# A Calibration Technique for a Two-Axis Magnetic Compass in Telematics Devices 

$\overline{\text { Seong Yun Cho and Chan Gook Park }}$

This paper presents an efficient algorithm for using the two-axis magnetic compass in portable devices. The general magnetic compass module consists of a three-axis magnetic compass and a two-axis inclinometer to calculate tilt-compensated azimuth information. In this paper, the tilt error is compensated using just a two-axis magnetic compass and two-axis accelerometer. The third-axis data of the magnetic compass is estimated using coordinate information that includes the extended dip angle and tilt information. The extended dip angle is estimated during the normalization process. This algorithm can be used to provide the tilt-compensated heading information to small portable devices such as navigation systems, PDAs, cell phones, and so on.

Keywords: Two-axis magnetic compass, tilt error compensation, extended dip angle.

[^0]
## I. Introduction

Azimuth is the angle between true north and the direction of movement. The azimuth can be measured using the following techniques: gyrocompassing using the gyroscopes in an inertial navigation system [1], [2], GPS application [3], and magnetometry using a magnetic compass [4]-[6]. When gyros are utilized for calculating the azimuth, the initial azimuth must be known. What is worse, an error can be increased because of the gyro bias. The multi-antenna method in a GPS application is suited to a large system such as an airplane, ship, and so on. A stand-alone antenna method can be used only while moving. On the other hand, a magnetic compass has a bounded error and the size of the compass is small. In particular, a magnetic compass can measure the absolute azimuth. Therefore, it is widely used in systems that need azimuth information.
If a magnetic compass is always horizontal to the earth's surface, a two-axis magnetic compass sensor module can calculate the azimuth accurately. Otherwise, a tilt error in the azimuth is generated. Generally, the magnetic compass module consists of three magnetic compass sensors and two inclinometers, all used to compensate the tilt error. The three magnetic compass sensors must be mutually orthogonal in the Cartesian reference frame [5], [6]. However, achieving this accurate orthogonality is difficult, and the sensors could be misaligned because of the vertical sensor. Also, the size of such a sensor module is comparatively large. Therefore, the application of the magnetic compass is limited. A small sensor module needs to be developed for mobile phones, PDAs, and so on, which need the azimuth information.
Recently, an azimuth calculation technique using a two-axis magnetic compass has been investigated [6]. In this technique, coordinate frames are defined using the Earth's dip angle
information. Then, the third-axis data of the magnetic compass is estimated based on the coordinate frames. Finally, the tilt compensated azimuth is calculated. The Earth's dip angle depends on the latitude. The measured value using the magnetic compass is, however, different from the calculated value using the latitude information because the magnetic compass is influenced by the surrounding magnetic field as well as Earth's magnetic field. It is necessary to calculate the accurate dip angle when the azimuth is calculated using the magnetic compass. In this paper, the dip angle with surrounding disturbance is referred to as the extended dip angle (EDA). An EDA searching algorithm is proposed to compensate the tilt error accurately in the two-axis magnetic compass sensor module. The dip angle is estimated simply by tilting the magnetic compass at any direction during the normalization process. After normalization containing the EDA searching process, the tilt compensated azimuth information can be calculated accurately in real-time.
This paper is organized into six sections. In section II, the Earth's magnetic field is briefly described. Then coordinate frames used in this paper are defined, and the relations between the frames are formulated using direction cosine matrices (DCMs). In section III, an azimuth calculation algorithm using a two-axis magnetic compass is introduced. Then, a dip angle searching algorithm is presented in section IV. In section V, the performance of the proposed algorithm is verified by some experiments, and conclusions are drawn in the final section.

## II. Earth's Magnetic Field and Coordinate Frames

The Earth's magnetic field intensity is about 0.5 to 0.6 gauss and can be approximated with the dipole model that points toward magnetic north. The magnetic field is parallel to the Earth's surface. Generally, a compass that is parallel to the Earth's surface measures an azimuth through the direction of the Earth's magnetic field [5]. The magnetic needle of the compass is, however, slant to the Earth except on the equator, and the slant angle changes according to the latitude. This angle is called the dip angle. The direction of the magnetic field does not lean toward the Earth's center. However, the dip angle is generated because the magnetic field is scattered all over the Earth. Figure 1 denotes the dip angle according to the latitude. The dip angle on the equator is zero and that on the magnetic north is 90 degrees. There is a location " $D$ " where the magnetic needle points to location "C". The dip angle changes at a high rate according to the latitude over the "AD" section, and the rate of change of the dip angle is slow over the "DB" section.
A magnetic compass is used for calculating the azimuth by measuring the intensity of the Earth's magnetic field. Several coordinate frames need to be defined when the magnetic compass is used to compute the azimuth. The coordinate


Fig. 1. Dip angle according to the latitude.
frames used in this paper are a body frame, horizontal frame, navigation frame, magnetic navigation frame, and magnetic frame. The body frame ( $X_{m c}, Y_{m c}, Z_{m c}$ ) has its origin at the magnetic compass module, and each axis points along each of the orthogonal sensitive axis of the magnetic compass. The horizontal frame ( $X_{h}, Y_{h}, Z_{h}$ ) is the body frame when the tilt angles are zero. The navigation frame $\left(N_{n}, E_{n}, D_{n}\right)$ is a locallevel frame with its axes pointing true North, East, and down. The magnetic navigation frame ( $N_{m n}, E_{m n}, D_{m n}$ ) is the rotated navigation frame by the declination about the vertical axis. And the magnetic frame ( $N_{m}, E_{m}, D_{m}$ ) is the rotated frame by the dip angle about the east axis of the magnetic navigation frame. The magnetic needle is flat on the $N_{m}-E_{m}$ plane.
Figure 2 shows the relations between the coordinate frames where $\phi, \theta$, and $\psi$ denote roll, pitch, and azimuth angle, respectively, while $\gamma$ and $\lambda$ denote declination and dip angles, respectively. The DCMs between the coordinate frames can be obtained from Fig. 1 as follows [7].

$$
\left[\begin{array}{c}
X_{h}  \tag{1}\\
Y_{h} \\
Z_{h}
\end{array}\right]=C_{b}^{h}\left[\begin{array}{c}
X_{m c} \\
Y_{m c} \\
Z_{m c}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \theta & \sin \theta \sin \phi & \sin \theta \cos \phi \\
0 & \cos \phi & -\sin \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi
\end{array}\right]\left[\begin{array}{c}
X_{m c} \\
Y_{m c} \\
Z_{m c}
\end{array}\right],
$$

$$
\left[\begin{array}{c}
N_{n}  \tag{2}\\
E_{n} \\
D_{n}
\end{array}\right]=C_{h}^{n}\left[\begin{array}{c}
X_{h} \\
Y_{h} \\
Z_{h}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
X_{h} \\
Y_{h} \\
Z_{h}
\end{array}\right],
$$

$$
\left[\begin{array}{l}
N_{m n} \\
E_{m n} \\
D_{m n}
\end{array}\right]=C_{n}^{m n}\left[\begin{array}{l}
N_{n} \\
E_{n} \\
D_{n}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \gamma & -\sin \gamma & 0 \\
\sin \gamma & \cos \gamma & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
N_{n} \\
E_{n} \\
D_{n}
\end{array}\right]
$$

$$
=C_{h}^{m n}\left[\begin{array}{c}
X_{h}  \tag{3}\\
Y_{h} \\
Z_{h}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \tilde{\psi} & -\sin \tilde{\psi} & 0 \\
\sin \widetilde{\psi} & \cos \widetilde{\psi} & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
X_{h} \\
Y_{h} \\
Z_{h}
\end{array}\right],
$$

$$
\left[\begin{array}{l}
N_{m}  \tag{4}\\
E_{m} \\
D_{m}
\end{array}\right]=C_{m n}^{m}\left[\begin{array}{l}
N_{m n} \\
E_{m n} \\
D_{m n}
\end{array}\right]=\left[\begin{array}{ccc}
\cos \lambda & 0 & \sin \lambda \\
0 & 1 & 0 \\
-\sin \lambda & 0 & \cos \lambda
\end{array}\right]\left[\begin{array}{l}
N_{m n} \\
E_{m n} \\
D_{m n}
\end{array}\right]
$$


(a) Body frame and horizontal frame

(b) Horizontal frame, navigation frame and magnetic navigation frame

(c) Navigation frame and magnetic frame

Fig. 2. Rotations between the coordinate frames.

## III. Algorithm for Azimuth Calculation

The output of the magnetic compass must be normalized
before applying the algorithm. At first, the outputs of the magnetic compass are measured during the compass turns on the z -axis on a horizontal plane. Then, the maximum and minimum values of the magnetic compass data are obtained. The normalization process considering the dip angle is performed with the following equation:

$$
\begin{gather*}
\bar{X}_{m c}=\left(X_{m c}-\text { Bias }_{x}\right) S F_{x},  \tag{5}\\
\text { Bias }_{x}=\frac{X_{\max }+X_{\min }}{2}, \text { and }  \tag{6}\\
S F_{x}=\frac{2 \cos \lambda}{X_{\max }-X_{\min }} \tag{7}
\end{gather*}
$$

where $X$ is the x-axis, $X_{m c}$ is the output of the magnetic compass, $\bar{X}_{m c}$ is the normalized x-axis magnetic compass, and $X_{\max }$ and $X_{\min }$ are the maximum and minimum values of the magnetic compass on a horizontal plane, respectively. Bias $_{x}$ denotes the bias of the x-axis magnetic compass and $S F_{x}$ the scale factor. The normalization of the $y$-axis is achieved by the same process as for the x -axis.
The output of the magnetic compass in the magnetic frame is normalized using (5) at any location as

$$
\left[\begin{array}{lll}
N_{m} & E_{m} & D_{m}
\end{array}\right]^{T}=\left[\begin{array}{lll}
1 & 0 & 0 \tag{8}
\end{array}\right]^{T} .
$$

With this prerequisite, we are able to state the main result of this section.
It is necessary to estimate the z -axis magnetic compass data in order to calculate the azimuth information when a two-axis magnetic compass is used. For normalized compass output, $\bar{X}_{m c}^{2}+\bar{Y}_{m c}^{2}+\bar{Z}_{m c}^{2}=1$, and $\bar{Z}_{m c}$ can be written as

$$
\begin{equation*}
\bar{Z}_{m c}= \pm \sqrt{1-\left(\bar{X}_{m c}^{2}+\bar{Y}_{m c}^{2}\right)} \tag{9}
\end{equation*}
$$

The sign of the solution cannot be determined with the proposed sensor set. This is the limitation of the conventional idea. The following Theorem 1 is proposed to overcome this limitation.

Theorem 1. Consider the normalized output of the magnetic compass and DCMs as stated in (1) through (4), where it is assumed that the dip angle and tilt information are known. Then, the third-axis measuring information of the two-axis magnetic compass is estimated as

$$
\begin{equation*}
\hat{\bar{Z}}_{m c}=\frac{\sin \lambda+\bar{X}_{m c} \sin \theta-\bar{Y}_{m c} \cos \theta \sin \phi}{\cos \theta \cos \phi} . \tag{10}
\end{equation*}
$$

Proof. For the proof of Theorem 1, the equation is formalized using the relations between the coordinate frames. The relation between the body frame and the magnetic frame can be made as

$$
\left[\begin{array}{c}
N_{m}  \tag{11}\\
E_{m} \\
D_{m}
\end{array}\right]=C_{b}^{m}\left[\begin{array}{c}
X_{m c} \\
Y_{m c} \\
Z_{m c}
\end{array}\right]=C_{m n}^{m} C_{h}^{m n} C_{b}^{h}\left[\begin{array}{c}
X_{m c} \\
Y_{m c} \\
Z_{m c}
\end{array}\right]
$$

Multiplying the inverse matrix of $C_{m n}^{m}$ to both sides of (11) yields

$$
\left(C_{m n}^{m}\right)^{-1}\left[\begin{array}{c}
N_{m}  \tag{12}\\
E_{m} \\
D_{m}
\end{array}\right]=C_{h}^{n} C_{b}^{h}\left[\begin{array}{c}
X_{m c} \\
Y_{m c} \\
Z_{m c}
\end{array}\right] .
$$

Substituting (1) through (4) into (12) gives

$$
\begin{align*}
& {\left[\begin{array}{ccc}
\cos \lambda & 0 & -\sin \lambda \\
0 & 1 & 0 \\
\sin \lambda & 0 & \cos \lambda
\end{array}\right]\left[\begin{array}{l}
N_{m} \\
E_{m} \\
D_{m}
\end{array}\right]} \\
& =\left[\begin{array}{ccc}
x_{1} \cos \theta & x_{1} \sin \theta \sin \phi-y_{1} \cos \phi & x_{1} \sin \theta \cos \phi+y_{1} \sin \phi \\
y_{1} \cos \theta & y_{1} \sin \theta \sin \phi+x_{1} \cos \phi & y_{1} \sin \theta \cos \phi-x_{1} \sin \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi
\end{array}\right] \\
& \quad \times\left[\begin{array}{l}
X_{m c} \\
Y_{m c} \\
Z_{m c}
\end{array}\right], \tag{13}
\end{align*}
$$

where

$$
\begin{equation*}
x_{1}=\cos \tilde{\psi}, y_{1}=\sin \tilde{\psi} \tag{14}
\end{equation*}
$$

The lower row vector of (13) can be extracted as

$$
\begin{align*}
& -X_{m c} \sin \theta+Y_{m c} \cos \theta \sin \phi+Z_{m c} \cos \theta \cos \phi \\
& =N_{m} \sin \lambda+D_{m} \cos \lambda \tag{15}
\end{align*}
$$

After normalization, this equation can be denoted as follows:

$$
\begin{equation*}
-\bar{X}_{m c} \sin \theta+\bar{Y}_{m c} \cos \theta \sin \phi+\bar{Z}_{m c} \cos \theta \cos \phi=\sin \lambda \tag{16}
\end{equation*}
$$

Because (16) does not have any information about azimuth, the $z$ axis data of the magnetic compass can be estimated as given in (10). Therefore, we can estimate the third-axis measuring information of the magnetic compass using a two-axis magnetic compass if the dip angle and tilt information are known. This result is not needed to solve the sign, unlike in (9).

The tilt information can be computed by using an
inclinometer. Nowadays, an accelerometer is widely adopted as an inclinometer. Using the two-axis accelerometer, the tilt angles of the magnetic compass at rest can be calculated as

$$
\begin{gather*}
\theta=\sin ^{-1}\left(\frac{a_{x}}{g}\right)  \tag{17a}\\
\phi=\sin ^{-1}\left(\frac{-a_{y}}{g \cos \theta}\right) \tag{17b}
\end{gather*}
$$

where $a_{\mathrm{y}}$ and $a_{\mathrm{y}}$ denote the outputs of the x -axis and the y -axis accelerometers, respectively, and $g$ indicates the gravity acceleration.

In (17), it is assumed that the tilt angle is smaller than 180 degrees.

When a two-axis magnetic compass is used, the magnetic compass must be at rest on the horizontal surface. This is the limitation of the conventional idea. The following Theorem 2 shows that a two-axis magnetic compass can calculate the azimuth angle accurately even if the tilt angle is not 0 .

Theorem 2. Consider the third-axis measuring information of the two-axis magnetic compass and the coordinate transformation equation (13). It is assumed that the declination angle, the dip angle, and the tilt information are known. Then, the azimuth calculation equation with a two-axis magnetic compass is as follows:

$$
\begin{equation*}
\psi=\tan ^{-1}\left(\frac{-\bar{Y}_{m c} \cos \phi+\hat{\bar{Z}}_{m c} \sin \phi}{\bar{X}_{m c} \cos \theta+\bar{Y}_{m c} \sin \theta \sin \phi+\hat{Z}_{m c} \sin \theta \cos \phi}\right)-\gamma \tag{18}
\end{equation*}
$$

Proof. For the proof of Theorem 2, the upper-two row vectors of (13) can be divided as

$$
\begin{gather*}
{\left[\begin{array}{ll}
x_{1} \cos \theta & x_{1} \sin \theta \sin \phi-y_{1} \cos \phi \\
y_{1} \cos \theta & x_{1} \sin \theta \cos \phi+y_{1} \sin \phi \\
y_{1} \sin \theta \sin \phi+x_{1} \cos \phi & y_{1} \sin \theta \cos \phi-x_{1} \sin \phi
\end{array}\right]} \\
 \tag{19}\\
\times\left[\begin{array}{c}
X_{m c} \\
Y_{m c} \\
Z_{m c}
\end{array}\right]=\left[\begin{array}{c}
N_{m} \cos \lambda-D_{m} \sin \lambda \\
E_{m}
\end{array}\right] .
\end{gather*}
$$

This equation can be rearranged to calculate the variables $x_{1}$ and $y_{1}$ as

$$
\left[\begin{array}{cc}
a & b  \tag{20}\\
-b & a
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
y_{1}
\end{array}\right]=\left[\begin{array}{c}
N_{m} \cos \lambda-D_{m} \sin \lambda \\
E_{m}
\end{array}\right]
$$

where

$$
\begin{gather*}
a=X_{m c} \cos \theta+Y_{m c} \sin \theta \sin \phi+Z_{m c} \sin \theta \cos \phi  \tag{21a}\\
b=-Y_{m c} \cos \phi+Z_{m c} \sin \phi \tag{21b}
\end{gather*}
$$

Note that $a$ and $b$ are equivalent to $X_{h}$ and $-Y_{h}$, respectively, in (1).

From (19), $x_{1}$ and $y_{1}$ can be obtained as

$$
\left[\begin{array}{l}
x_{1}  \tag{22}\\
y_{1}
\end{array}\right]=\left[\begin{array}{l}
\left(a N_{m} \cos \lambda-a D_{m} \sin \lambda-b E_{m}\right) /\left(a^{2}+b^{2}\right) \\
\left(b N_{m} \cos \lambda-b D_{m} \sin \lambda+a E_{m}\right) /\left(a^{2}+b^{2}\right)
\end{array}\right] .
$$

Therefore, the azimuth can be calculated as

$$
\begin{equation*}
\psi=\tan ^{-1}\left(\frac{y_{1}}{x_{1}}\right)=\tan ^{-1}\left(\frac{b N_{m} \cos \lambda-b D_{m} \sin \lambda+a E_{m}}{a N_{m} \cos \lambda-a D_{m} \sin \lambda-b E_{m}}\right) . \tag{23}
\end{equation*}
$$

After, normalization (23) can be denoted as,

$$
\begin{align*}
\psi & =\tan ^{-1}\left(\frac{b}{a}\right) \\
& =\tan ^{-1}\left(\frac{-\bar{Y}_{m c} \cos \phi+\hat{\bar{Z}}_{m c} \sin \phi}{\bar{X}_{m c} \cos \theta+\bar{Y}_{m c} \sin \theta \sin \phi+\hat{\bar{Z}}_{m c} \sin \theta \cos \phi}\right) . \tag{24}
\end{align*}
$$

Therefore, we can calculate the azimuth information using a two-axis magnetic compass if the dip angle and the tilt information are known. However, the azimuth estimated by (24) has an error by the declination angle as can be seen in Fig. 2(b). The declination angle can be compensated. First, the declination angle information can be downloaded from [8] and can be used in a wide area without change. Second, the inclination angle and the various error angles can be estimated using an in-motion alignment filter when the magnetic compass is utilized in navigation systems [9]. Therefore, the azimuth can be calculated.
In this section, it is proved that the tilt-compensated azimuth can be calculated using a two-axis magnetic compass and a two-axis accelerometer if the dip angle is known.

## IV. Algorithm for Extended Dip Angle Estimation

The dip angle is necessary to calculate the tilt-compensated azimuth when a two-axis magnetic compass is utilized. The dip angle varies according to the latitude and is fixed to the location. Usually, the dip angle is calculated using the latitude information as

$$
\begin{equation*}
\lambda=\tan ^{-1}(2 \tan L) \tag{25}
\end{equation*}
$$

where, $L$ denotes the latitude.

The measured dip angle is, however, different from the known value because of the surrounding magnetic field of the magnetic compass. The following Theorem 3 is proposed to estimate the dip angle distorted by the surrounding magnetism.
Theorem 3. Consider Theorem 1 and Theorem 2. If either the roll angle or the pitch angle is not zero, and the azimuth information is known, the EDA is then calculated as follows:

$$
\begin{equation*}
\lambda_{e}=\tan ^{-1}\left(\frac{\hat{X}_{m c}(\sin \theta \sin \phi-\cos \phi \tan \psi)-\hat{Y}_{m c} \cos \theta}{\sin \theta \cos \phi \tan \psi-\sin \phi}\right), \tag{26}
\end{equation*}
$$

where

$$
\begin{equation*}
\hat{X}_{m c}=\left(X_{m c}-\operatorname{Bias}_{x}\right) \overline{S F}_{x} \tag{27}
\end{equation*}
$$

$$
\begin{equation*}
\overline{S F}_{x}=\frac{2}{X_{\max }-X_{\min }} . \tag{28}
\end{equation*}
$$

Proof. Inserting (10) into (18) yields

$$
\begin{equation*}
\psi=\tan ^{-1}\left(\frac{\sin \lambda_{e} \sin \phi+\bar{X}_{m c} \sin \theta \sin \phi-\bar{Y}_{m c} \cos \theta}{\sin \lambda_{e} \sin \theta \cos \phi+\bar{X}_{m c} \cos \phi}\right), \tag{29}
\end{equation*}
$$

where $\lambda_{e}$ is the EDA. The normalization cannot be accomplished when the EDA is unknown. During normalization through rotating the magnetic compass on the horizontal plane, (5) can be represented as follows:

$$
\begin{equation*}
\bar{X}_{m c}=\left(X_{m c}-\operatorname{Bias}_{x}\right) \overline{S F}_{x} \cos \lambda_{e}=\hat{X}_{m c} \cos \lambda_{e} . \tag{30}
\end{equation*}
$$

Substituting (30) into (29) gives

$$
\begin{equation*}
\tan \psi=\frac{x_{2}\left(\hat{X}_{m c} \sin \theta \sin \phi-\hat{Y}_{m c} \cos \theta\right)+y_{2} \sin \phi}{x_{2} \hat{X}_{m c} \cos \phi+y_{2} \sin \theta \cos \phi} \tag{31}
\end{equation*}
$$

where

$$
\begin{align*}
& x_{2}=\cos \lambda_{e}  \tag{32a}\\
& y_{2}=\sin \lambda_{e} . \tag{32b}
\end{align*}
$$

Equation (31) can be rearranged as

$$
\begin{align*}
& y_{2}(\sin \theta \cos \phi \tan \psi-\sin \phi) \\
& \quad=x_{2}\left\{\hat{X}_{m c}(\sin \theta \sin \phi-\cos \phi \tan \psi)-\hat{Y}_{m c} \cos \theta\right\} . \tag{33}
\end{align*}
$$

Therefore, the EDA can be calculated using (32) and (33) as follows:

$$
\begin{align*}
\lambda_{e} & =\tan ^{-1}\left(\frac{y_{2}}{x_{2}}\right) \\
& =\tan ^{-1}\left(\frac{\hat{X}_{m c}(\sin \theta \sin \phi-\cos \phi \tan \psi)-\hat{Y}_{m c} \cos \theta}{\sin \theta \cos \phi \tan \psi-\sin \phi}\right) . \tag{34}
\end{align*}
$$

In (34), there are two problems. First, the denominator may be zero if both tilt angles are zero. Second, the azimuth information is necessary. The two problems are solved as follows: The twoaxis magnetic compass can calculate the azimuth information accurately on the horizontal plane after normalization without dip angle. Then, the magnetic compass is rotated along the x -axis or $y$-axis for any fixed azimuth to avoid the singular problem. Therefore, the EDA is calculated using (26).

## V. Experimental Results

An experiment was carried out to verify the performance of the proposed algorithm. At first, an experimental magnetic compass module is implemented as shown in Fig. 3. In this module, the extra $z$-axis magnetic compass is used to compare with the estimated third-axis measuring information. The biaxial magnetic compass and the biaxial accelerometer used in this experiment are TMC2000 (Tokin) and ADXL202E (analog devices), respectively. The magnetic compass is a fluxgate type detector to detect terrestrial magnetism.
The EDA at the experimental position is about 48.72 degrees. This value is calculated by seeking the pitch angle that maximizes the x -axis output of the magnetic compass. Figures 4 and 5 show the results of the EDA estimation. When the EDA is calculated using (26), the tilt angles must not be zero degrees. It becomes apparent from the diagram in Fig. (5) that the error of the calculated EDA is decreased with increasing the tilt angle. In this paper, the calculated EDA is adopted when the tilt angle is over 30 degrees. The result of the EDA estimation is presented in Fig. 5 and Table 1. From these results, it can be seen that the calculated EDA has a small error of less than 1 degree.

A second experiment was carried out. The pitch angle is fixed on 30 degrees. Then the magnetic compass module is rotated along the $z$-axis. Figure 6 shows the azimuth error when the tilt error is not compensated. As can be seen in this figure, the azimuth information cannot be used unless the tilt error is compensated. Figures 7 and 8 show the performance of the proposed algorithm. At first, the dip angle is calculated using (25). The calculated dip angle is $56.91^{\circ}$ because the latitude of Seoul is about $37.5^{\circ}$. The result is denoted in Fig. 7. Then, the dip angle is estimated by the proposed EDA


Fig. 3. Block diagram of the experimental magnetic compass module.


Fig. 4. EDA error according to pitch angle.
searching algorithm. The result is shown in Fig. 7. In this experiment, the z -axis magnetic compass was additionally equipped as in Fig. 3. Figures 7(a) and 8(a) show the experimental $z$-axis data, the estimated $z$-axis data, and the estimation error. As can be seen in these figures, the dip angle calculated by using the latitude information makes the inaccurate z -axis data of the magnetic compass. This error also makes the azimuth error as can be seen in Fig. 7(b). However, the EDA estimated by the proposed algorithm makes the small azimuth error as can be seen in Fig. 8(b).

## VI. Conclusion

In this paper, an efficient error compensation algorithm for a two-axis magnetic compass is proposed. The third-axis data of the magnetic compass is estimated using the extended dip angle and the tilt information. And the extended dip angle is estimated simply by tilting the magnetic compass module during the normalization process. The validity of the proposed algorithm is analyzed by some experiments. In these


Fig. 5. Results of the EDA estimation.

Table 1. Estimation results of the EDA.

| Figure | Condition | Result |
| :---: | :---: | :---: |
| $5(\mathrm{a})$ | Azimuth $=45^{\circ}$ | $49.01^{\circ}$ |
|  | Roll angle $=0$, Pitch angle change |  |
| $5(\mathrm{~b})$ | Azimuth $=135^{\circ}$ | $49.22^{\circ}$ |
|  | Roll angle $=0$, Pitch angle change |  |
| $5(\mathrm{c})$ | Azimuth $=225^{\circ}$ | $47.64^{\circ}$ |
|  | Roll angle $=0$, Pitch angle change | $49.85^{\circ}$ |
| $5(\mathrm{~d})$ | Azimuth $=315^{\circ}$ |  |
|  | Roll angle $=0$, Pitch angle change | $49.77^{\circ}$ |
| $5(\mathrm{e})$ | Azimuth $=45^{\circ}$ |  |
|  | Pitch, Roll angle change | $1.09^{\circ}$ |
|  | Azimuth $=315^{\circ}$ | $48.72^{\circ}$ |

$\qquad$


Fig. 6. Azimuth error when tilt error is not compensated.
experiments, it is confirmed that the proposed EDA estimation algorithm estimates the EDA accurately and easily. Moreover, the azimuth is calculated without tilt error. It is expected that the proposed algorithm will be useful in the systems that need a small-size magnetic compass module such as a portable navigation system, mobile phone, PDA, virtual reality system, and so on.


Fig. 7. Dip angle is calculated using the latitude.


Fig. 8. Dip angle is calculated by the proposed algorithm.

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Seong Yun Cho was born in Jinju, Korea, in 1974. He received the $\mathrm{BS}, \mathrm{MS}$, and PhD degrees in Department of Control and Instrumentation Engineering from Kwangwoon University, Korea in 1998, 2000, and 2004. In 2003, he was a research assistant with the Automation and System Research Institute (ASRI), Seoul National University, Korea where he worked on the development of a tilt compensation algorithm for a magnetic compass with Samsung Advanced Institute of Technology (SAIT). In 2004, he was with the School of Mechanical and Aerospace Engineering, Seoul National University, Korea, where he was a postdoctoral fellow, BK21. Since 2004, he has been with Electronics and Telecommunications Research Institute (ETRI), Korea, as a member of Senior Research Staff. His research interests include the development of navigation systems (INS, INS/GPS, PNS), filter design for nonlinear systems, and development of telematics application systems.


Chan Gook Park was born in Seoul, Korea, in 1961. He received the $\mathrm{BS}, \mathrm{MS}$, and PhD degrees in control and instrumentation engineering from the Seoul National University, Korea, in 1985, 1987, and 2003. He worked as a postdoctoral fellow at the Engineering Research Center for Advanced Control and Instrumentation, Seoul National University, in 1993. From 1994 to 2003, he was with the Kwangwoon University, Korea, as an Associate Professor. In 2003, he joined the faculty of the School of Mechanical and Aerospace Engineering at Seoul National University, Korea, where he is currently an Associate Professor. In 1998, he worked with Professor Jason L. Speyer on peak seeking control for formation flight at University of California, Los Angeles (UCLA), as a visiting scholar. His research interests include filtering techniques, inertial navigation systems, GPS/INS integration, and personal navigation systems.


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    Seong Yun Cho (phone: +82 42869 1636, email: sycho@etri.re.kr) is with Telematics \& USN Research Division, ETRI, Daejeon, Korea.
    Chan Gook Park (email: chanpark@snu.ac.kr) is with the School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, Korea.

