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Development of Thermal Error Model with Minimum Number of Variables Using Fuzzy Logic Strategy

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Thermally-induced errors originating from machine tool errors have received significant attention recently because high speed and precise machining is now the principal trend in manufacturing processes using CNC machine tools. Since the thermal error model is generally a function of temperature, the thermal error compensation system contains temperature sensors with the same number of temperature variables. The minimization of the number of variables in the thermal error model can affect the economical efficiency and the possibility of unexpected sensor fault in a error compensation system. This paper presents a thermal error model with minimum number of variables using a fuzzy logic strategy. The proposed method using a fuzzy logic strategy does not require any information about the characteristics of the plant contrary to numerical analysis techniques, but the developed thermal error model guarantees good prediction performance. The proposed modeling method can also be applied to any type of CNC machine tool if a combination of the possible input variables is determined because the error model parameters are only calculated mathematically based on the number of temperature variables.

Key Words : Thermal Error Model, Temperature Variable, Model Performance, Fuzzy Logic, CNC Machine Tool

1. Introduction

The recent trend in manufacturing processes is demanding high spindle speeds and precision machining to improve productivity and product quality. This high speed operation of machine tools induces thermal errors, which are known to be key contributors to machine tool errors (Bryan, 1990).

Error compensation systems have received wide attention in relation to their ability to cost effectively improve machine accuracy. However, the effectiveness of an error compensation system relies on the prediction accuracy of the error model, which in turn is affected by the modeling method applied, appropriate selection of input variables, etc.

There are two main research areas related to thermal error modeling. The first area is numerical analysis techniques, such as the finite element method (Weck and Zangs, 1975 ; Venugopal and Barash, 1986) and finite difference method (Moriwaki, 1988). Yet, these techniques are restricted to a qualitative analysis of machine thermal behavior because the boundary conditions and heat transfer characteristics can not be clearly identified (Bryan, 1990).

The second area is empirical modeling based on the measurement of thermal errors and temperatures at several representative points on the machine tools. Examples include engineering judgement (Yang et al., 1996), regression analysis (Fan et al., 1992 ; Soons et al., 1994), and neural networks (Hatamura, 1993 ; Yang et al., 1996). Empirical models have been demonstrated to show satisfactory prediction accuracy in many

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applications even though each method has its own inherent shortcomings.

Although various empirical modeling methods have been developed for identifying thermal errors in CNC machine tools, scant attention has been paid to the application of a fuzzy logic strategy. Moreover, an error compensation system contains temperature sensors with the same number of temperature variables because the thermal error model is generally a function of temperature. The minimization of the number of variables in a thermal error model can affect the economical efficiency and the possibility of unexpected sensor fault in an compensation system. Accordingly, this study uses a fuzzy logic strategy to develop a thermal error model with minimum number of variables for a CNC machine tool. Of course, the prediction performance of the error model must be guaranteed even though the number of variables is minimized.

Fuzzy logic originates from the research of Zadeh (1965), who introduced a possibility model based on the analysis of a fuzzy set. Thereafter, Tanaka *et al.* (1982) proposed a linear regression analysis using the concept of possibility and further developed the possibility model.

In the conventional regression model, the difference between observed data and values predicted by the model is regarded as the observation error, whereas, in a fuzzy linear regression analysis based on the possibility model, this difference is regarded as the fuzziness of the system itself. As such, the fuzzy linear regression analysis is the formulation for the possibility distribution of the inferred output from the viewpoint of possibility and the membership function can be regarded as the possibility distribution.

The fuzzy implication used in this study is quite simple as it is based on a fuzzy partition of the input space. In each fuzzy subspace, a linear input-output relation is formed. The output of the fuzzy reasoning is given by the aggregation of the values inferred by certain implications applied to the input.

In contrast to a numerical analysis method, the proposed method using a fuzzy logic strategy does not require any detailed information about the characteristics of the plant. The thermal error modeling method proposed in this study can be widely applied to any type of CNC machine tool if a combination of the possible input variables is determined because the parameters of the error model are only calculated mathematically based on the number of temperature variables.

2. Fuzzy Implication and Reasoning

In this study, the membership function of a fuzzy set A is denoted as $A(x)$, $x \in X$. The truth value of a proposition "x is A and y is B" is expressed by

$$|x \text{ is A and } y \text{ is B}| = A(x) \wedge B(y)$$

Takagi and Sugeno (1983) has proposed the fuzzy implication $L_i (i=1, \cdots n)$ for a multi input single output system (MISO) with input variables $x_1, x_2, \cdots, x_k$, and output variable y.

input	$x_1 = x_1^0$ and $x_2 = x_2^0$ and $\cdots$ and $x_k = x_k^0$
implication L_1	If $x_1 = A_{11}$ and $\cdots$ and $x_k = A_{1k}$, then $y = g_1(x_1, \cdots, x_k)$
implication L_2	If $x_1 = A_{21}$ and $\cdots$ and $x_k = A_{2k}$, then $y = g_2(x_1, \cdots, x_k)$
·	· · ·
·	· · ·
·	· · ·
implication L_n	If $x_1 = A_{n1}$ and $\cdots$ and $x_k = A_{nk}$, then $y = g_n(x_1, \cdots, x_k)$
output	$y = y^*$

The inferred value y_i^* by the i-th implication is defined as

$$y_i^* = g_i(x_1^0, x_2^0, \cdots, x_k^0) \tag{1}$$

The truth value ω_i of the proposition $y_i^* = y^*$ is calculated by the equation

$$\omega_i = A_{i1}(x_1^0) \wedge \cdots \wedge A_{ik}(x_k^0) \tag{2}$$

The final output y^* inferred from n implications is given as weighted combination.

$$y^* = \frac{\sum_{i=1}^{n} \omega_i y_i^*}{\sum_{i=1}^{n} \omega_i} \tag{3}$$

In the implication by Takagi and Sugeno (1983), "and" connectives and linguistic expressions are used in the premise and the consequence is general function relation. If the function g_i in the consequence has the format of a linear function, the implication is written as

$$L_i: \text{If } x_1 \text{ is } A_{i1} \text{ and } \cdots \text{ and } x_k \text{ is } A_{ik} \quad \text{then } y = p_{i0} + p_{i1}x_1 + \cdots + p_{ik}x_k \tag{4}$$

3. Experiment

The experimental apparatus was composed of a machining center and sensing units, as shown in Fig. 1. The sensing units were thermal sensors and a displacement sensor. Thermocouples were used as thermal sensors to detect the variation of temperature in the machine structure and a capacitance sensor was used as the displacement sensor to detect spindle drift errors on the z-axis.

Fourteen thermocouples were initially mounted on the machine to detect the temperature field, as shown in Fig. 1. Sensors T1 and T2 provided temperature readings on the nut and leadscrew bearing of the x-axis, sensors T3 and T4 on the nut and leadscrew bearing of the y-axis, sensors T5 and T6 on the nut and leadscrew bearing of the z-axis, sensors T7 and T8 on the spindle, sensors T9, T10, T11, and T12 on the column, and sensors T13 and T14 on the x-axis bed and z-axis bed, respectively. A non-contacting displacement sensor was also settled on the z-axis bed.

To investigate the thermal behavior of this machining center, tests were performed under the following conditions.

- The running conditions were divided into 3 classes to consider the actual environment in the industrial shop : stop(spindle 0 rpm, feedrate 0 mm/min), low speed(spindle 600 rpm, feedrate 508 mm/min), and high speed (spindle 3000 rpm, feedrate 2006 mm/min)
- The running conditions were arbitrarily combined to remove the effect of an experimental sequence. The temperature field and spindle drift error were recorded every 30 minutes by moving the X, Y, Z tables in a body -diagonal direction.

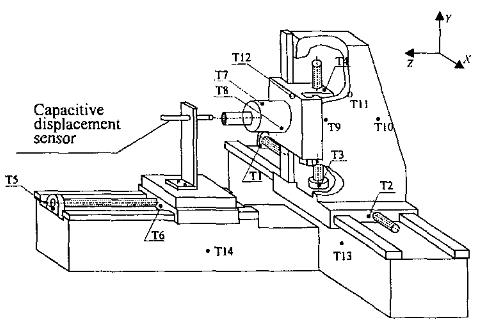

Fig. 1 Experimental apparatus with machining center and sensing units

4. Modeling and Discussion

In the proposed thermal error modeling, the temperature difference ΔT from a reference temperature is assigned as the variable of the thermal error model. Since the temperature measurement from the sensor T14 fluctuated the least, this temperature measurement was used as the reference temperature.

Figure 2 represents the proposed modeling algorithm and the following gives a detailed description of each step. For a fuzzy model consisting of a certain number of implications that are of the format in Eq. (4), the following three items must be determined by the input-output data of an objective system.

4.1 Choice of premise structure

There are two problems concerned with the

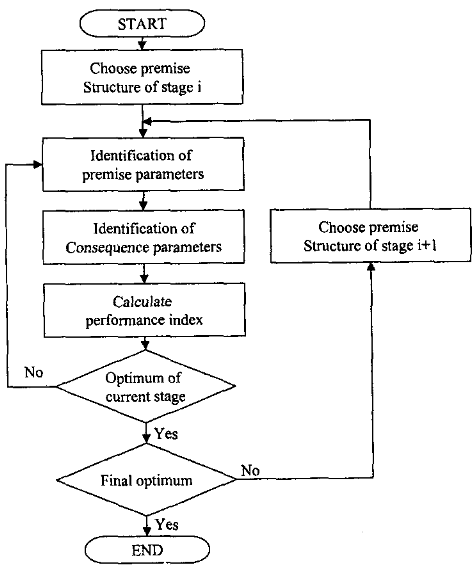

Fig. 2 Flowchart of modeling algorithm

choice of premise structure. The first is the choice of a combination of premise variables for possible input variables. The second is the choice of a variable for dividing the space and the number of divisions.

With the first problem, the choice of a combination of input variables is a key difficulty for the thermal error modeling. As a solution of this problem, a statistical optimization method was used in the previous study (Hwang et al., 1999). In general, a statistical regression model shows a tendency increasing the prediction accuracy if the number of variables for the model is increased. Yet, the number of cases requiring consideration will be too high if all the variables are used for the modeling, furthermore, a disadvantage will arise from a viewpoint of the number of sensors. Even in practical aspect, it is more beneficial to develop a thermal error compensation system with fewer temperature sensors.

Since the main purpose of this study is the development of a thermal error model with minimum number of variables, a result of the statistical optimization method (Hwang et al., 1999) is adopted as a starting model for reducing the number of variables. From the starting model, the variable with less contribution to error prediction will be eliminated step by step. The number of variables by the statistical optimization method was four, so the prediction accuracy will be decreased in a general regression model if the number of variables is less than four. In this study, the fuzzy logic strategy will complement the reduction effect of the number of variables.

With the second problem of choosing a variable for dividing the space and the number of divisions, there would seem to be no theoretical approach currently available. Therefore, a heuristic search method (Takagi and Sugeno, 1985) is applied with a performance index to judge the best partition of variables. The performance index is defined as in the following Eq. (5) (Horikawa et al., 1992).

$$E=\sqrt{(E_A+E_B)}+UC \tag{5}$$

$$E_A=\Sigma(y_A-\hat{y}_A)^2,\ E_B=\Sigma(y_B-\hat{y}_B)^2$$

$$UC=\sqrt{\Sigma(y_A-\hat{y}_{AB})^2+\Sigma(y_B-\hat{y}_{BA})^2}$$

where, UC (unbiasedness criterion) (Ivakhnenko, 1971) represents the generality of the model. To calculate the UC value, the output data of the objective system are divided into two groups. The data scattering within the two groups is almost the same. Therefore, since the maximum to minimum range of both groups is nearly the same, a data group must not be composed of data from only one specified operating condition i. e. high speed operation or low speed operation. The measured data are thus divided into group A and group B using the average and variance. In the expression of UC, $\hat{y}_{AB}$ means the estimated value of group A based on the model using group B data and vice versa for the $\hat{y}_{BA}$.

The heuristic search method proceeds in the following steps. Suppose that a fuzzy model of k-inputs x_1, x_2, $\cdots$, x_k and single-output system is built.

STEP 1 : The range of any one variable is divided into two fuzzy subspaces "big" and "small", whereas the ranges of the other variables are not divided, which means that only the variable with a divided subspace appears in the premises of the implications.

STEP 2 : For the model established in STEP 1, the optimal premise parameters and consequence parameters are identified using the performance index. Once the optimal model with the least performance index is determined from the k-models, the variable in this optimal model is divided into more than two subspaces. The calculation of the parameters and the performance index is then repeated. When a final optimal model is obtained, this is called a stable state.

STEP 3 : Starting from the stable state at STEP 2, where only the variable x_i appears in the premises, take all the combinations of x_i-x_j ($j=1, 2\ \cdots, k$) and divide the range of variable x_j into fuzzy subspaces step by step.

STEP 4 : Repeat STEP 2 and STEP 3 until a

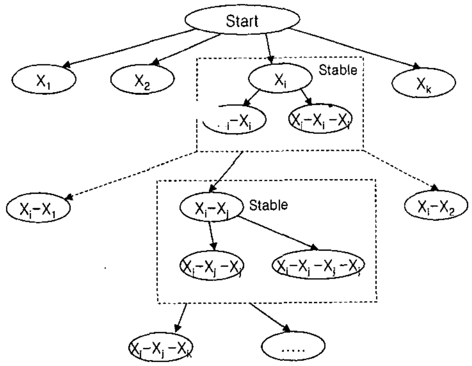

Fig. 3 Schematic diagram of heuristic search method for the best partition of premise variables

final search solution is obtained.

A schematic diagram of the heuristic search method for the best partition of premise variables is represented in Fig. 3.

4.2 Premise and consequence parameters identification

This step identifies the optimal premise parameters for the premise variables chosen at Sec. 4.1. The identification of premise parameters is for determining the membership functions. The temperature variation of a CNC machine tool in this study has an almost exponential tendency in relation to machine tool operation. As such, a bell-type membership function is used for the thermal error modeling in this study. The mathematical expression of a bell-type membership function is shown in Eq. (6).

$$A(x) = \frac{1}{\left(1+\left|\frac{x-c}{a}\right|\right)^{2b}} \quad (6)$$

where, constant c represents the symmetry point, constant a is the distance from the symmetry point to the point having function value of 0.5, and the exponent b determines the function shape.

Based on assuming the values of the premise parameters, the optimal consequence parameters can be obtained together with the performance index. As a result, the problem of identifying the optimal premise parameters can be reduced to a nonlinear programming problem of minimizing the performance index. In this study, backpropagation gradient decent method is used for the optimal solution.

The consequence parameters that produce the lowest performance index are determined using the least square method for the premise variables given in section 4. 1 and premise parameters as above.

4.3 Application to experimental data

The thermal error model by a statistical optimization method for the CNC machine tool used in this study is composed of four variables, ΔT_2 ,ΔT_6, ΔT_7, ΔT_8. Based on the contribution analysis of the variables to error prediction performance (Lee and Yang, 2001), two cases are investigated. The first case has variables ΔT_2, ΔT_7, ΔT_8 as the premise variables (CASE I). This case has the same variables as a previous study (Lee et al., 2000) using a backward elimination method. The second case has variables ΔT_2, ΔT_7 as the premise variables (CASE II).

Table 1 is a summary of the fuzzy partition results using the heuristic search method. Based on the variation of performance index E, the optimal model with the minimum performance index is attained when the variable ΔT_7 is divided into two subspaces. Figure 4 plots the membership function using the premise parameters of the optimal model. Using this premise structure, the consequence parameters are then optimized by the least square method. The final fuzzy model for thermal error is shown in Eq. (7).

If ΔT_7 is small,
then $\delta=-7.11+2.33*\Delta T_2+1.80*\Delta T_7-39.47*\Delta T_8$
If ΔT_7 is big, (7)
then $\delta=11.56-8.51*\Delta T_2+7.31*\Delta T_7+72.09*\Delta T_8$

Table 2 is a summary of the fuzzy partition results in case of two variables, and Fig. 5 shows the membership function using the premise parameters in Table 2 for the optimal model with two variables. For this case, the optimal model is obtained when the variable ΔT_2 is divided into three subspaces. The final fuzzy model in case of two variables is shown in Eq. (8).

If ΔT_2 is small,
then $\delta=-12.92+4.82*\Delta T_2-23.59*\Delta T_7$

Table 1 Summary of fuzzy partition results for CASE I

Step	Fuzzy partition ΔT_2	Fuzzy partition ΔT_7	Fuzzy partition ΔT_8	Performance index E	Premise parameters [a b c] in Eq.(6)
1	2	1	1	90.536	[3.295 2 1.39] [3.295 2 7.98]
	1	2	1	50.865	[10.48 2 3.82] [10.48 2 24.77]
	1	1	2	81.147	[6.005 2 4.21] [6.005 2 16.22]
	1	3	1	96.124	[5.237 2 3.82] [5.237 2 14.3] [5.237 2 24.77]
	1	4	1	265.390	[3.492 2 3.82] [3.492 2 10.8] [3.492 2 17.79] [3.492 2 24.77]
2	2	2	1	72.259	[3.295 2 1.39] [3.295 2 7.98] [10.48 2 3.82] [10.48 2 24.77]
	1	2	2	92.040	[10.48 2 3.82] [10.48 2 24.77] [6.005 2 4.21] [6.005 2 16.22]

Table 2 Summary of fuzzy partition results for CASE Ⅱ

Step	Fuzzy partition ΔT_2	Fuzzy partition ΔT_7	Performance index E	Premise parameters [a b c] in Eq.(6)
1	2	1	56.974	[3.295 2 1.39] [3.295 2 7.98]
	1	2	73.840	[10.48 2 3.82] [10.48 2 24.77]
	3	1	55.681	[1.648 2 1.39] [1.648 2 4.685] [1.648 2 7.98]
	4	1	101.635	[1.098 2 1.39] [1.098 2 3.587] [1.098 2 5.783] [1.098 2 7.98]
2	3	2	130.325	[1.594 2.004 1.344] [1.617 2.021 4.666] [1.674 1.984 7.961] [10.47 2.014 3.821] [10.48 1.978 24.77]

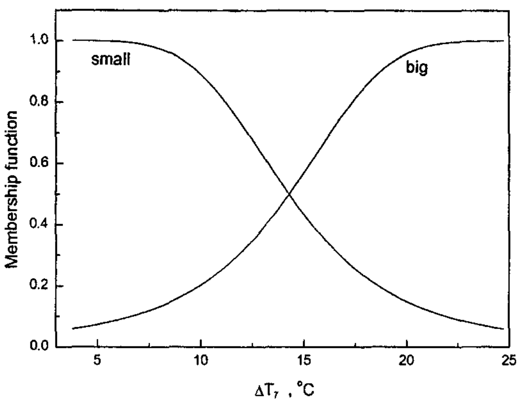

Fig. 4 Membership function for the variable ΔT_7

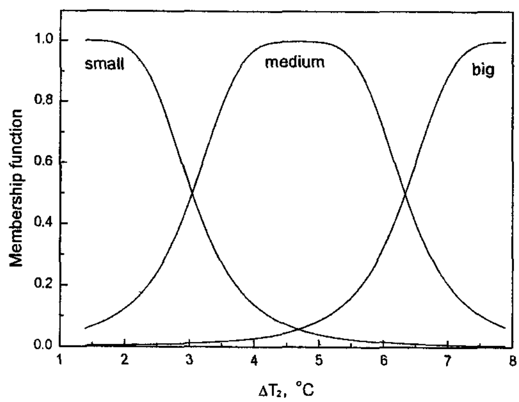

Fig. 5 Membership function for the variable ΔT_2

If ΔT_2 is medium,

then $\delta = -24.03 + 9.00 * \Delta T_2 + 10.30 * \Delta T_7$ (8)

If ΔT_2 is big,

then $\delta = 13.32 - 2.23 * \Delta T_2 + 17.24 * \Delta T_7$

4.4 Analysis of model performance

To evaluate the developed thermal error model, new experimental data were obtained using the same manner as described in the previous section. Fig. 6 and Fig. 7 represent a comparison between the measured thermal error data, the values predicted by the developed fuzzy model and two comparative models: an engineering judgement model (Yang et al., 1996) and a linear regression model (Hwang et al., 1999). Table 3 lists the prediction performance of the models used for comparison.

Figures 6 and 7 and Table 3 show that the prediction performance of the fuzzy models is very good even though the number of variables used in the fuzzy models is less than that of two other models. In practical perspective, two

Table 3 Prediction performance of the models used for comparison

Model / Term	Fuzzy model		Engineering judgement model	Linear regression model
	CASE I	CASE II		
Variables	ΔT_2, ΔT_7, ΔT_8	ΔT_2, ΔT_7	ΔT_6, ΔT_7, ΔT_{13}, ΔT_6^2, ΔT_7^2, ΔT_{13}^2	ΔT_2, ΔT_6, ΔT_7, ΔT_8
R^2	0.9673	0.9704	0.9612	0.9672
Standard error (μm)	5.77	5.49	6.27	5.78

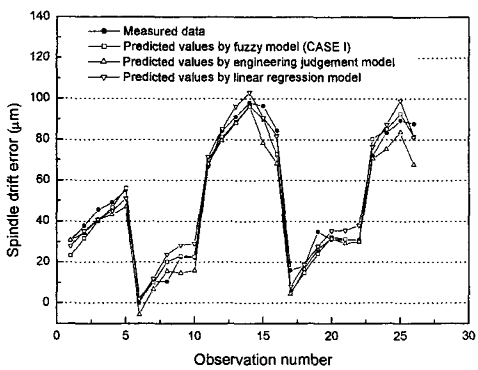

Fig. 6 Comparison between measured thermal error, values predicted by developed fuzzy model (CASE I) and two comparative models

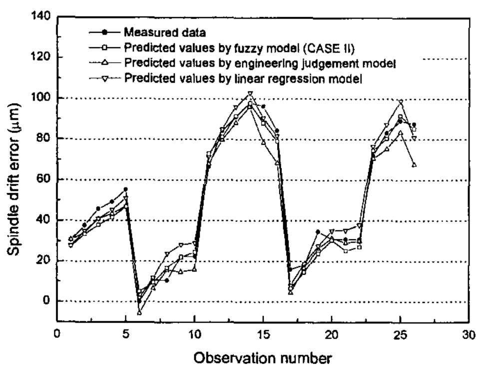

Fig. 7 Comparison between measured thermal error, values predicted by developed fuzzy model (CASE II) and two comparative models

variables may be the minimum number for the error model. However, if the prediction performance of the fuzzy model with two variables is poor than that of the fuzzy model with three variables, it can not be said that two variables is the minimum number. Yet, the above results show that the minimum number of variables is two.

In this study, a thermal error model with two variables as the minimum number could be obtained using a fuzzy logic strategy. The reduction of the number of variables achieved by fuzzy partition and the developed fuzzy model could guarantee the prediction performance. The economical efficiency and the reduction effect of sensor uncertainty such as a unexpected sensor fault can be regarded as the typical advantages in developing or using a thermal error compensation system. The thermal error modeling method proposed in this study can be widely applied to any type of CNC machine tool if a combination of the possible input variables is determined because the parameters of the error model are only calculated mathematically based on the number of temperature variables.

5. Conclusions

A thermal error model for a CNC machine tool with minimum number of variables was developed using a fuzzy logic strategy. Based on this study, the following conclusions were made :

(1) A thermal error model with two variables as the minimum number could be obtained using a fuzzy logic strategy. The reduction of the number of variables could be complemented by fuzzy partition. The developed fuzzy model showed very good prediction performance even though the number of variables in the fuzzy model was less than that of other comparative models, such as an engineering judgement model and a linear regression model.

(2) The thermal error model with fewer variables can produce the reduction of temperature sensors, therefore, the economical efficiency and the reduction effect of sensor uncertainty, such as a unexpected sensor fault can be expected in developing or using a thermal error compensation system.

(3) The thermal error modeling method proposed in this study can be applied to any type of CNC machine tool if a combination of the possible input variables is determined because the error model parameters are only calculated

mathematically based on the number of temperature variables.

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References

Bryan, J. B. 1990, "International Status of Thermal Error Research," *Annals of the CIRP*, Vol. 39, No. 2, pp. 645~656.

Fan, K. C., Lin, J. F. and Lu, S. S., 1992, "Measurement and Compensation of Thermal Error on a Machining Center," *29th MATADOR Conf., England*, April, pp. 261~268.

Hatamura, Y., 1993, "Development of an Intelligent Machining Center Incorporating Active Compensation for Thermal Distortion," *Annals of the CIRP*, Vol. 42, No. 1, pp. 549~552.

Horikawa, S., Furuhashi, T. and Uchikawa, Y., 1992, "On Fuzzy Modeling Using Fuzzy Neural Networks with the Back-Propagation Algorithm," *IEEE Trans. on Neural Networks*, Vol. 3, No. 5, pp. 801~806.

Hwang, S. H., Lee, J. H. and Yang, S., 1999, "Optimal Variable Selection in a Thermal Error Model for Real Time Error Compensation," *Journal of the Korean Society of Precision Engineering*, Vol. 16, No. 3, pp. 215~221.(in Korea)

Ivakhnenko, A. G., 1971, "Polynomial Theory of Complex Systems," *IEEE Trans. on Systems, Man, and Cybernetics*, Vol. SMC-1, No. 4, pp. 364~378.

Lee, Jae-Ha, Lee, Jin-Hyeon and Yang, Seung-Han, 2000, "Thermal Error Modeling of a Horizontal Machining Center Using the Fuzzy Logic Strategy," *Transactions of the Korean Society of Mechanical Engineers*, Vol. 24, No. 10, pp. 2589~2596.(in Korea)

Lee Jin-Hyeon and Yang, Seung-Han, 2001, "Statistical Optimization and Assessment of a Thermal Error Model for CNC Machine Tools," *Int. J. Mach. Tools & Manufact.*, Vol. 42, No. 1, pp. 147~155, 2001.

Moriwaki, T., 1988, "Thermal Deformation and its On-line Compensation of Hydrostatically Supported Precision Spindle," *Annals of the CIRP*, Vol. 37, No. 1, pp. 283~286.

Soons, J. A., Spaan, H. A. and Schellekens, P. H., 1994, "Thermal Error Models for Software Compensation of Machine Tools," *Proc. 9th Ann. Meet. American Society for Precision Engineering*, October, pp. 69~75.

Takagi, T. and Sugeno, M., 1983, "Derivation of Fuzzy Control Rules from Human Operator's Control Actions," *Proc. of the IFAC Symposium on Fuzzy Information, Knowledge Representation, and Decision Analysis*, Marseille, France, July, pp. 50~60.

Takagi, T. and Sugeno, M., 1985, "Fuzzy Identification of Systems and Its Applications to Modeling and Control," *IEEE Trans. on Systems, Man, and Cybernetics*, Vol. SMC-15, No. 1, pp. 116~132.

Tanaka, H., Uejima, S. and Asai, K., 1982, "A Linear Regression Analysis with Fuzzy Functions," *Journal of the Operations Research Society of Japan*, Vol. 25, No. 2, pp. 162~174.

Venugopal, R. and Barash, M., 1986, "Thermal Effect on the Accuracy of Numerically Controlled Machine Tool," *Annals of the CIRP*, Vol. 35, No. 1, pp. 255~285.

Weck, M. and Zangs, L., 1975, "Computing the Thermal Behavior of Machine Tools Using the Finite Element Method-Possibilities and Limitations," *Proceedings of the 16th MTDR Conference*, Vol. 16, pp. 185~194.

Yang, S., Yuan, J. Ni, J., 1996, "Accuracy Enhancement of a Horizontal Machining Center by Real-Time Error Compensation," *Journals of Manufacturing Systems*, Vol. 15, No. 2, pp. 113~118.

Yang, S., Yuan, J. and Ni, J., 1996, "The Improvement of Thermal Error Modeling and Compensation on Machine Tools by CMAC Neural Network," *Int. J. Mach. Tools & Manufact.*, Vol. 36, No. 4, pp. 527~537.

Zadeh, L. A., 1965, "Fuzzy Sets," *Information and Control*, Vol. 8, pp. 338~353.