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NOTES ON $\overline{WN_{n,0,0_{[2]}}}$ II

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Abstract. The Weyl-type non-associative algebra $\overline{WN_{g_n,m,s_r}}$ and its subalgebra $\overline{WN_{n,m,s_r}}$ are defined and studied in the papers [2], [3], [9], [11], [12]. We find the derivation group $Der_{non}(\overline{WN_{1,0,0_{[2]}}})$ the non-associative simple algebra $\overline{WN_{1,0,0_{[2]}}}$.

1. Preliminaries

Let $\mathbf{N}$ be the set of all non-negative integers and $\mathbf{Z}$ be the set of all integers. Let $\mathbf{F}$ be a field of characteristic zero. Let $\mathbf{F}^{\bullet}$ be the multiplicative group of non-zero elements of $\mathbf{F}$. Let $\mathbf{F}[x_1, \cdots, x_{m+s}]$ be the polynomial ring with the variables $x_1, \cdots, x_{m+s}$. Let $g_1, \cdots, g_n$ be given polynomials in $\mathbf{F}[x_1, \cdots, x_{m+s}]$. For $n, m, s \in \mathbf{N}$, let us define the commutative, associative $\mathbf{F}$-algebra $F_{g_n,m,s} = \mathbf{F}[e^{\pm g_1}, \cdots, e^{\pm g_n}, x_1^{\pm 1}, \cdots, x_m^{\pm 1}, x_{m+1}, \cdots, x_{m+s}]$ which is called a stable algebra in the paper [5] with the standard basis

$$\mathbf{B} = \{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} | a_1, \cdots, a_n, i_1, \cdots, i_m \in \mathbf{Z}, \\ i_{m+1}, \cdots, i_{m+s} \in \mathbf{N}\}$$

and with the obvious addition and the multiplication [5], [8] where we take appropriate $g_1, \cdots, g_n$ so that $\mathbf{B}$ can be the standard basis of $F_{g_n,m,s}$. ∂_w, $1 \leq w \leq m+s$, denotes the usual partial derivative with respect to x_w on $F_{g_n,m,s}$. For partial derivatives $\partial_u, \cdots, \partial_v$ of $F_{g_n,m,s}$,

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the composition $\partial_u^{j_u} \circ \cdots \circ \partial_v^{j_v}$ of them is denoted $\partial_u^{j_u} \cdots \partial_v^{j_v}$ where $j_u, \cdots, j_v \in \mathbf{N}$. Let us define the vector space $WN(g_n, m, s)$ over $\mathbf{F}$ which is spanned by the standard basis

$$(1) \qquad \{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} | a_1, \cdots, a_n, i_1, \cdots, i_m \in \mathbf{Z},$$
$$i_{m+1}, \cdots, i_{m+s} \in \mathbf{N}, j_u, \cdots, j_v \in \mathbf{N}, 1 \leq u, \cdots, v \leq m+s\}$$

Thus we may define the multiplication $*$ on $WN(g_n, m, s)$ as follows:

$$(2) \qquad e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} *$$
$$e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}} \partial_h^{j_h} \cdots \partial_w^{j_w}$$
$$= e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v}$$
$$(e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}}) \partial_h^{j_h} \cdots \partial_w^{j_w}$$

for any basis elements $e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v}$ and $e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}} \partial_h^{j_h} \cdots \partial_w^{j_w} \in WN(g_n, m, s)$. Thus we can define the Weyl-type non-associative algebra $\overline{WN_{g_n,m,s}}$ with the multiplication $*$ in (2) and with the set $WN(g_n, m, s)$ [1], [14]. For $r \in \mathbf{N}$, let us define the the non-associative subalgebra $\overline{WN_{g_n,m,s_r}}$ of the non-associative algebra $\overline{WN_{g_n,m,s}}$ spanned by

$$\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_s^{i_s} \partial_u^{j_u} \cdots \partial_v^{j_v} | a_1, \cdots, a_n, i_1, \cdots, i_m \in \mathbf{Z},$$
$$i_{m+1}, \cdots, i_s \in \mathbf{N}, j_u, \cdots, j_v \in \mathbf{N},$$
$$(3) \qquad j_u + \cdots + j_v \leq r, 1 \leq u, \cdots, v \leq m+s\}$$

The non-associative subalgebra $\overline{WN_{g_n,m,s_1}}$ of the non-associative algebra $\overline{WN_{g_n,m,s}}$ is the the non-associative algebra $\overline{N_{g_n,m,s}}$ in the paper [1]. There is no left or right identity of $\overline{WN_{g_n,m,s}}$. The the non-associative algebra $\overline{WN_{g_n,m,s}}$ is $\mathbf{Z}^n$-graded as follows:

$$(4) \qquad \overline{WN_{g_n,m,s}} = \bigoplus_{(a_1,\cdots,a_n)} WN_{(a_1,\cdots,a_n)}$$

where $WN_{(a_1,\cdots,a_n)}$ is the vector subspace of $\overline{WN_{g_n,m,s}}$ with the basis

$$\{e^{a_1g_1}\cdots e^{a_ng_n}x_1^{i_1}\cdots x_{m+s}^{i_{m+s}}\partial_u^{j_u}\cdots\partial_v^{j_v}|i_1,\cdots,i_m\in\mathbf{Z},$$
$$i_{m+1},\cdots,i_{m+s},j_u,\cdots,j_v\in\mathbf{N},1\leq u,\cdots,v\leq m+s\}.$$

An element in $WN_{(a_1,\cdots,a_n)}$ is called an $(a_1,\cdots,a_n)$-homogenous element and $WN_{(a_1,\cdots,a_n)}$ is called the $(a_1,\cdots,a_n)$-homogeneous component. For any basis element $e^{a_1g_1}\cdots e^{a_ng_n}x_1^{i_1}\cdots x_{m+s}^{i_{m+s}}\ \partial_t$ of $\overline{WN_{g_n,m,s}}$, let us define the homogeneous degree $deg_N(e^{a_1g_1}\cdots e^{a_ng_n}x_1^{i_1}\cdots x_{m+s}^{i_{m+s}}\partial_u^{j_u}\cdots\partial_v^{j_v})$ of it as follows:

$$deg_N(e^{a_1g_1}\cdots e^{a_ng_n}x_1^{i_1}\cdots x_{m+s}^{i_{m+s}}\partial_u^{j_u}\cdots\partial_v^{j_v})=\sum_{u=1}^{m+s}|i_u|$$

where $|i_u|$ is the absolute value of i_u, $1\leq u\leq m+s$. Throughout this paper, for any basis element $e^{a_\mu g_\mu}\cdots e^{a_\nu g_\nu}x_\lambda^{i_\lambda}\cdots x_\sigma^{i_\sigma}\partial_u^{j_u}\cdots\partial_v^{j_v}$, we write it such that $1\leq\mu\leq\cdots\leq\nu\leq n$, $1\leq\lambda\leq\cdots\leq\sigma\leq m$, and $1\leq u\leq\cdots\leq v\leq m+s$. For any element $l\in\overline{WN_{g_n,m,s}}$, we may define $deg_N(l)$ as the highest homogeneous degree of the basis terms of l. Thus for any basis elements l_1 and l_2 of $\overline{WN_{0,0,s}}$, we may write l_1+l_1 or l_2+l_1 well orderly with unambiguity. For any element $l\in\overline{WN_{0,0,s}}$, we may define $deg_N(l)$ as the highest homogeneous degree of each monomial of l. For any $l\in\overline{WN_{g_n,m,s}}$, let us define $\#(l)$ as the number of different homogeneous components of l. $\overline{WN_{n,m,s}}$ (resp. $\overline{WN_{g_n,m,s_r}}$) has the subalgebra WT (resp. WT_r) spanned by $\{\partial_u^{j_u}\cdots\partial_v^{j_v}|(resp.\ \ j_u+\cdots+j_v\leq r),\ \ j_u,\cdots,j_v\in\mathbf{N},1\leq u,v\leq s_1\}$ which is the right annihilator of $\overline{WN_{g_n,m,s}}$ (resp. $\overline{WN_{g_n,m,s_r}}$). Let us define the non-associative subalgebra $\overline{WN_{n,0,0_{[r]}}}$ of the non-associative algebra $\overline{WN_{g_n,m,s}}$ is spanned by $\{e^{a_1x_1}\cdots e^{a_nx_n}\partial_u^r|1\leq u\leq n\}$. The non-associative algebra $\overline{WN_{n,0,0_{[r]}}}$ is $\mathbf{Z}^n$-graded as follows:

$$\overline{WN_{n,0,0_{[r]}}}=\bigoplus_{(a_1,\cdots,a_n)}N_{(a_1,\cdots,a_n)}\tag{5}$$

where $N_{(a_1,\cdots,a_n)}$ is the vector subspace of $\overline{WN_{n,0,0_{[r]}}}$ with the basis $\{e^{a_1x_1}\cdots e^{a_nx_n}\partial_v^r|1 \le v \le n\}$. The non-associative algebra $\overline{WN_{g_n,m,s}}$ contains the matrix ring $M_n(\mathbf{F})$ [1]. A non-associative algebra A is simple, if it has no proper two sided ideal which is not zero ideal [14]. For any element l in a non-associative algebra A, l is full, if the ideal $< l >$ generated by l is A. Generally, the algebra $\overline{WN_{0,0,s_r}}$ or $\overline{WN_{0,0,s}}$ is not Lie admissible [1], [8], since the Jacobi identity does not hold using the commutator of the non-associative algebra $\overline{WN_{0,0,s_r}}$ or the non-associative algebra $\overline{WN_{0,0,s}}$ for $r > 1$. For any $\mathbf{F}$-algebra A and an element $l \in A$, an element $l_1 \in A$ is a left (resp. right) stabilizing element of l, if $l_1 * l = cl$ (resp. $l * l_1 = cl$) where $c \in \mathbf{F}$. For any element $l_1 \in A$, $l \in A$ is a locally left (resp. right) unity of $l_1 \in A$, if $l * l_1 = l_1$ (resp. $l_1 * l = l_1$) holds and throughout the paper, we read it as that l is a left unity of l_1, etc.. The Weyl-type non-associative algebra $\overline{WN_{g_n,m,s_r}}$ and its subalgebra $\overline{WN_{n,m,s_r}}$ contains the matrix ring $M_s(\mathbf{F})$, i.e., $x_u\partial_v$ in The Weyl-type non-associative algebra $\overline{WN_{g_n,m,s_r}}$ or its subalgebra $\overline{WN_{n,m,s_r}}$ corresponds e_{uv} where e_{uv} is the unit matrix of $M_s(\mathbf{F})$ such that its uv-entry is one and its other terms are zero.

2. Simplicity of $\overline{WN_{n,0,0_{[r]}}}$

Even if the non-associative algebra $\overline{WN_{n,0,0_{[r]}}}$ has right annihilators, we have the following results. The non-associative algebra $\overline{WN_{n,0,0_{[r]}}}$ has no idempotent. $\overline{WN_{1,0,0_{[r]}}}$ does not have a right identity and a left identity. For the set $W = \{w_a|a \in \mathbf{Z}\}$, let us define the addition $+$ as follows: for any $w_a, w_b \in W$,

$$w_a + w_b = w_{a+b}$$

and the multiplication $\cdot$ as follows:

$$w_a \cdot w_b = b^2 w_{a+b}$$

Then $W_2 =$ is an non-associative algebra with the usual scalar multiplication. If we define the usual addition and Lie bracket on $W = \{w_a | a \in \mathbf{Z}\}$ as follows:

$$[w_a, w_b] = (b^2 - a^2) w_{a+b}$$

Then $W_{2[,]} =$ is a semi-Lie algebra. The semi-Lie algebra $W_{2[,]}$ has one dimensional center spanned by w_0.

Remark 1. An (non-associative, Lie, or associative) algebra A is simple if and only if every element of the (non-associative, Lie, or associative) algebra A is full.

Lemma 1. *For any ∂_u^r, $1 \leq u \leq n$, in the non-associative algebra $\overline{WN_{n,m,s}}_{[r]}$, ∂_u^r is full.*

Proof. The proof of this lemma is standard, so let us omit it. □

Theorem 1. *The non-associative algebra $\overline{WN_{n,m,s}}_{[r]}$ is simple.*

Proof. The proof of this theorem is also standard, so let us omit it. □

Corollary 1. *The non-associative algebra $\overline{WN_{1,m,s}}_{[r]}$ is simple.*

Proof. The proof of the corollary is straightforward by Theorem 1. Thus let us omit the proof of the corollary. □

Theorem 2. *If r is odd, then the semi-Lie algebra $\overline{WN_{n,m,s}}_{[r][,]}$ is simple.*

Proof. The proof of the theorem is straightforward by Theorem 1. So let omit it. □

Corollary 2. *The Lie algebra $\overline{WN_{0,0,n}}_{1[,]}$ is simple.*

Proof. Since the Lie algebra $\overline{WN_{n,0,0}}_{1[,]}$ is isomorphic to the Lie algebra $\overline{WN_{0,0,n}}_{1[,]}$, the Lie algebra $\overline{WN_{0,0,n}}_{1[,]}$ is simple. □

The semi-Lie algebra $\overline{WN_{0,0,n}}_{[r]_{[,]}}$ is called the Witt type semi-Lie algebra [13].

3. Derivations of $\overline{WN_{1,0,0}}_{[2]}$ and Isomorphism

Note that the **F**-algebra $\mathbf{F}[x, x^{-1}]$ is isomorphic to the **F**-algebra $\mathbf{F}[e^{\pm x}]$ as **F**-algebras.

Definition 1. *Let A be an **F**-algebra. An additive **F**-map D from A to A is a derivation if $D(l_1 * l_2) = D(l_1) * l_2 + l_1 * D(l_2)$ for any $l_1, l_2 \in A$.*

Example 5. The usual partial derivative ∂_u, $1 \le u \le n$, on $\mathbf{F}[x_1, \cdots, x_n]$ is a well known derivation on $\mathbf{F}[x_1, \cdots, x_n]$.

Note 1. For any basis element $e^{kx}\partial^2$ of the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$, if we define **F**-additive linear map D_c of the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$ as follows:

$$D_c(e^{kx}\partial^2) = cke^{kx}\partial^2$$

then D_c can be linearly extended to a derivation of the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$ where $c \in \mathbf{F}$.

Lemma 2. *For any derivation D of the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$, if $D(\partial^2) = 0$, then D is the derivation D_c which is defined in Note 1.*

Proof. Let D be the derivation of the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$ in the lemma. By $D(\partial^2 * \partial^2) = 0$, we have that $D(\partial^2) = c\partial^2$ where $c \in \mathbf{F}$. By $D(\partial^2 * e^x\partial^2) = D(e^x\partial^2)$, we have that

$$D(\partial^2) * e^x\partial^2 + \partial^2 * D(e^x\partial^2) = D(e^x\partial^2) \tag{6}$$

This implies that

$$D(e^x\partial^2) = c_1e^{rx}\partial^2 + c_2e^{-sx}\partial^2 + \#_1 \tag{7}$$

with appropriate scalars and $r > s \in \mathbf{N}$ where $\#_1$ is the sum of remaining terms. By (6), we have that $s = 1$, $r = 1$, and $c = 0$, i.e., $D(\partial^2) = 0$. By $D(e^{2x}\partial^2) = D(e^x\partial^2 * e^x\partial^2)$, we have that

$$D(e^{2x}\partial^2) = 2c_1e^{2x}\partial^2 + 2c_2\partial^2$$

By $D(\partial^2 * 1e^{2x}\partial^2) = 4D(1e^{2x}\partial^2)$, we have that $D(e^{2x}\partial^2) = 2c_1e^{2x}\partial^2$. By induction on k of $e^{kx}\partial^2$, we can prove that

$$D(e^{kx}\partial^2) = kc_1e^{kx}\partial^2$$

This implies that D is the derivation D_c in Note 1. Therefore we have proven the lemma. □

Theorem 3. *For any derivation D of the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$, $D = \sum_{c\in\mathbf{F}} D_c$ where D_c is the derivation which is defined in Note 1.*

Proof. The proof of the theorem is straightforward by Lemma 2. So let us omit its remaining steps of the proof. □

By Theorem 2, we know that every derivation of the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$ is the sum of scalar derivations. All the derivations of the non-associative algebras $\overline{WN_{1,0,0}}_{[2]}$, $1 \leq r \leq 3$, are found in the papers [1], [11], [12]. Thus it is an interesting problem to find all the derivations of the non-associative algebras $\overline{WN_{n,0,0}}_{[r]}$. Also it is an interesting problem to find the non-associative algebra automorphism group $Aut_{non}(\overline{WN_{n,0,0}}_{[r]})$ of the non-associative algebras $\overline{WN_{n,0,0}}_{[r]}$ [1], [7], [10].

Proposition 1. *The non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$ is not isomorphic to the non-associative algebra $\overline{WN_{1,0,0}}_{[1]}$.*

Proof. Let us assume that there is an isomorphism θ from $\overline{WN_{1,0,0}}_{[1]}$ to $\overline{WN_{1,0,0}}_{[2]}$. This implies that $\theta(\partial) = c\partial^2$ for $c \in \mathbf{F}^\bullet$. By $\theta(\partial * e^x\partial) = \theta(e^x\partial)$, we have that $\theta(e^x\partial) = de^{ax}\partial^2 + \#_1$ where $\#_1$ is the sum of the remaining terms of $\theta(e^x\partial)$ and $e^{ax}\partial^2$ is its maximal term with respect to the order. We have that $ca^2 = 1$. Since θ is an automorphism and $\overline{WN_{1,0,0}}_{[2]}$ is infinite dimensional, we can take $a_1 >> a$, then we have that $ca_1^2 = 1$ where $a_1 >> a$ represents a_1 is a sufficiently larger number than a. These imply that $a^2 = a_1$. This contradiction shows that there is no isomorphism between two algebras. This completes the proof of the proposition. □

Proposition 2. *The semi-Lie algebra* $\overline{WN_{1,0,0}}_{[2]_{[,]}}$ *is not isomorphic to the semi-Lie algebra* $\overline{WN_{1,0,0}}_{[1]_{[,]}}$ *as semi-Lie algebras.*

Proof. Let us assume that there is an isomorphism θ from $\overline{WN_{1,0,0}}_{[1]_{[,]}}$ to $\overline{WN_{1,0,0}}_{[2]_{[,]}}$. There are l_1 and l_2 in $\overline{WN_{1,0,0}}_{[1]_{[,]}}$ such that $\theta(l_1) = e^{ax}\partial^2$ and $\theta(l_2) = e^{-ax}\partial^2$. This implies that $\theta([l_1, l_2]) = [e^{ax}\partial^2, e^{-ax}\partial^2] = 0$. This contradicts the fact that $e^{ax}\partial^2$ and $e^{-ax}\partial^2$ are linearly independent. Thus there is no isomorphism θ from $\overline{WN_{1,0,0}}_{[1]_{[,]}}$ to $\overline{WN_{1,0,0}}_{[2]_{[,]}}$. □

Corollary 3. *The semi-Lie algebra* $\overline{WN_{1,0,0}}_{[3]_{[,]}}$ *contains 3 dimensional Lie subalgebra isomorphic to the Lie algebra* $sl_2(\mathbf{F})$.

Proof. Obviously, the Lie subalgebra the semi-Lie algebra $\overline{WN_{1,0,0}}_{[3]_{[,]}}$ spanned by $\{e^{ax}\partial^3, \partial^3, e^{-ax}\partial^3 |\ a \in \mathbf{Z}\}$ is isomorphic to the Lie algebra $sl_2(\mathbf{F})$. □

Since $\overline{WN_{1,0,0}}_{[2]}$ has a lot of infinite dimensional subalgebras, it is an interesting problem to find its all finite dimensional subalgebras.

Proposition 3. *There is the unique finite dimensional subalgebra of the non-associative algebra* $\overline{WN_{1,0,0}}_{[2]}$ *spanned by* ∂^2.

Proof. Obviously, the subalgebra spanned by ∂^2 is one dimension. Conversely, it is easy to prove that any one dimensional subalgebra of $\overline{WN_{1,0,0}}_{[2]}$ spanned by ∂^2. Let A be a finite dimensional subalgebra of

$\overline{WN_{1,0,0}}_{[2]}$. Let l be a non-zero element of A. We may assume that $l \neq \partial^2$ where $c \in \mathbf{F}^\bullet$. Also, we may assume that l is the sum of more than one basis term of $\overline{WN_{1,0,0}}_{[2]}$. Thus l can be written as follows:

$$l = c_a e^{ax}\partial^2 + \#_1$$

where $\#_1$ is the sum of remaining terms of l and $e^{ax}\partial^2$ is the maximal with respect to its order with appropriate scalars and $a \neq 0$. This implies that the subalgebra A_1 of A generated by l is infinite. This contradicts the fact that A_1 is finite. This shows that $l = c_1\partial^2$ for $c_1 \in \mathbf{F}^\bullet$. thus we have proven the proposition. □

Corollary 4. *The matrix ring $M_n(\mathbf{F})$ is not embedded in the the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$ for $n \leq 2$.*

Proof. The proof of the corollary straightforward by Proposition 2. Let us omit it. □

Corollary 5. *A finite dimensional subalgebra of the semi-Lie algebra $\overline{WN_{1,0,0}}_{[3]_{[,]}}$ has dimension one, two, or three.*

Proof. Since the semi-Lie algebra $\overline{WN_{1,0,0}}_{[3]_{[,]}}$ is self-centralizing, the proof of the corollary is similar to the proof of Proposition 3. Let us omit it. □

Proposition 4. *The non-associative algebra W_2 is isomorphic to the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$. The semi-Lie algebra $W_{2[,]}$ is isomorphic to the semi-Lie algebra $\overline{WN_{1,0,0}}_{[2]_{[,]}}$. The quotient algebra of the semi-Lie algebra $W_{2[,]}/$ of $W_{2[,]}$ by the one-dimensional space $< w_0 >$ spanned by w_0 is simple.*

Proof. Let us define an $\mathbf{F}$-map θ from the non-associative algebra W_2 is isomorphic to the non-associative algebra $\overline{WN_{1,0,0}}_{[2]}$ as follows:

$$\theta(w_a) = e^{ax}\partial^2$$

for any $w_a \in W_2$. Then it is easy to prove that θ is an isomorphism between map. Similarly, the **F**-map θ can be an isomorphism from to the semi-Lie algebra W_2 is isomorphic to the semi-Lie algebra $\overline{WN_{1,0,0}}_{[2]_{[,]}}$. The remaining statement of the proposition is obvious, because $[w_a, w_b] = 0$ if and only if $b = a$ or $b = -a$. This completes the proof of the proposition. □

By Proposition 4, the algebras $\overline{WN_{1,0,0}}_{[2]}$ and $\overline{WN_{1,0,0}}_{[2]_{[,]}}$ have different notations, but in many variables case, still the polynomial notation seems more convenient to handle it.

Open Question. Find all the derivations of $\overline{WN_{n,0,0}}_{[r]}$. □

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