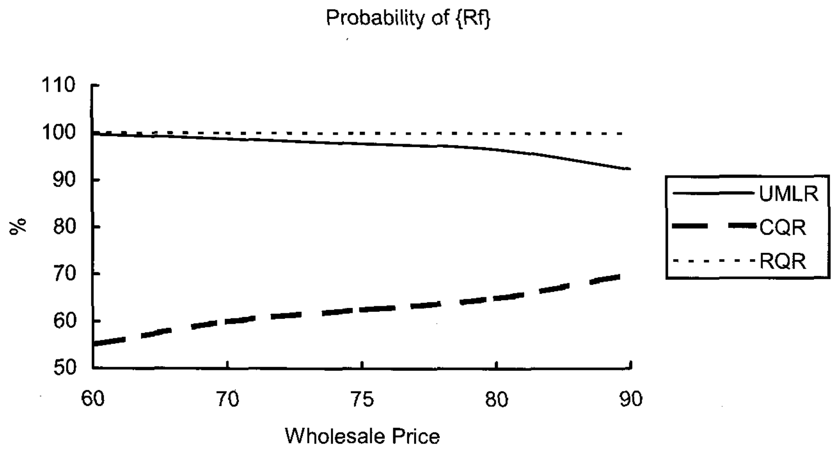

Figure 2. Probabilities of the Occurrence of Event $\{Rf\}$

4. CONCLUSION

This paper considers a returns contract that can significantly increase the manufacturer's expected profit with very small decreases in the risk-free probability. When a supply chain is coordinated by a returns policy, the retailer's (manufacturer's) expected profit function is a decreasing (increasing) function of the returns rebate (see Pasternack [9]). Thus, a manufacturer could push a returns rebate as high as possible to increase her individual profit share at the expense of the retailer's profit share. This result leads us to consider a supply contract in which a manufacturer could maximize her individual expected profit by designing a supply chain coordinating returns contract (SCRC) that consists of (i) *a channel coordinating returns policy that maximizes the supply chain joint expected profit, and* (ii) *a profit sharing arrangement (via manipulation of the returns contract terms) that gives the retailer an expected profit only slightly higher than that in the no returns case so that it is just enough to induce the retailer to accept the SCRC.* By doing so, the manufacturer captures as high a percentage as possible of the jointly maximized supply chain profit (via channel coordination), and earns an expected profit higher than that earned from any other contracts that are acceptable to the retailer (with the retailer's expected profits no less than those in the no returns case). However, even though realization of profit can be sufficiently attractive on average, it can also lead to an undesirable situation in which extremely high returns rebates stem from unexpectedly low demands. Having discovered these shortcomings, we have proposed a SCRC that takes risk sensitivity into consideration, and aim to maximize the probability that the manufacturer's profit is at least as high as in the event of no returns. In the model, labeled Multi-levels Returns Policy (MLR), a manufacturer adds a more returns schedule $\{q_j\}_{j=1}^{n}$ into a QR policy to either reduce or increase the returns quantity. A numerical experiment was provided to illustrate the performance of the upper bound of the MLR (UMLR) policy. The result reveals that the manufacturer's expected profits are significantly higher than those in the no returns case, and the probabilities of $\{Rf\}$ occurring are only slightly lower than when there is no risk.

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